AN ENHANCED CAVITY MODEL FOR CHARACTERIZATION OF MICROSTRIP ANTENNAS

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An enhanced cavity model for analyzing microstrip patch antenna is presented. The predicted resonant frequency and resonant resistance of the antenna are in good agreement with measured data. Theoretical results of the enhanced model are also compared with some commonly used models to determine their range of validity.

I Introduction

A conventional microstrip antenna usually comprises of a metallic patch deposited on one side of the substrate and ground plane on the other side. Over the years, many models [1-6] have been used to analyze microstrip patch geometry. Among them, the cavity model with perfect magnetic conducting (PMC) walls has been useful in providing insight to the radiation mechanisms of microstrip patch but inaccurate prediction of its resonant frequent and resonant resistance. In this model, the thickness t of the microstrip patch antenna's substrate has been assumed to be electrically thin (usually on the order of $0.01\lambda_o$) and also a low dielectric constant has been used. These two assumptions made by the analytical models no longer hold as the microstrip patch antenna is increasingly used in the millimeter wave region. To achieve miniaturization and larger bandwidth, thicker and higher dielectric substrate is also commonly used. Hence, a better analytical model or technique is needed to characterize the microstrip patch antenna.

II Enhanced Model

The enhanced Cavity Model is based on Carver and Coffey's [2, 3] design equations formulated for microstrip patch resonator using the modal expansion technique. In this approach, the patch is viewed as a thin TM_z-mode cavity supporting quasi-discrete TM_{mn} modes transverse to z, where m and n are the mode numbers associated with the x- and y-directions, respectively. The field between the patch and the ground plane is expanded in terms of a series of eigenfunctions with their corresponding eigenvalues. For a non-radiating cavity, these eigenvalues are positive and real, defined as $k_x = n\pi/a$ and $k_y = m\pi/b$. For the cavity to radiate, the interior fields have to be related to the exterior fields. This is achieved by imposing impedance boundary conditions at the four walls, making use of fictitious complex wall admittances Y_w to represent the external stored and radiated energy effects. Consider a microstrip patch antenna (shown in Fig. 1) of resonant length b ($\approx \lambda_d/2$) along the y-direction and width a ($\approx 2b$), the transcendental equation is obtained [2]. Carver [2] suggested solving the transcendental equation by an iterative method. However, a result with better accuracy could be obtained by solving numerically the transcendental equation for its eigenvalues k_y. This is the main contribution of the enhanced model.

For the eigenvalue of a particular TM_{mn} mode to be found accurately, an appropriate value has to be chosen for k_o. A suitable value of k_o is the real and positive eigenvalue for the case of a non-radiating cavity since the complex eigenvalue is expected to have a magnitude of 1%-5% lower than that in the non-radiating case. Subsequently, the iteration will stop when $|f(k_{n+1})| \leq \epsilon$, where the error tolerance ϵ is set to be 10^{-5}. The resonant frequency,f_r, of a rectangular microstrip patch can be found from the complex resonant radian frequency. In a similar manner, the transcendental equation of a circular patch [3] can be solved numerically to obtain a more accurate prediction of the resonant frequency and the resonant resistance.

III Test Antennas

Rectangular and circular patches were fabricated to test the validity of the enhanced model. The design parameters for both microstrip antennas are shown in Tables 1 and 2. Eight cases

Table 1: Design parameters for rectangular patch

Case	ϵ_r	$\tan\delta$	d (cm)	d/λ_o	b (cm)	a (cm)	F (cm)	W_f (cm)
1	10.2	0.0023	0.127	0.01	2.00	3.00	0.65	0.119
2	10.2	0.0023	0.127	0.02	0.95	1.50	0.32	0.119
3	10.2	0.0023	0.254	0.02	1.90	3.00	0.65	0.238
4	10.2	0.0023	0.254	0.04	0.90	1.50	0.32	0.238
5	2.22	0.0009	0.079	0.01	2.50	4.00	0.40	0.242
6	2.22	0.0009	0.079	0.02	1.25	2.00	0.20	0.242
7	2.22	0.0009	0.152	0.02	2.50	4.00	0.40	0.466
8	2.22	0.0009	0.152	0.04	1.20	2.00	0.20	0.466

Table 2: Design parameters for circular patch

Case	ϵ_r	$\tan\delta$	d (cm)	d/λ_o	radius (cm)	λ_o(cm)
1	10.2	0.0023	0.127	0.02	0.5308	0.113
2	10.2	0.0023	0.254	0.04	0.9920	0.216
3	2.33	0.0013	0.079	0.01	1.4845	0.415
4	2.20	0.0009	0.079	0.02	0.7502	0.2268
5	2.33	0.0013	0.1575	0.02	1.4528	0.432
6	2.20	0.0009	0.1575	0.04	0.6868	0.2285

with different electrical thickness and dielectric constants are examined. For a microstrip patch on low dielectric substrate, Rogers RT/duroid 5880 (ϵ_r=2.200.002, $\tan\delta$=0.0009) and Taconic TLY-3 (ϵ_r=2.330.02, $\tan\delta$=0.0013) have been used. On the other hand, Rogers RT/duroid 6010 (ϵ_r=10.20.25, $\tan\delta$=0.0023) is used as the substrate for patch on high dielectric constant material. Three types of feeds are used in each case of the rectangular patch. They are, namely, microstripline feed at the radiating edge (case a), probe feed (case b), and microstripline feed at the non-radiating edge (case c). For the circular patch, only probe feed is considered. The fabricated patches are measured using the Anritsu Vector Network Analyzer (VNA) to obtain the resonant frequency and resonant resistance.

IV Numerical and Experimental Results

In Table 3, predictions of the resonant frequency and resonant resistance by the enhanced model were compared with measured results and three other established models taken from Pozar's paper [10]. Comparisons between the cavity models are first examined. The Carver's model [2] is generally able to predict the measured resonant frequency within 3% error. However, its prediction of the resonant resistance has been unreliable, especially for the case of electrically thick substrate (in cases 4 and 8). It gives better prediction for thin probe-feed type antenna. The cavity model of Illinois's group [4] has assumed the PMC walls for the cavity and incorporates an effective loss tangent to take into account the effect of a radiating cavity. This model has led to better prediction of the resonant frequency as compared to the Carver's model citeCarver, with about 2% error based on measured results. For low dielectric substrate (ϵ_r = 2.22), the model in [4] is able to provide sufficiently accurate prediction with an error of less than 1%. However, the Illinois' model [4] is unable to provide reliable prediction on the resonant resistance, and incur substantial error for cases with high dielectric constant. The enhanced model is generally capable of estimating the measured resonant frequency within 2% error. Its prediction of the resonant frequency is superior to the Carver's model [2] (except for case 3a) and comparable to the Illinois' model [4]. Its strength lies in its ability of giving a comparatively more accurate characterization of the resonant frequency for a patch on high dielectric constant (ϵ_r=10.2) to within 2% error; and in most cases, it leads to an improved prediction on the resonant resistance (except for cases 1a, 3a, 5a, 6b, 7b) among the cavity models.

Table 3: Comparison of calculated and measured results for rectangular patch

Case	Measured		Cavity Model[2]				Cavity Model(Enhanced)				Moment Method			
	Fr (GHz)	Rr	Fr (GHz)	% Error	Rr	% Error	Fr (GHz)	% Error	Rr	% Error	Fr (GHz)	% Error	Rr	% Error
1a	2.26	335	2.20	-2.7	245	-26.9	2.234	-1.15	239	-28.7	2.25	-0.4	350	4.5
1b	2.26	85	2.20	-2.7	67	-21.2	2.234	-1.15	72	-15.3	2.28	0.9	100	17.6
1c	2.26	56	2.20	-2.6	67	19.6	2.235	-1.11	56	0	2.29	1.3	50	10.7
2a	4.43	339	4.35	-1.8	200	-41.0	4.431	-0.02	224	-33.9	4.50	1.6	350	3.2
2b	4.49	53	4.35	-3.1	48	-9.4	4.431	-1.31	56	5.66	4.58	2.0	75	41.5
2c	4.52	40	4.35	-6.5	48	20.0	4.432	-1.95	38	-5.0	4.58	1.3	50	25
3a	2.18	363	2.18	0	200	-44.9	2.215	1.61	224	-38.3	2.33	6.9	420	15.7
3b	2.24	80	2.18	-2.7	45	-43.8	2.215	-1.12	53	-33.8	2.29	2.2	75	-6.3
3c	2.23	34	2.18	-2.2	45	-32.4	2.216	-0.63	34	0	2.29	2.7	40	17.6
4a	-	-	3.90	-	116	-	4.035	-	180	-	4.55	-	345	-
4b	-	-	3.90	-	22	-	4.035	-	35	-	4.50	-	50	-
4c	4.23	9	3.90	-7.8	22	144	4.033	-4.66	2	-78	4.50	6.4	36	300
5a	3.92	136	3.84	-2.0	107	-21.3	3.896	-0.61	108	-20.6	3.92	0	130	-4.4
5b	3.94	89	3.84	-2.5	82	-7.9	3.896	-1.12	83	-6.7	3.89	-1.3	101	13.5
5c	3.94	62	3.84	-2.5	82	32.3	3.897	-1.09	79	27.4	3.89	-1.3	55	-11.3
6a	7.56	152	7.42	-1.9	100	-34.2	7.532	-0.37	105	-30.9	7.60	0.5	160	5.3
6b	7.65	99	7.42	-3.0	77	-22.2	7.532	-1.54	81	-18.2	7.61	-0.5	130	31.3
6c	7.66	50	7.42	-3.1	77	-54.0	7.533	-1.66	71	42.0	7.61	-0.7	58	16.0
7a	3.82	119	3.71	-2.9	100	-16.0	3.776	-1.15	105	-11.8	3.80	-0.5	143	20.2
7b	3.84	87	3.71	-3.4	77	-11.5	3.776	-1.67	81	-6.9	3.81	-0.8	127	46.0
7c	3.82	44	3.71	-2.9	77	75.0	3.777	-0.50	72	63.6	3.81	-0.3	58	31.8
8a	7.72	69	7.12	-7.8	80	15.9	7.269	-5.84	81	17.4	7.75	0.4	145	110
8b	-	-	7.12	-	60	-	7.269	-	51	-	7.55	-	112	-
8c	-	-	7.12	-	60	-	7.268	-	44	-	7.55	-	55	-

Next, the enhanced model is compared to a more rigorous technique, in this case, the method of moments [5, 6]. In general, the moment method is able to predict the resonant frequency to within 2% from measured results. For low dielectric constant substrate, this full-wave method is capable of predicting the resonant frequency with an accuracy of less than 1% from the measured results. In comparison, for higher dielectric constant, the enhanced model gives a closer estimate of the resonant frequency with 2% error. The method of moments generally gives better and more consistent prediction of the resonant resistance, as compared to models in [2] and [4]. However, in cases 1b, 1c, 2b, 2c, 3c, 4c, 5b, 6b, 7a, 7b and 8a, the enhanced model is found to give even better prediction of the resonant resistance than the moment method. In cases 4 and 8 where most models in [2], [9] and [5, 6] fail, the enhanced model gives improved results and better insights on how the resonant resistance would change as the substrate thickness increases electrically, and as the feed location changes.

In Table 4, comparisons of measured and theoretical results for circular patches of various

Table 4: Comparison of measured and calculated values of circular patch

Case	Measured			Moment Method (Ensemble)				Cavity Model (Enhanced)			
	Fr (GHz)	Rr	$\|S_{11}\|$ (dB)	Fr	% Error	Rr	% Error	Fr (GHz)	% Error	Rr	% Error
1	4.60	41	-16	5.08	10.4	78	90.3	4.71	2.4	50	22
2	2.71	55	-25	2.68	-1.1	46	16.8	2.50	-7.7	50	-9.1
3	3.72	49	-37	3.80	2.2	55	11.3	3.81	2.4	50	2.0
4	7.38	47	-31	7.73	4.7	57	22.0	7.60	3.0	50	6.4
5	3.76	46	-24	3.86	2.5	51	11.1	3.81	1.3	50	8.9
6	7.75	49	-27	8.23	6.2	51	3.1	7.90	1.9	50	2.0

electrical thickness and dielectric constants are shown. The circular patches in all the six cases are first-cut design using the enhanced cavity model. The predicted results of the enhanced model are compared to the simulated results using Ansoft Ensemble 8 and the measured results. From the measured results, the enhanced cavity model has been observed to give closer estimate of the resonant resistance than the moment method results from Ensemble. Except in case 2, the resonant frequency generally predicts within 3% or less error. It is noted that when high dielectric constant and electrically thick substrate is used, as in case 2, the enhanced model for the circular patch has substantial error in the predicted resonant frequency. However, when a low dielectric constant and electrically thick substrate is used as in case 8, the enhanced model is able to predict the resonant frequency to within 2% based on the measured result, better than the moment method result which is about 6.2% in relative error.

V Conclusions

In this thesis, an enhanced Cavity Model based on modal expansion technique has been obtained. The improvement is due to two techniques adopted by the enhanced model. The first technique make use of numerical root finding method to solve for the complex eigenvalues by "forcing" the equation $f(k_{n+1})$ to be as close to zero as possible. This resulted in an accurate evaluation of the complex eigenvalues. The second technique is discovered in the process of implementing the model. This technique make use of the patch antenna's input impedance plot to determine the resonant frequency and resonant resistance. It is observed that this approach leads to a good prediction of the measured resonant frequency and resonant resistance, especially for feeding at the non-radiating edge where higher order modes are excited.

From the comparison with measured results and theoretical models, the enhanced model has led to an improvement of Carver's Cavity Model [2]. Its prediction of resonant frequency is within 2% of error which is comparable to the Illinois's group Cavity Model. However, the Illinois' group cavity model and Carver's cavity models have failed to give a good prediction of the patch resonant resistance, especially when the substrate permittivity used is high. When this problem arises, the enhanced model is found to give better results.

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