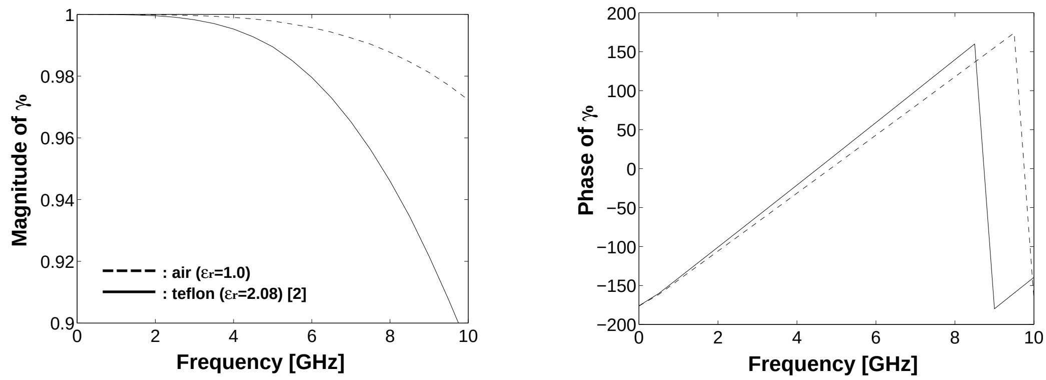
Scattering Analysis of a Flared Coaxial Line Radiating into a Dielectric Slab

Dong H. Shin and Hyo J. Eom
Department of Electrical Engineering
Korea Advanced Institute of Science and Technology
373-1, Kusong Dong, Yusong Gu, Taejon, Korea
E-mail : hjeom@ee.kaist.ac.kr

1 Introduction

An open-ended coaxial line has been commonly utilized to measure the dielectric constant of a slab medium at microwave frequencies due to its easy application. An open-ended coaxial line with a small aperture is, however, very ineffective to distinguish the low permittivity materials at low frequencies. This is because the reflection coefficient magnitude is almost unity and its



(a) (b)

Figure 1: Comparison of the reflection coefficients for the several permittivity materials. (inner conductor radius : 1.4264 mm, outer conductor radius : 4.7250mm [2], and $\epsilon = 2.08$)

phase remains unchanged for two different materials as shown in Fig. 1. A large aperture size of an open-ended coaxial line is thus necessary for the accurate estimation for the reflection coefficients of low permittivity materials at low frequencies [1]. To achieve accurate permittivity estimation, we will introduce a flared coaxial line that is composed of a conical waveguide with a large aperture size. A conical waveguide is connected to the terminal of a coaxial line to increase an aperture size and reduce the reflection. The geometrical structure is shown in Fig. 2. In order to calculate the reflection coefficient, the rigorous scattering analysis based on the boundary condition is necessary. In sections 2 and 3, we shall solve the boundary-value problem of radiation from a flared coaxial line. A flared coaxial line is described into multiply-stepped coaxial line for scattering analysis. The Hankel transform and mode matching is used to obtain the modal coefficients for multiply-stepped coaxial lines. In section 4, computations are performed to show the behavior of reflection coefficients.

2 Field Representations

We shall consider a flared coaxial line that radiates into a dielectric slab backed by air as shown in Fig. 2 (a). A flared section of a coaxial line is modeled in terms of the N-step coaxial lines with constant radii a and r. N-step model is illustrated in Fig. 2 (b). An incident TEM mode exicites the coaxial line. In region (X) $(a < r < b, z < -t_1)$, the incident and reflected H fields are

$$H_\phi^i(r,z) = A_0 \frac{e^{i\beta_\xi(z+t_1)}}{r} \tag{1}$$

$$H_\phi^r(r,z) = \gamma_0 A_0 \frac{e^{-i\beta_\xi(z+t_1)}}{r} + \sum_{n=1}^{\infty} \gamma_n A_n F_n(r) e^{-ik_{zn}(z+t_1)} \tag{2}$$

where $\beta_\xi = \omega\sqrt{\mu\epsilon_\xi}$, $k_{zn} = \sqrt{{\beta_\xi}^2 - {\lambda_n}^2}$, $F_n(r) = J_1(\lambda_n r)N_0(\lambda_n b) - N_1(\lambda_n r)J_0(\lambda_n b)$, and $J_m(\cdot)$ and $N_m(\cdot)$ are the mth order Bessel and Neumann functions, respectively. Note that λ_n is the eigenvalue satisfying the characteristic equation, $J_0(\lambda_n a)N_0(\lambda_n b) = N_0(\lambda_n a)J_0(\lambda_n b)$. The normalization factors of TM_{0n} modes of the coaxial line are $A_0 = 1/\sqrt{\ln(b/a)}$, $A_n = \pi\lambda_n / \sqrt{2 - 2\frac{J_0^2(\lambda_n b)}{J_0^2(\lambda_n a)}}$.

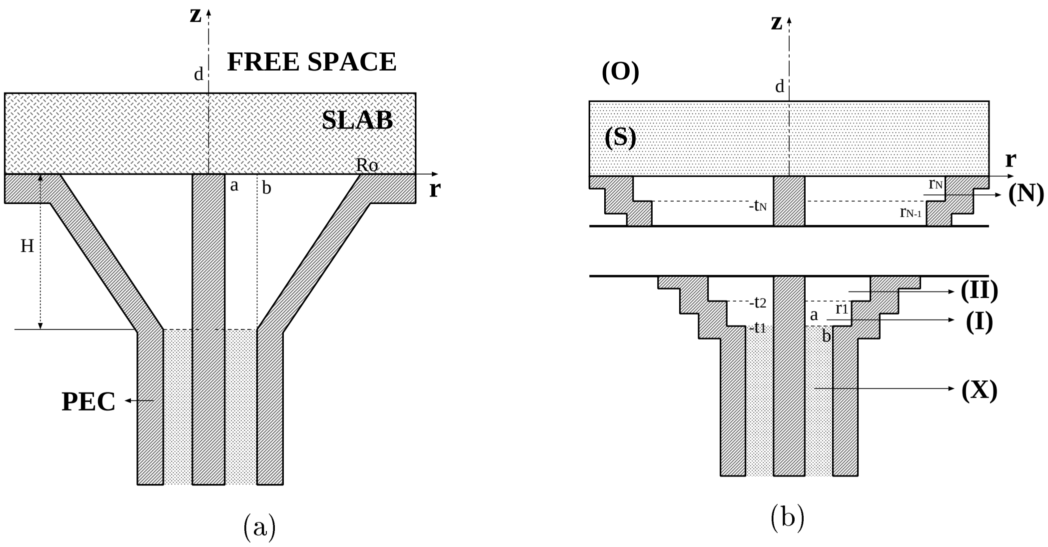


Figure 2: (a) Longitudinal section of a flared coaxial line with a flange (b) Geometry of N-step model for a flared coaxial line.

In region (p) $(a < r < r_p, -t_p < z < -t_{p+1}, 1 \le p \le N)$, H field consists of the TEM mode and higher $\mathrm{TM}_{0n}^{(p)}$ modes as

$$\begin{aligned} H_\phi^{(p)}(r,z) &= d_0^{(p)} B_0^{(p)} \frac{e^{i\beta_p(z+t_p)}}{r} + \sum_{n=1}^{\infty} d_n^{(p)} B_n^{(p)} G_n^{(p)}(r) e^{ik_n^{(p)}(z+t_p)} \\ &+ e_0^{(p)} B_0^{(p)} \frac{e^{-i\beta_p(z+t_p)}}{r} + \sum_{n=1}^{\infty} e_n^{(p)} B_n^{(p)} G_n^{(p)}(r) e^{-ik_n^{(p)}(z+t_p)} \end{aligned} \tag{3}$$

where $G_n^{(p)}(r) = J_1(\lambda_n^{(p)} r)N_0(\lambda_n^{(p)} r_p) - N_1(\lambda_n^{(p)} r)J_0(\lambda_n^{(p)} r_p)$, $\beta_p = \omega\sqrt{\mu\epsilon_p}$, and $k_n^{(p)} =$

$\sqrt{\beta_p{}^2 - \lambda_n^{(p)2}}$. Similarly, $\lambda_n^{(p)}$ is the eigenvalue of the coaxial line with the equation, $J_0(\lambda_n^{(p)}a)N_0(\lambda_n^{(p)}r_p) = N_0(\lambda_n^{(p)}a)J_0(\lambda_n^{(p)}r_p)$.

The normalization factors are given as $B_0^{(p)} = 1/\sqrt{\ln(r_p/a)}$, $B_n^{(p)} = \pi\lambda_n^{(p)}/\sqrt{2 - 2\frac{J_0^2(\lambda_n^{(p)}r_p)}{J_0^2(\lambda_n^{(p)}a)}}$.

In region (S) ($0 < z < d$) and region (O) ($z > d$), the fields are represented in terms of the continuous mode in the spectral domain as

$$H_\phi^s(r,z) = \int_0^\infty \left[\tilde{H}_s^+(\zeta)e^{i\kappa_s z} + \tilde{H}_s^-(\zeta)e^{-i\kappa_s z}\right]\zeta J_1(\zeta r)d\zeta \tag{4}$$

$$H_\phi^o(r,z) = \int_0^\infty \tilde{H}_o(\zeta)e^{i\kappa_o(z-d)}\zeta J_1(\zeta r)d\zeta \tag{5}$$

where $\kappa_s = \sqrt{\beta_s^2 - \zeta^2}$, $\beta_s = \omega\sqrt{\mu\epsilon_s}$, $\kappa_o = \sqrt{\beta_o^2 - \zeta^2}$ and $\beta_o = \omega\sqrt{\mu\epsilon_o}$.

3 Enforcement of Boundary Conditions

To obtain the simultaneous equations for unknown coefficients, we enforce the boundary conditions on the field continuities at $z = -t_1$, $z = -t_p (p \geq 2)$, $z = d$, and $z = 0$, respectively. The boundary conditions to be enforced are

$$E_r^{(1)}(r,-t_1) = \begin{cases} E_r^i(r,-t_1) + E_r^r(r,-t_1), & a < r < b \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

$$H_\phi^{(1)}(r,-t_1) = H_\phi^i(r,-t_1) + H_\phi^r(r,-t_1), \quad a < r < b \tag{7}$$

$$E_r^{(p)}(r,-t_p) = \begin{cases} E_r^{(p-1)}(r,-t_p), & a < r < r_{p-1} \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

$$H_\phi^{(p)}(r,-t_p) = H_\phi^{(p-1)}(r,-t_p), \quad a < r < r_{p-1} \tag{9}$$

$$E_r^o(r,d) = E_r^s(r,d) \tag{10}$$

$$H_\phi^o(r,d) = H_\phi^s(r,d) \tag{11}$$

$$E_r^s(r,0) = \begin{cases} E_r^{(N)}(r,0), & a < r < r_N \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

$$H_\phi^s(r,0) = H_\phi^{(N)}(r,0), \quad a < r < r_N \tag{13}$$

Using the above boundary conditions, it is possible to constitute a system of simultaneous equations for the modal coefficients γ_n, $(d_n^{(1)}, e_n^{(1)})$, $(d_n^{(2)}, e_n^{(2)})$, $\cdots$, $(d_n^{(N)}, e_n^{(N)})$. The reflection coefficient for the TEM mode is given by γ_0.

4 Numerical Computations

The dimensions of the structure are $a = 0.815mm$, $b = 2.655mm$, $R_o = 30mm$, and $H = 20mm$. A flared region is modeled with 20 steps ($N = 20$) of coaxial line with different outer conductor radii. For computations, it is necessary to truncate the number of modes in the simultaneous equations. Three higher order modes ($n = 3$) in each step are included in computation to achieve numerical accuracy. The behavior of reflection coefficients for air half space and teflon

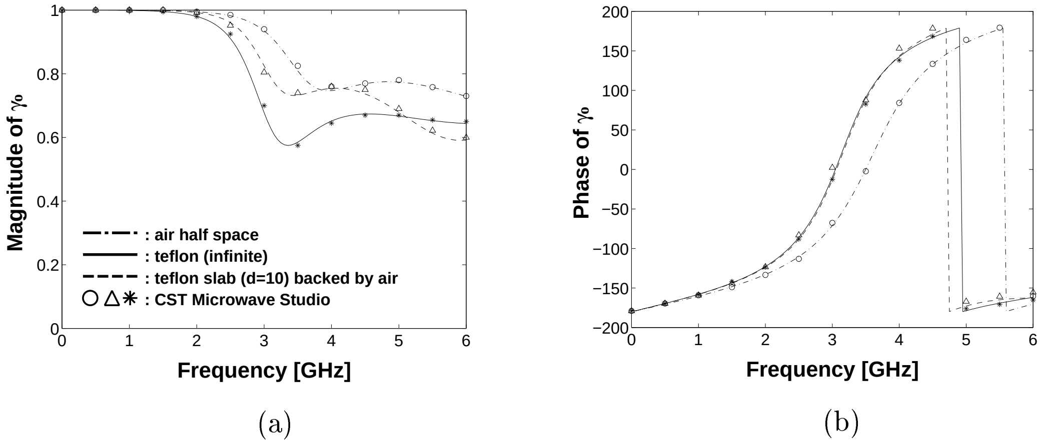


Figure 3: Reflection coefficient versus frequency for different permittivity materials. The computation uses a=0.815mm, b=2.655mm, R_o=30mm, and H=20mm.

slab is plotted in Fig. 3. In order to check the validity of our method, we compare our results with the data obtained from the MW STUDIO, FIM (finite integration method) simulator. The favorable agreement is observed. The curves illustrate that the difference in magnitude of the reflection coefficient between air half space and teflon slab (infinite) is large near 3 GHz. A larger sensitivity $\partial\gamma_0/\partial\epsilon_r$ is obtained, which indicates that the flared coaxial line is less sensitive to an error in estimating ϵ from the measured γ_0.

5 Conclusion

A problem of a flared coaxial line radiating into a dielectric slab with a flange is solved using the Hankel transform and mode matching technique. Numerical computations are performed to illustrate the reflection behavior. A sensitivity of the reflection coefficient to a change in permittivity increases when the flared coaxial line is used. A flared coaxial line is useful to accurately estimate the permittivity of a dielectric slab medium.

REFERENCES

[1] P. D. Langhe, L. Martens, and D. D. Zutter, "Design rules for an experimental setup using an open-ended coaxial probe based on theoretical modeling," *IEEE Trans. Instrum. Meas.*, vol. 43, no. 6, pp. 810-817, Dec. 1994.
[2] J. H. Lee, H. J. Eom, and K. H. Jun, "Reflection of a coaxial line radiating into a parallel plate," *IEEE Trans. Microwave and Guided Wave Letters*, vol. 6, no. 3, pp. 135-137, Mar. 1996.
[3] Y. C. Noh and H. J. Eom, "Radiation from a flanged coaxial line into a dielectric slab," *IEEE Trans. Microwave Theory and Techniques*, vol. 47, no. 11, pp. 2158-2161, Nov. 1999.