Efficient evaluation of the two-dimensional generalized exponential and Hankel integrals for the microstrip structures

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Abstract

An integration scheme for evaluating the MoM matrix elements in conjunction with closed-form Green's functions is considered for analyzing general microstrip structures. The use of these functions in an MoM with the MPIE formulation enables all the integrands appearing in the integral calculation procedure of all the matrix elements to be cast into the function type of a 2-D(two-dimensional) generalized exponential function and Hankel function. The 2-D integrations for these integrals are efficiently reduced to the 1-D integrations by use of an integration scheme for a polar coordinate.

I. Introduction

Recently, in order to accelerate the calculation of spatial domain Green's functions for multi-layered planar structures, various closed-form Green's function methods have been proposed [1-3] for application to the method of moments(MoM). As already well known in this area, the expressions of closed-form Green's functions generally have three terms that correspond to the source dipole itself of type e^{-jkr}/r, surface wave pole(SWP) contributions expressed as a Hankel function, and complex images expressed as the sum of complex exponentials.

When closed-form Green's functions are used in conjunction with rooftop-pulse subsectional basis functions and the razor testing function in an MoM with an MPIE formulation, the integrals appearing in the calculation procedure of the diagonal matrix elements are of two types. The first is 2-D generalized exponential integral for the contribution of both the source dipole itself and complex images, while the other is Hankel integral for the contribution of the surface wave pole. Adopting a polar coordinate for the integrals not only removes the singularities but also drastically reduces the evaluation time for the numerical integration. In addition, the above numerical efficiency is also retained for the off-diagonal elements.

II. Theory

Following the previous closed-form Green's functions method[1-3], vector and scalar potential Green's functions for general microstrip structure can be derived as follows :

$$\frac{4\pi}{\mu_0} G_A = 4\pi\varepsilon G_q = G_0 + G_{sw} + \sum_{k=1}^{N} G_{ck} \tag{1}$$

where $G_0 (= e^{-jkr_0}/r_0\ , r_0 = \sqrt{\rho^2 + (z-z')^2}\ , \rho = \sqrt{(x-x')^2 + (y-y')^2})$ represents the contribution from source dipole itself, $G_{sw} (= C_1 H_0^{(2)}(k_{\rho p}\rho))$ represents the contribution of surface wave pole($k_{\rho p}$), and $G_{ck} (= a_k e^{-jkr_k}/r_k,\quad r_k = \sqrt{\rho^2 + (z - z' - jb_k)^2})$ corresponds to the contribution of the k-th complex image. When closed-form Green's functions of Eq. (1) are used in conjunction with the rooftop(pulse) subsectional basis functions for x-directed current cells (charge cells with dimension $a \times b$), the integrals appearing in the calculation procedure of the diagonal matrix elements are of following type:

$$I_{q,ck} = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \frac{e^{-jk\sqrt{x'^2 + y'^2 + (jb_k)^2}}}{\sqrt{x'^2 + y'^2 + (jb_k)^2}}\, dx'\, dy' \tag{2}$$

$$I_{A,ck} = \int_{-b/2}^{b/2} \int_{-a}^{a} (1 - \frac{|x'|}{a}) \frac{e^{-jk\sqrt{x'^2 + y'^2 + (jb_k)^2}}}{\sqrt{x'^2 + y'^2 + (jb_k)^2}}\, dx'\, dy' \tag{3}$$

$$I_{q,sw} = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} H_0^{(2)}(k_{\rho p}\rho) dx'\, dy' \tag{4}$$

$$I_{A,sw} = \int_{-b/2}^{b/2} \int_{-a}^{a} (1 - \frac{|x'|}{a}) H_0^{(2)}(k_{\rho p}\rho) dx'\, dy' \tag{5}$$

If $b_k = 0$ in Eq. (2) and (3), the expressions are reduced to singular integral of G_0 type. The integrals also in Eq. (4) and (5) have logarithmic singularity of Hankel function. It is observed that the numerical integration results of these integrals in the rectangular coordinate system are very slowly convergent (refer to Fig. 1 and 2). So the integration corresponding to finite number of complex images as well as the burdensome integrations for the singular integral of type G_0 and Hankel function should be performed carefully.

As a solution to resolve these problematic aspects, an integration scheme for a polar coordinate is employed[4]. If the integrations are expressed using a polar coordinate, the final expressions for the above integrals take the following form:

$$I_{q,ck}=\frac{j}{k}\int_0^{2\pi}\left[e^{-jk\sqrt{\rho_c(\theta)^2+(jb_k)^2}}-e^{-jk\sqrt{(jb_k)^2}}\right]d\theta \tag{6}$$

$$I_{A,ck}=I_{q,ck}+\frac{j}{ka}\int_{-b/2}^{b/2}\left[e^{-jk\sqrt{y'^2+(jb_k)^2}}-e^{-jk\sqrt{a^2+y'^2+(jb_k)^2}}\right]dy' \tag{7}$$

$$I_{q,sw}=\frac{1}{k_{\rho p}}\int_0^{2\pi}H_1^{(2)}\{k_{\rho p}\rho_{c(\theta)}\}\rho_c(\theta)\,d\theta-j\frac{4}{k_{\rho p}^2} \tag{8}$$

where $\rho_c(\theta)$ is the radial distance from the origin of the coordinate system to the contour of the integration region, as defined in [4]. $I_{A,ck}$ of Eq. (7) is expression for the integration range of the falling half rooftop function, and $I_{A,sw}$ can be easily derived by use of the partial integration in Eq. (5). As a result, it is seen that aside from elimination singularity, the original two-dimensional integrals (Eqs. (2), (3), (4), and (5)) are reduced to the single integrals type of Eqs. (6), (7), and (8). It can be easily found that the above numerical efficiency is also retained for the off-diagonal elements.

III. Numerical Results

In order to examine the convergence of the integrals appearing in the calculation procedure of $I_{A,ck}$ for the case of h=0.8㎜, ε_r=4.34(epoxy), a=b=6.666㎜, Fig. 1 shows the integration results for $I_{A,ck}$ obtained by a Gaussian quadrature using a polar coordinate versus the magnitude of the complex exponent $|jb_k|$ as compared with those obtained using a rectangular coordinate. As shown in this figure, the integrations in the polar coordinate converged very rapidly with only a few Gauss-points(i. e., only 3 points for a single variable θ or y' were needed to achieve an accuracy of better than 0.1%) independent of the magnitude of b_k(even for $b_k=0$). Also, Fig. 2 shows the magnitude of the integral $I_{q,sw}$ versus the number of points for Gaussian integration. Once again, only 3 points in case of polar coordinate were needed to achieve results accurate to better than 0.1%.

IV. Conclusion

An integration scheme for calculating impedance matrix elements in conjunction with closed-form Green's functions was presented for analyzing the general microstrip structures. Adopting a polar coordinate for the integrals appearing in the calculation procedure of the all matrix elements not only removes the singularities but also drastically reduces the evaluation time for the numerical integration. Accordingly, the present method may help in analyzing general microstrip structures.

References

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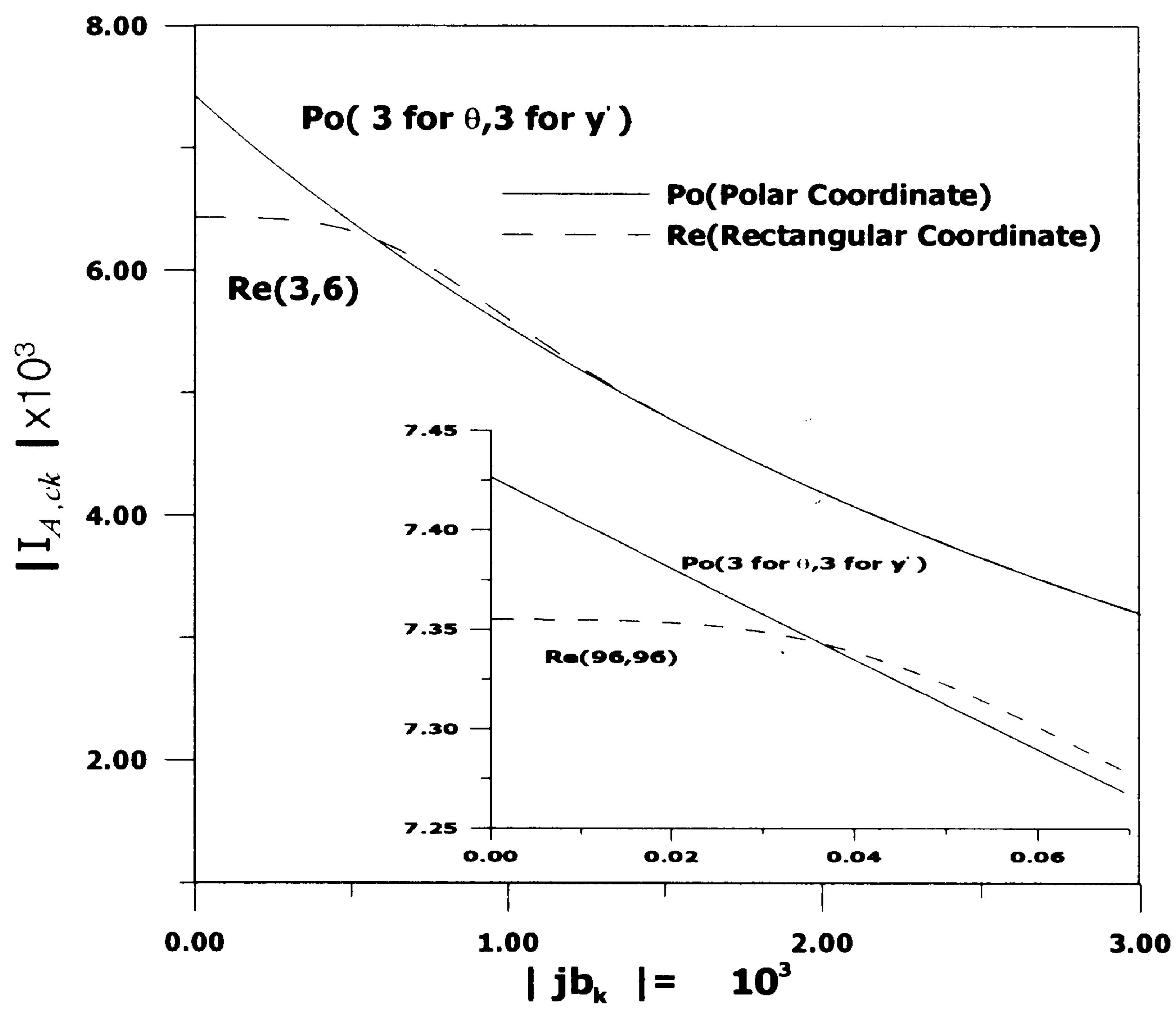

Fig. 1 $|I_{A,ck}|$ versus $|jb_k|$

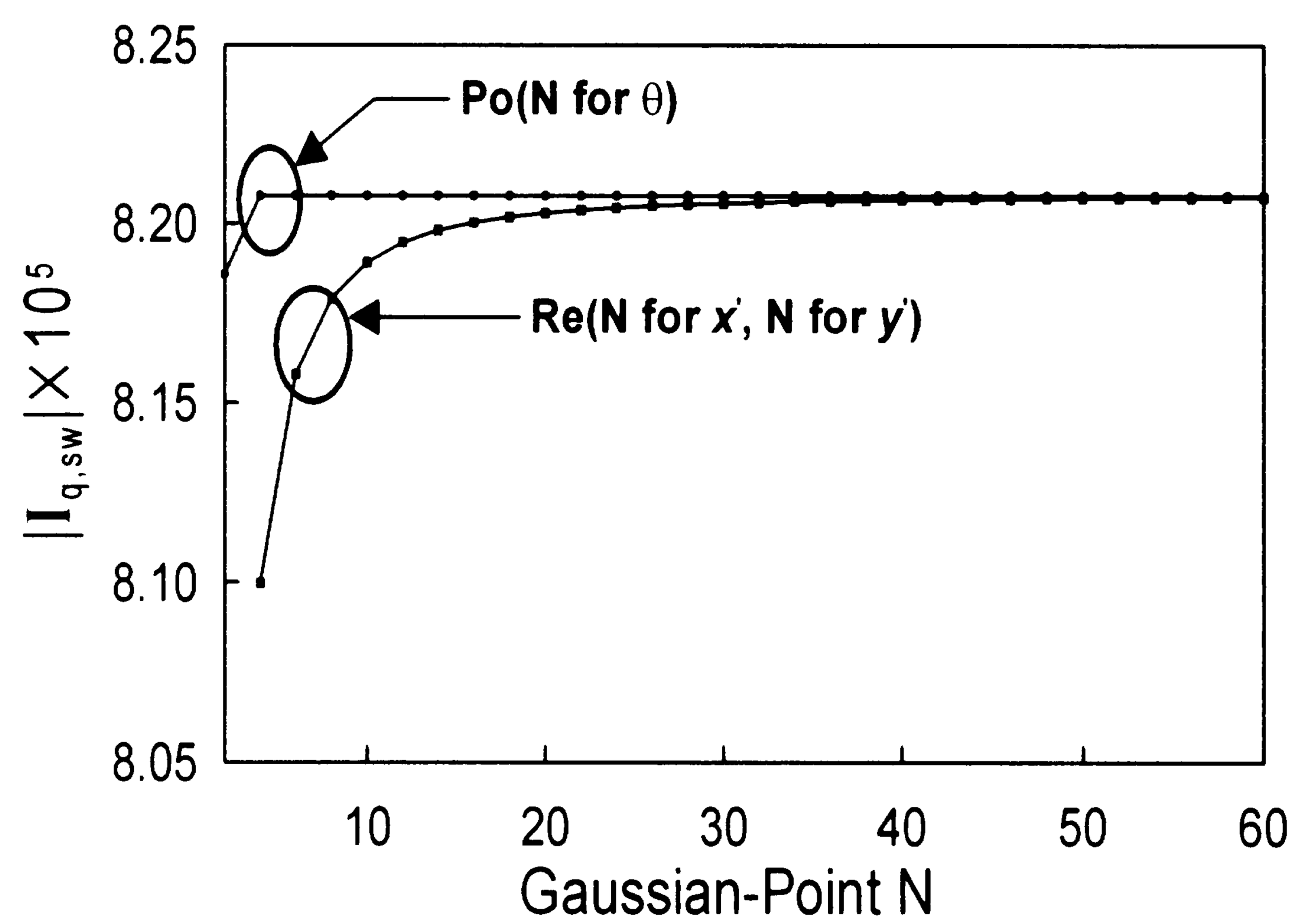

Fig. 2 $|I_{q,sw}|$ versus N