Applicability of Insensitivity Properties of MEI Coefficients : Scalar-field Approach

N.M.Alam CHOWDHURY[1], Jun-ichi TAKADA[1] and Masanobu HIROSE[2]

[1]Tokyo Institute of Technology, Graduate School of Science & Engineering, O-okayama, Meguro-ku, Tokyo 152-8552, Japan, {nmalam@mobile.ss, takada@ide}.titech.ac.jp
[2]EM Waves Division, NMIJ, AIST, Ibaraki 305-8568, Japan, mahirose@etl.go.jp

1 Introduction

The Measured Equation of Invariance (MEI) concept was proposed by Mei *et al.* [1] for the electromagnetic (EM) wave scattering with effective truncation of the mesh boundary near the object surface for the Finite Difference (FD) method by preserving the matrix sparsity which can save computational time and memory requirements. By considering these advantages, this concept was later applied by Rius *et al.* [2] directly on the object surface with the surface integral equation as Integral Equation formulation of MEI (IE-MEI) method to 2D EM scattering problem. Recently, the IE-MEI concept was also applied to 3D scalar-field scattering problem by Chowdhury *et al.* [3] and get the same advantages of sparse matrices.

In this paper, we focus on the insensitivity properties of MEI coefficients with respect to the small modification of the scatterer shape. In the proposal, during scattering computation by using MEI technique, the MEI coefficients are derived for the whole scatterer and stored. After that, if any portion of the scatterer is changed or modified due to any reason, then recalculate the MEI coefficients only around the modified area and for the other portion of the scatterer the previous coefficients are used from the stored data. Due to insensitivity properties of the MEI coefficients this new set of data give the same scattering results, as of results obtained by considering the whole modified structure.

Here, we describe this property only with the Scalar-field approach of IE-MEI (SIE-MEI) method for the 3D problem, which can also be implemented to IE-MEI method without any modification.

2 Scalar-field approach of IE-MEI method

In Scalar-field approach of IE-MEI method, derive the *Scalar Reciprocity Relation* from Green's theorem by using 3D scalar Helmholtz equation as

$$\int_V \left(\phi_2(\boldsymbol{r})g_1(\boldsymbol{r}) - \phi_1(\boldsymbol{r})g_2(\boldsymbol{r})\right) dV = \oint_{\partial V} \left(\phi_1(\boldsymbol{r})\frac{\partial \phi_2(\boldsymbol{r})}{\partial n} - \phi_2(\boldsymbol{r})\frac{\partial \phi_1(\boldsymbol{r})}{\partial n}\right) dS, \tag{1}$$

where ϕ_1, ϕ_2 and $\frac{\partial \phi_1}{\partial n}$, $\frac{\partial \phi_2}{\partial n}$ are the scalar fields and their normal derivatives, respectively, due to the source distributions g_1 and g_2, respectively.

Let us consider a closed region V^+ very near to the scatterer which is bounded by the surfaces ∂V^+ and ∂V^+_∞ and assume that the region V^+ includes only the source g_2 which is represented by the equivalent monopole source ρ_2 and the dipole moment $\boldsymbol{\mu}_2$ as shown in Fig.1(a). According to the Ref. [3], this leads to the *Scalar-field Integral Equation*

$$\oint_{\partial V} \left(\phi_1(\boldsymbol{r})\tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n}\tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}}\right) dS = 0, \tag{2}$$

where ($\tilde{\cdot}$) terms represent the equivalent sources near the scatterer.

SIE-MEI postulates [3] are then applied to Eq. (2). By discretizing and solving this integral equation repeatedly for the whole scatterer surface with suitable set of equivalent sources called *metrons*, two sparse matrices $\mathbf{A}$, $\mathbf{B}$ of MEI coefficients are obtained (Eq. (3a)). Therefore, by using Dirichlet boundary condition, the Eq. (3a) can be represented as the equation of equivalent surface source $\frac{\partial \phi}{\partial n}$ on the scatterer (Eq. (3b)).

$$\mathbf{A}\,[\boldsymbol{\phi}_1] - \mathbf{B}\left[\frac{\partial \boldsymbol{\phi}_1}{\partial n}\right] = 0, \quad \text{(a)} \qquad \left[\frac{\partial \boldsymbol{\phi}}{\partial n}\right] = \left[\frac{\partial \boldsymbol{\phi}^{\text{inc}}}{\partial n}\right] - \mathbf{B}^{-1}\mathbf{A}\left[\boldsymbol{\phi}^{\text{inc}}\right], \quad \text{(b)} \tag{3}$$

where ϕ_1, $\frac{\partial \phi_1}{\partial n}$ are the scattered field and its normal derivative, respectively, due to the metrons for some certain incident fields and ϕ^{inc}, $\frac{\partial \phi^{\text{inc}}}{\partial n}$ are the incident field and its normal derivative, respectively.

3 Insensitivity Properties of MEI Coefficients

By now, it is assumed that the MEI coefficients are dependent on the whole scatterer geometry. But in fact, they do not depend so much on the whole scatterer geometry. The MEI coefficients of each node are derived from the metron fields of that node for the possible sets of metrons, with the interaction of metron fields of the neighboring nodes associated in the local region. Thus, they have the locality properties in nature. That is, they depend mainly on the local geometry of the scatterer and do not give significant effect on the other portion of the scatterer. This local geometrical dependency, i.e. insensitivity properties of MEI coefficients, can be applied for the scattering computation of modified structure to save the computational time.

4 Numerical Implementation

Let us consider a cube on which a plane wave ϕ^{inc} is incident to $+y$ direction as shown in Fig.1(b) and the scattered field ϕ^{sc} generated by the metrons are

$$\phi^{\text{inc}}(\boldsymbol{r}) = e^{-jky}, \quad \text{(a)} \quad \text{and} \quad \phi^{\text{sc}}(\boldsymbol{r}) = \oint_{\partial V} \rho(\boldsymbol{r}')\, G(\boldsymbol{r}, \boldsymbol{r}')\, dS', \quad \text{(b)} \tag{4}$$

where k is the wave propagation number, $\rho(\boldsymbol{r}')$ is a metron obtained from the spherical wave function given in Eq. (27) of Ref. [3], $G(\boldsymbol{r}, \boldsymbol{r}')$ is the free space 3D Green's function, and $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of observation point and source point, respectively.

Figures 1(c), (d), and (e) show the same cube with 0.1λ, 0.2λ, and 0.3λ modification at the corner of $x - z$ and $y - z$ plane. All of these cases, consider the same type of plane wave is incident (Eq. (4a)) from the same direction.

Figures 2(a), (b), and (c) show the 2D plot of equivalent surface source along the perimeter of section-A (Figs.1(b),(c),(d),(e)) for the side of $l = 1\lambda$. Each of the graphs contains the results of *full cube* by using conventional solution technique and modified cube by using *conventional* and *proposed* solution technique, in the SIE-MEI method. These results are also compared with the numerical solution using Combined-field Method of Moments (*CfMoM*), which is applied on the modified body with the conventional solution process. Results show a good agreement with the CfMoM solution. Also there is some error in the shadowed region, but it does not give any significant effect on the scattering computation.

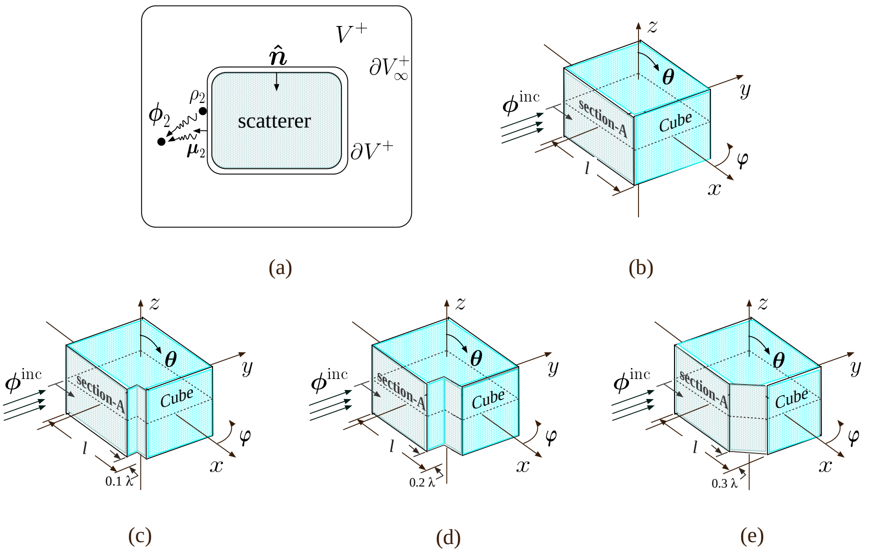

Figure 1: (a) Region V^+ very near to the scatterer. Plane wave incident on a (b) Cube, (c) 0.1λ Modified Cube, (d) 0.2λ Modified Cube, and (e) 0.3λ Modified Cube.

5 Savings in Computational Time

In SIE-MEI method, most of the computational time is spent in the integration process to derive the MEI coefficients. In the proposed solution technique, we compute the MEI coefficients only around the modified region for the rest of the part use the stored data. Thus the new solution technique can save a large amount of computational time which increases rapidly as the size of the scatterer increases with the same modification.

Table 1: Time Comparison for the computation of MEI coefficients by using SIE-MEI method

Scatterer	Cube Side (λ)	Time (sec)	No. of MEI Coefficients
Full Cube	1	202	6,000
	2	11,280	24,000
Modified Cube	1	179	5,640
	2	10,743	23,340
Modified Area	1	32	940
	2	754	1,640

Table 1 shows the time required for the computation of MEI coefficients of full cube, modified cube, and around the modified area by using SIE-MEI method. In the comparison we use cube of 1λ side length with 10 segment per wavelength. From the comparison, it is clear that, by using proposed solution technique we can easily save the computational time for the computation of MEI coefficients of modified structured body.

6 Conclusion

A new solution technique is proposed for the scattering from modified structured body based on the insensitivity properties of MEI coefficients. This technique is implemented on the scatterer when its ge-

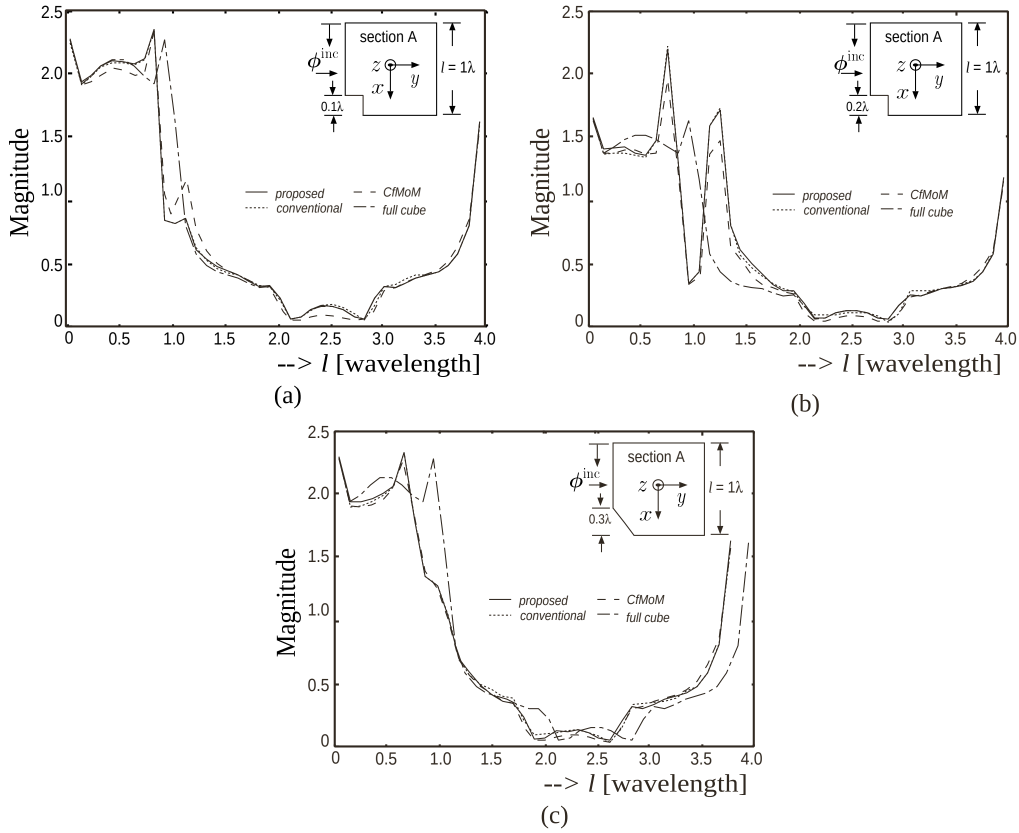

Figure 2: Equivalent surface source on the (a) 0.1λ, (b) 0.2λ, and (b) 0.3λ Modified Cube.

ometry is modified due to any reason and need to be recalculate the scattering performance. By avoiding the computing of MEI coefficients we can save large amount of CPU time with the same accuracy in the result. To verify our proposed technique the results also compared with the available numerical solution and they have a good agreement.

If the scatterer size is larger compared to its modified area, then the technique gives more reasonable result with more savings in CPU time. This technique can also be applicable to IE-MEI method for the EM scattering computation of the modified body. We are in progress to validate our comments to other 3D scalar-field problems as well as EM problems.

References

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