ON IMPROVING THE RECURSIVE UNITARY ESPRIT FOR ITERATIVE DOA ESTIMATION

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1 Introduction

Many array signal processing techniques have been proposed for application in mobile communication systems[1][2]. Particularly, much attention is now focused on adaptive and signal processing antenna arrays operating as a space-time equalizer (or optimal filter)[3].

As is well known, the mobile radio propagation is much complicated because of interference and multipath waves deteriorating seriously the quality of radio communications. In order to understand the mobile radio propagation structures and also consider the signal recovering techniques in the mobile environments, it is most effective to estimate the signal parameters (DOA and TOA etc.) of the individual incoming waves. Furthermore, utilizing the estimated DOA information in the adaptive array based on the criterion like DCMP[4], we can form easily optimum beam patterns of antenna arrays. Note that the optimum patterns can be used not only for the receiving arrays but also for the transmitting arrays in the mobile communication systems.

Recently, we proposed the recursive Unitary ESPRIT[5] which is the unitary-transformed version of the recursive Standard ESPRIT[6][7] to realize much higher computation efficiency. We are aiming at using this algorithm for the adaptive beamforming. However, the conventional recursive Unitary ESPRIT has to repeat twice the updating process for each of the real and imaginary parts of input snapshot data. It is time-consuming and hence it is supposed that the conventional algorithm still remains to be improved.

In this paper, therefore, we will modify the recursive Unitary ESPRIT to be more computationally efficient, and we will show some computer simulation results to demonstrate the effectiveness of the proposed algorithm through the DOA estimation.

2 Receiving System and Input Data Formulation

Figure 1 shows a K-element equispaced linear antenna array with the element spacing of d, and L waves are assumed to be incident on the array with the different DOAs: $\theta_1, \ldots, \theta_L$.

For the recursive ESPRIT, the data matrix at t time instant $\boldsymbol{X}(t)$ is defined by

$$\boldsymbol{X}(t) \triangleq \left[\; \alpha^{1/2}\boldsymbol{X}(t-1) \quad (1-\alpha)^{1/2}\boldsymbol{x}(t) \; \right] \qquad (t = 1, 2, \cdots) \tag{1}$$

where $\boldsymbol{x}(t) \in C^{K\times 1}$ is the array snapshot vector at t time instant and $\alpha (0 < \alpha < 1)$ is the forgetting factor[7].

In the case of Unitary ESPRIT, the array snapshot vector $\boldsymbol{x}(t)$ is transformed to the following vectors[5]:

$$\boldsymbol{y}_1(t) \triangleq \mathrm{Re}\left[\boldsymbol{Q}_K^H \boldsymbol{x}(t)\right] \in R^{K\times 1}, \qquad \boldsymbol{y}_2(t) \triangleq \mathrm{Im}\left[\boldsymbol{Q}_K^H \boldsymbol{x}(t)\right] \in R^{K\times 1} \tag{2}$$

where $\boldsymbol{Q}_K$ is the unitary matrix given by

$$\boldsymbol{Q}_K = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{I}_q & j\boldsymbol{I}_q \\ \boldsymbol{\Pi}_q & -j\boldsymbol{\Pi}_q \end{bmatrix} \qquad \text{if } K = 2q \text{ (even number)} \tag{3}$$

$$\boldsymbol{Q}_K = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{I}_q & \boldsymbol{0} & j\boldsymbol{I}_q \\ \boldsymbol{0}^T & \sqrt{2} & \boldsymbol{0}^T \\ \boldsymbol{\Pi}_q & \boldsymbol{0} & -j\boldsymbol{\Pi}_q \end{bmatrix} \qquad \text{if } K = 2q+1 \text{ (odd number)} \tag{4}$$

with the q-dimensional identity matrix $\boldsymbol{I}_q$ and

$$\boldsymbol{\Pi}_q = \begin{bmatrix} \boldsymbol{O} & & & 1 \\ & & 1 & \\ & \iddots & & \\ 1 & & & \boldsymbol{O} \end{bmatrix} \in R^{q\times q} \tag{5}$$

Also, the data matrix for the recursive Unitary ESPRIT is defined by

$$\boldsymbol{Y}(t) \triangleq \left[\ \alpha^{1/2}\boldsymbol{Y}(t-1) \quad (1-\alpha)^{1/2}\boldsymbol{y}_1(t) \quad (1-\alpha)^{1/2}\boldsymbol{y}_2(t) \ \right] \qquad (t = 1, 2, \cdots) \tag{6}$$

which has the following relationship in the covariance domain with $\boldsymbol{X}(t)$:

$$\boldsymbol{Y}(t)\boldsymbol{Y}^T(t) = \mathrm{Re}\left[\boldsymbol{Q}_K^H \boldsymbol{X}(t)\boldsymbol{X}^H(t)\boldsymbol{Q}_K\right] \tag{7}$$

3 Principle of Modified Recursive Unitary ESPRIT

3.1 BiSVD subspace tracking algorithm

Bi-Iteration SVD (BiSVD) subspace tracking method[6] is used to carry out iteratively SVD of the array data matrix (6) and obtain recursively the signal subspace eigenvector matrix $\boldsymbol{Q}_A \in R^{K\times L}$ which is utilized in the Unitary ESPRIT.

The conventional recursive Unitary ESPRIT requires two recursions of updating process to produce $\boldsymbol{Q}_A(t)$ from $\boldsymbol{Q}_A(t-1)$ because there are two snapshot vectors $\boldsymbol{y}_1(t)$ and $\boldsymbol{y}_2(t)$ at t time instant, which is shown in Fig.2[5]. For this reason, the conventional algorithm is slightly time-consuming.

On the other hand, the modified version proposed in this paper expresses the data matrix of (6) as follows:

$$\boldsymbol{Y}(t) = \left[\ \alpha^{1/2}\boldsymbol{Y}(t-1) \quad (1-\alpha)^{1/2}\boldsymbol{Y}_c(t) \ \right] \qquad (t = 1, 2, \cdots) \tag{8}$$

$$\boldsymbol{Y}_c(t) \triangleq [\boldsymbol{y}_1(t) \quad \boldsymbol{y}_2(t)] \ \in R^{K\times 2} \tag{9}$$

Using this data matrix expression in the BiSVD scheme, a direct time update for the signal subspace eigenvector matrix $\boldsymbol{Q}_A(t)$ can be established as follows[6]:

$$\boldsymbol{Q}_A(t) = \boldsymbol{Q}_A(t-1)\boldsymbol{\Theta}_A(t) + \overline{\boldsymbol{Y}}_\perp(t)\boldsymbol{F}^T(t) \tag{10}$$

$$\overline{\boldsymbol{Y}}_\perp(t) \ \triangleq \ \boldsymbol{Y}_\perp(t)\boldsymbol{V}(t)\boldsymbol{\Sigma}^{-1/2}(t) \tag{11}$$

$$\boldsymbol{Y}_\perp(t) \ \triangleq \ \boldsymbol{Y}_c(t) - \boldsymbol{Q}_A(t-1)\boldsymbol{U}(t) \tag{12}$$

$$\boldsymbol{U}(t) \ \triangleq \ \boldsymbol{Q}_A^T(t-1)\boldsymbol{Y}_c(t) \tag{13}$$

where $\boldsymbol{V}(t)$ and $\boldsymbol{\Sigma}(t)$ are the eigenvector matrix and diagonal eigenvalue matrix of $\boldsymbol{Y}_\perp^T(t)\boldsymbol{Y}_\perp(t)$, respectively. Since $\boldsymbol{Y}_\perp^T(t)\boldsymbol{Y}_\perp(t)$ is a symmetric matrix, $\boldsymbol{V}(t)$ is an orthogonal matrix. Therefore, $\overline{\boldsymbol{Y}}_\perp(t)$ satisfies $\overline{\boldsymbol{Y}}_\perp^T(t)\overline{\boldsymbol{Y}}_\perp(t) = \boldsymbol{I}_2$ in addition to $\overline{\boldsymbol{Y}}_\perp(t) \perp \boldsymbol{Q}_A(t-1)$. It is a key technique of the proposed algorithm. Also, $\boldsymbol{\Theta}_A(t)$ and $\boldsymbol{F}(t)$ in (10) are matrices extracted from a matrix $\boldsymbol{G}_A(t)$ as shown below:

$$\boldsymbol{G}_A^T(t) = \begin{array}{c} L\left\{ \vphantom{\boldsymbol{\Theta}_A} \right. \\ \\ \end{array} \underbrace{\begin{bmatrix} \overbrace{\boldsymbol{\Theta}_A(t)}^{L} & * \\ \hline \boldsymbol{F}^T(t) & * \end{bmatrix}}_{} \left.\vphantom{\begin{bmatrix} a \\ b \end{bmatrix}}\right\} L+2 \quad \in R^{(L+2)\times(L+2)} \tag{14}$$

where "$*$" stands for uninteresting quantities. $\boldsymbol{G}_A(t)$ is an orthogonal matrix that is obtained from the QR-decomposition expressed in (15).

$$\boldsymbol{G}_A^T(t)\begin{bmatrix} \boldsymbol{R}_A(t) \\ \boldsymbol{O} \end{bmatrix} = \begin{bmatrix} \alpha\boldsymbol{R}_A(t-1)\boldsymbol{H}_R(t) + (1-\alpha)\boldsymbol{U}(t)\boldsymbol{U}_R^T(t) \\ (1-\alpha)\boldsymbol{\Sigma}^{1/2}(t)\boldsymbol{V}^T(t)\boldsymbol{U}_R^T(t) \end{bmatrix} \tag{15}$$

$$\boldsymbol{H}_R(t) \ \triangleq \ \boldsymbol{H}(t)\boldsymbol{R}_B^{-1}(t) \tag{16}$$

$$\boldsymbol{H}(t) \ \triangleq \ \boldsymbol{R}_B(t-1)\boldsymbol{\Theta}_A(t-1) \tag{17}$$

$$\boldsymbol{U}_R^T(t) \ \triangleq \ \boldsymbol{U}^T(t)\boldsymbol{R}_B^{-1}(t) \tag{18}$$

In the above equations, $\boldsymbol{R}_A(t)$ and $\boldsymbol{R}_B(t)$ are $L \times L$ upper triangular matrices. $\boldsymbol{R}_A(t)$ is updated by (15) itself and $\boldsymbol{R}_B(t)$ is updated by the following QR-decomposition:

$$\boldsymbol{G}_B^T(t)\begin{bmatrix} \boldsymbol{R}_B(t) \\ \boldsymbol{O} \end{bmatrix} = \begin{bmatrix} \alpha^{1/2}\boldsymbol{R}_B(t-1)\boldsymbol{\Theta}_A(t-1) \\ (1-\alpha)^{1/2}\boldsymbol{U}^T(t) \end{bmatrix} \tag{19}$$

$$\left(\boldsymbol{G}_B(t) \in R^{(L+2)\times(L+2)} : \text{orthogonal matrix}\right)$$

The recursive flow of the proposed algorithm is shown in Fig.3.

In this paper, the initial values are given by

$$\boldsymbol{Q}_A(0) = \begin{bmatrix} p\boldsymbol{I}_L \\ \boldsymbol{O} \end{bmatrix} (p\text{: constant}),\ \boldsymbol{R}_B(0) = \boldsymbol{\Theta}_A(0) = \boldsymbol{I}_L\ ,\ \boldsymbol{R}_A(0) = \boldsymbol{O} \tag{20}$$

on the assumption that L is known or estimated by the other methods[5].

3.2 DOA estimation stage in the recursive Unitary ESPRIT

The recursive Unitary ESPRIT[5] employs the QRD-based LS Unitary ESPRIT procedure to extract the DOA estimates from $\boldsymbol{Q}_A(t)$. For this purpose, the same recursive technique as BiSVD is used which applies the QR-decomposition of coefficient matrix in solving the rotational invariance $\boldsymbol{K}_1\boldsymbol{Q}_A(t)\boldsymbol{\Psi}(t) = \boldsymbol{K}_2\boldsymbol{Q}_A(t)$ with respect to $\boldsymbol{\Psi}(t)$[8]. The matrices $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are represented by

$$\boldsymbol{K}_1 \triangleq \mathrm{Re}\left[\boldsymbol{Q}_{K-1}^H\boldsymbol{J}_2\boldsymbol{Q}_K\right], \qquad \boldsymbol{K}_2 \triangleq \mathrm{Im}\left[\boldsymbol{Q}_{K-1}^H\boldsymbol{J}_2\boldsymbol{Q}_K\right] \tag{21}$$

$$\boldsymbol{J}_2 \quad \triangleq \quad [\boldsymbol{0}\ \ \boldsymbol{I}_{K-1}] \ \in R^{(K-1)\times K} \tag{22}$$

$\boldsymbol{\Psi}(t)$ obtained in this way has the following eigenstructure[8]:

$$\boldsymbol{\Psi}(t) = \boldsymbol{T}^{-1}(t)\boldsymbol{\Omega}(t)\boldsymbol{T}(t) \tag{23}$$

where $\boldsymbol{T}^{-1}(t)$ is an eigenvector matrix and $\boldsymbol{\Omega}(t)$ is a diagonal eigenvalue matrix of $\boldsymbol{\Psi}(t)$. The eigenvalue matrix $\boldsymbol{\Omega}(t)$ includes the DOA information: $\theta_1(t), \ldots, \theta_L(t)$ in the following form[8]:

$$\boldsymbol{\Omega}(t) = \mathrm{diag}\left\{\tan\left(-\frac{\pi}{\lambda}d\sin\theta_1(t)\right), \ldots, \tan\left(-\frac{\pi}{\lambda}d\sin\theta_L(t)\right)\right\} \qquad (\lambda\text{: wavelength}) \tag{24}$$

4 Computer Simulation

In this section, computer simulation is carried out using a linear array of isotropic elements with the element spacing of a half wavelength ($d = \lambda/2$). There are two waves arriving at the array ($L = 2$), and they are uncorrelated with each other. The Gaussian internal noises (thermal noises) of equal power exist at all antenna elements, and they are statistically independent of incident waves. Detail of the radio environment is described in Table 1. In this simulation, the control parameter p which gives the initial value of $\boldsymbol{Q}_A(t)$ is 10^{-8}, the forgetting factor α is 0.9, and the input SNR which is defined for the first wave is 20dB.

Total RMSEs of estimates for the two waves are computed from 200 independent trials and used for sample performance statistics. Figure 4 shows the time variation of RMSEs. For comparison, the estimation results by the conventional recursive Unitary ESPRIT and TLS Unitary ESPRIT (non-recursive type) are plotted in the same figure. It is found from the figure that RMSE performance of TLS Unitary ESPRIT is the best. However, the difference among the three algorithms is much small and so we can say the proposed algorithm also provides accurate estimation.

Next, we measured the computation time of the three algorithms. Figure 5 shows the computation time for one update. Obviously, the proposed algorithm is much improved over the conventional recursive one. In addition, while the computation time of the TLS Unitary ESPRIT increases as the number of elements gets large, the proposed algorithm preserves constantly less computational load.

5 Conclusion

In this paper, we have proposed the modified recursive Unitary ESPRIT for estimating more efficiently DOAs of the multiple waves. Via computer simulation, we have shown that the proposed algorithm is much improved in computation time. The improved performance will enable us to use the proposed algorithm as the real-time adaptive processor in the mobile communication systems.

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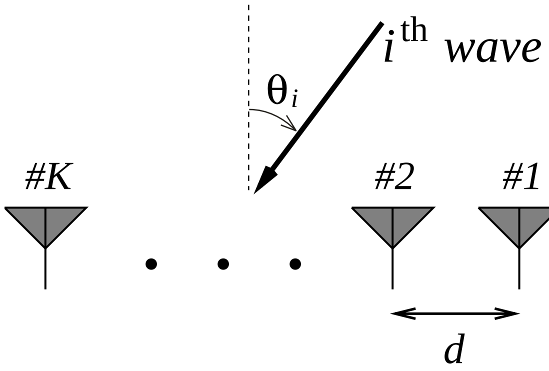

Figure 1: K-element linear array

Table 1: Radio environment

	θ[deg]	power[dB]
wave 1	−30	0
wave 2	50	0

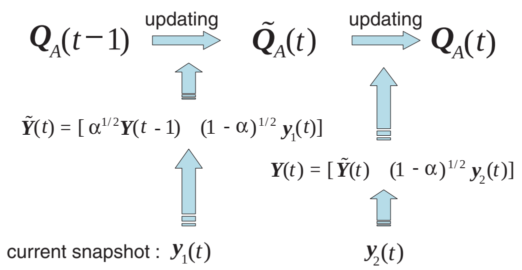

Figure 2: Recursive flow of conventional algorithm

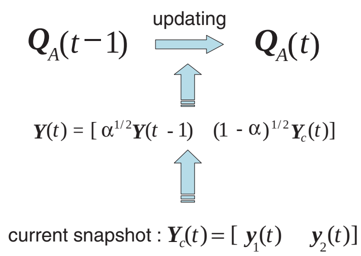

Figure 3: Recursive flow of proposed algorithm

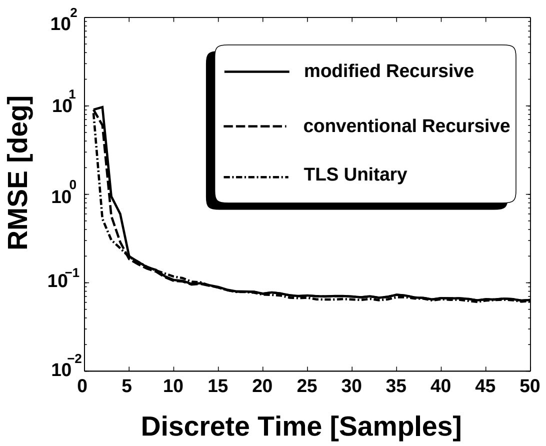

Figure 4: RMSE versus discrete time (samples)

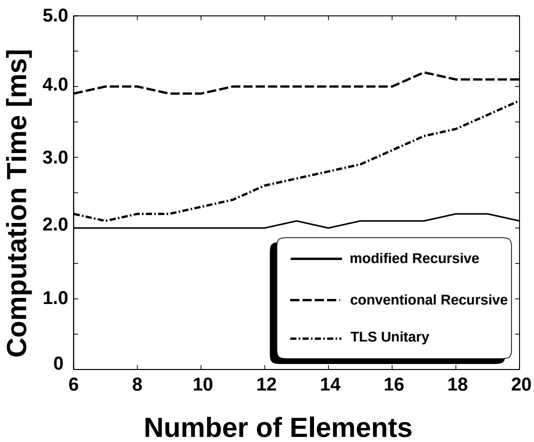

Figure 5: Computation time for one update