# DIFFRACTION FROM AN ANISOTROPIC CHIRAL SLAB WITH A PERIODICALLY-APERTURED PLANE

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## 1 Introduction

There has been an increasing interest in the electromagnetic waves interacting with chiral media which are mainly characterized by eigenmodes with right and left-handed polarization and different velocity [1]-[3]. Research work on chirality introduced in periodic structures also has gained attention in recent years [4]、[5]. One of the potential applications is to design the polarization of diffracted modes of gratings. In the frequency region of radio waves, planar structures can be alternatives to periodically-shaped or modulated chiral media. A FSS structure with an isotropic chiral slab has been reported [6]. Artificial chiral materials in this frequency region however inevitably show the electrical anisotropy, and the optical axes can also be the design parameters. Authors have analyzed an isotropic and uniaxial anisotropic chiral planar structures with strip gratings where the possibility of designing gratings which diffract modes with specific polarization has been investigated [7], [8], [9]. In this paper, diffraction from an anisotropic chiral slab with a periodically-apertured conducting plane are analyzed in $4\times 4$ matrix form [9]. In the numerical computation, the polarization characteristics of diffracted waves are investigated considering the design parameters.

## 2 Description of the problem

The configuration of the problem is shown in the Figure 1. A lossless chiral slab region 2 (anisotropic region in general) with thickness $D$ is assumed to be placed between semi-infinite isotropic achiral dielectric regions 1 and 3 where the relative permeability is unity all through the regions. Considering that isotropic dielectric media are particular cases of anisotropic chiral ones, the chirality admittance and relative permittivity for the region $l$ ($l = 1, 2, 3$) are generally expressed by $3 \times 3$ tensors $(\xi_l)$ and $(\epsilon_l)$ where both $(\xi_1)$ and $(\xi_3)$ are null tensors and $(\epsilon_1)$ and $(\epsilon_3)$ are diagonal ones with elements of $\epsilon_{l,xx} = \epsilon_{l,yy} = \epsilon_{l,zz} = \epsilon_l$ ($l = 1, 3$). A thin conducting plane with periodic array of rectangular apertures with periodicity $\Lambda_Y$ and $\Lambda_Z$ and sizes $L_Y$ and $L_Z$ in Y and Z direction respectively is assumed to be placed on the boundary $X = 0$. XZ plane is assumed to be the plane of incidence where a TE or TM plane wave illuminates under the incidence angle $\theta$. In the following anaysis the time harmonic dependence $\exp(i\omega t)$ is assumed. Electromagnetic fields in the region $l$ ($l = 1, 2, 3$) satisty the constitutive relations as follows [1].

$$\boldsymbol{D} = \epsilon_0((\epsilon_l) + (\tau_l)^2)\boldsymbol{E} - i\sqrt{\epsilon_0\mu_0}(\tau_l)\boldsymbol{H}, \qquad \boldsymbol{B} = i\sqrt{\epsilon_0\mu_0}(\tau_l)\boldsymbol{E} + \mu_0\boldsymbol{H}, \tag{1}$$

$$(\tau_l) = Z_0(\xi_l), \quad Y_0 = \frac{1}{Z_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}. \tag{2}$$

These equations and Maxwell's equations are combined to the following relations:

$$\mathrm{curl}\sqrt{Y_0}\boldsymbol{E} = (\tau_l)\sqrt{Y_0}\boldsymbol{E} - i\sqrt{Z_0}\boldsymbol{H}, \quad \mathrm{curl}\sqrt{Z_0}\boldsymbol{H} = i[(\epsilon_l) + (\tau_l)^2]\sqrt{Y_0}\boldsymbol{E} + (\tau_l)\sqrt{Z_0}\boldsymbol{H} \tag{3}$$

where the space variables expressed by upper cases $X$, $Y$ and $Z$ are normalized by wave number in vacuum $k_0$ to be transformed into lower-case ones, putting $x = k_0 X$, $y = k_0 Y$ and $z = k_0 Z$. The same normalization is made in the operator *curl*

## 3 General solutions for fields

The total fields in each region are given by the superposition of the primary electric and magnetic fields $E_t^p$ and $H_t^p$ $(t = x, y, z)$ which exists even if the apertured plane are replaced by a ground plane and the scattered fields $E_t^s$ and $H_t^s$ $(t = x, y, z)$ from magnetic currents in the apertures which can be expanded in terms of Floquet modes. The conditions of phase matching at the boundaries for the incidence of a plane wave give the forms of the fields as

$$\sqrt{Y_0} E_t^p(x, y, z) = e_t^p(x) e^{-is_0 z}, \quad \sqrt{Z_0} H_t^p(x, y, z) = h_t^p(x) e^{-is_0 z}, \tag{4}$$

$$\sqrt{Y_0} E_t^s(x,y,z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e_{tmn}^s(x) e^{-i(q_m y + s_n z)}, \quad \sqrt{Z_0} H_t^s(x,y,z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h_{tmn}^s(x) e^{-i(q_m y + s_n z)}, \tag{5}$$

$$q_m = m\frac{\lambda}{\Lambda_Y}, \quad s_n = s_0 + n\frac{\lambda}{\Lambda_Z}, \quad s_0 = \sqrt{\epsilon_1}\sin\theta \quad (t = x, y, z) \tag{6}$$

where $\lambda$ is the wave length in vacuum. The coefficients $e_t^p(x)$, $h_t^p(x)$ in Eq.(4) and $e_{tmn}^s(x)$, $h_{tmn}^s(x)$ in Eq.(5) $(t = y, z)$ are expressed here in vector form as

$$\boldsymbol{f}^p(x) = \begin{pmatrix} e_y^p \\ e_z^p \\ h_y^p \\ h_z^p \end{pmatrix}, \quad \boldsymbol{f}^{pn}(x) = \begin{pmatrix} e_x^p \\ h_x^p \end{pmatrix}, \quad \boldsymbol{f}_m^s(x) = \begin{pmatrix} e_{ymn}^s \\ e_{zmn}^s \\ h_{ymn}^s \\ h_{zmn}^s \end{pmatrix}, \quad \boldsymbol{f}_m^{sn}(x) = \begin{pmatrix} e_{xmn}^s \\ h_{xmn}^s \end{pmatrix}. \tag{7}$$

Introducing the above scattered fields into Eq.(3) derives the following coupled wave equations for the $(m,\ n)$th Floquet mode in the region $l$ $(l = 1, 2, 3)$ as follows [8]-[9]:

$$\frac{d}{dx}\boldsymbol{f}_{mn}^s = i(R_{l,mn})\boldsymbol{f}_{mn}^s, \quad \boldsymbol{f}_{mn}^{sn} = (K_{l,mn})\boldsymbol{f}_{mn}^s \tag{8}$$

where $(R_{l,mn})$ and $(K_{l,mn})$ are $4 \times 4$ and $2 \times 4$ matrices which show the mode coupling in each region. The solutions of Eq.(8) in the region $l$ $(l = 1, 2, 3)$ are given in matrix form as

$$\boldsymbol{f}_{mn}^s(x) = (U_{l,mn}) \begin{pmatrix} e^{i\kappa_{l,mn}^{R+}(x-w_l^+)} & & & (O) \\ & e^{i\kappa_{l,mn}^{L+}(x-w_l^+)} & & \\ & & e^{i\kappa_{l,mn}^{R-}(x-w_l^-)} & \\ (O) & & & e^{i\kappa_{l,mn}^{L-}(x-w_l^-)} \end{pmatrix} \boldsymbol{g}_{l,mn}^s, \quad \boldsymbol{g}_{l,mn}^s = \begin{pmatrix} \boldsymbol{g}_{l,mn}^{s+} \\ \boldsymbol{g}_{l,mn}^{s-} \end{pmatrix}, \quad \boldsymbol{g}_{l,mn}^{s\pm} = \begin{pmatrix} g_{l,mn}^{s,R\pm} \\ g_{l,mn}^{s,L\pm} \end{pmatrix}, \tag{9}$$

$$(U_{l,mn}^{\pm}) = \begin{pmatrix} (U_{l,mn}^{e+}) & (U_{l,mn}^{e-}) \\ (U_{l,mn}^{h+}) & (U_{l,mn}^{h-}) \end{pmatrix} = (\boldsymbol{v}_{l,mn}^{R+}\ \boldsymbol{v}_{l,mn}^{L+}\ \boldsymbol{v}_{l,mn}^{R-}\ \boldsymbol{v}_{l,mn}^{L-}), \quad w_1^{\pm} = w_2^{-} = 0, \quad w_3^{\pm} = w_2^{+} = -k_0 D. \tag{10}$$

$\kappa_{l,mn}^{R+}$, $\kappa_{l,mn}^{L+}$, $\kappa_{l,mn}^{R-}$, $\kappa_{l,mn}^{L-}$ in Eq.(9) and $\boldsymbol{v}_{l,mn}^{R+}$, $\boldsymbol{v}_{l,mn}^{L+}$, $\boldsymbol{v}_{l,mn}^{R-}$, $\boldsymbol{v}_{l,mn}^{L-}$ in the $4 \times 4$ matrix $(U_{l,mn})$ are eigenvalues and eigenvectors of the matrix $(R_{l,mn})$ respectively. The superscripts $R\pm$ and $L\pm$ denote right and left-handed elliptically-polarized eigenmodes propagating in $\pm x$ directions respectively. Each $\boldsymbol{g}_{l,mn}^{s\pm}$ is an unknown column vector defined at $x = w_l^{\pm}$ where $\boldsymbol{g}_l^{p\pm}$ for the primary fields is also defined. The solutions of the primary fields $\boldsymbol{f}^p(x)$ with unknowns $\boldsymbol{g}_l^p$ in region $l (l = 1, 2, 3)$ are also derived by setting $m = n = 0$ in Eq.(9). In an isotropic achiral region, eigenmodes are separated into TE and TM linearly-polarized modes or right and left-handed circular polarization modes with the same wave numbers [7].

## 4 Method of solution

The primary fields are distributed in the region 1 $(x \geqq 0)$ and satisfy the condition of y and z components of electric fields being equal to zero at $x = 0$. Considering this condition in the form of Eq.(9) yields linear equations by which the unknowns $\boldsymbol{g}_1^{p+}$ are determined assuming $\boldsymbol{g}_1^{p-} = (1\,0)^T$ and $(0\,1)^T$ for TE and TM incidence respectively. The scattered fields depend on the magnetic currents $M_y(0^\pm, y, z)$ and $M_z(0^\pm, y, z)$ in the apertures at $x = 0^\pm$ which can be expanded in terms of Floquet modes with coefficients $c_{ymn}(0^\pm)$ and $c_{zmn}(0^\pm)$ respectively in the same form as in Eq.(5):

$$\sqrt{Y_0}M_t(0^\pm, y, z) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} c_{tmn}(0^\pm)e^{-i(q_m y + s_n z)}, \quad c_{tmn}(0^\pm) = e^s_{umn}(0) \times (\pm \boldsymbol{i}_x) \qquad (11)$$

where$(t, u) = (y, z)$ *or* $(z, y)$ and $\boldsymbol{i}_x$ is the unit vector in x direction. The scatterd fields satisfy the conditions of continuity of y and z components of fields at $X = -D$ and those of waves in the regions 1 and 3 propagating only in $+x$ and $-x$ directions respectively. These conditions in the form of Eq.(9) yield linear equations to determine the solutions $\boldsymbol{g}^s_{l,mn,y}$ and $\boldsymbol{g}^s_{l,mn,z}$ $(l = 1, 2, 3)$ for the assumption that $(c_{ymn}(0^+),\ c_{zmn}(0^+)) = (1, 0)$ and $(0, 1)$ respectively. Combining these and Eq.(9) leads to the expression of the scattered fields in the region $l$ $(l = 1, 2, 3)$:

$$\boldsymbol{f}^s_{mn}(x) = \begin{cases} \boldsymbol{G}_{l,mn,y}(x)c_{ymn}(0^+) + \boldsymbol{G}_{l,mn,z}(x)c_{zmn}(0^+) & (x \geqq 0) \\ \boldsymbol{G}_{l,mn,y}(x)c_{ymn}(0^-) + \boldsymbol{G}_{l,mn,z}(x)c_{zmn}(0^-) & (x \leqq 0) \end{cases}, \qquad (12)$$

$$\boldsymbol{G}_{l,mn,t}(x) = \begin{pmatrix} G^e_{l,ymnt}(x) \\ G^e_{l,zmnt}(x) \\ G^h_{l,ymnt}(x) \\ G^h_{l,zmnt}(x) \end{pmatrix} = (U_{l,mn}) \begin{pmatrix} e^{i\kappa^{R+}_{l,mn}(x-w_l^+)} & & & (O) \\ & e^{i\kappa^{L+}_{l,mn}(x-w_l^+)} & & \\ & & e^{i\kappa^{R-}_{l,mn}(x-w_l^-)} & \\ (O) & & & e^{i\kappa^{L-}_{l,mn}(x-w_l^-)} \end{pmatrix} \boldsymbol{g}^s_{l,mn,t}, \quad t = y,\ z. \qquad (13)$$

The remaining unknown magnetic currents $M_y(0^\pm, y, z)$ and $M_z(0^\pm, y, z)$ in the apertures at $x = 0^\pm$ can be approximated by the expansion in terms of a set of linear independent basis functions $\Phi_{tk^t}(y, z)$ $(k^t = 1, \ldots, N^t,\ t = y, z)$ defined in the apertures with unknown coefficients $C_{tk^t}$ $(k^t = 1, 2, \ldots, N^t,\ t = y, z)$. The primary fields and the scattered fields Eq.(12) with the approximated currents form the boundary condition of y and z components of magnetic fields being continuous in the apertures at $x = 0^\pm$. Applying the Galerkin's Method to this condition in the spectral domain yields a system of linear equations to determine the unknown coefficients for the magnetic currents.

## 5 Numerical results

In the following numerical calculation the Root Top basis functions are used for the Galerkin's procedure as in [10] where $16 \times 16$ segmentation is made for each of the rectangular apertures, and Floquet mode expansion in Eq.(5) is truncated at $m,\ n = \pm 90$ where graphical representation of results are possible. A uniaxial chiral medium is assumed for the region 2 where the optical axis is in XZ plane and makes the angle $\alpha = 42$ (degree) to the positive X direction with Z axis. The values of the chirality and the relative permittivity along the optical axis and those along the directions normal to it are assumed to be $\xi_e = 7.0 \times 10^{-4}$(S), $\epsilon_e = 1.35$ and $\xi_o = 1.0 \times 10^{-4}$(S), $\epsilon_o = 1.2$ respectively. The periodicity and the sizes of the apertures are assumed to be $\Lambda_Y = 0.7\lambda$ and $\Lambda_Z = 1.1\lambda$, $L_Y/\Lambda_Y = L_Z/\Lambda_Z = 0.6$ respectively. The regions 1 and 3 are assumed to be air. Figure 2 (a), (b) and (c) show the ellipticity and the orientation angles of the polarization as defined in [8] and the diffraction efficiencies of the transmitted (0, 0)th and (0, -1)th modes respectively for TE incidence with $\theta = 30$ (degree). At around $D = 2.9\lambda$, ellipticity angles of both modes are almost zero and difference of the orientation angles comes to be 90 degree with the same extent of the diffraction efficiencies. This implies the transmitted wave is almost evenly splitted into the dominat and the diffracted modes with orthogonal linear polarization. Other examples of design parmeters will be shown in the presentation.

# 6 Conclusion

Diffracted waves from an anisotropic chiral slab with a periodically-apertured conducting plane have been analyzed in $4 \times 4$ matrix form. Numerical results have shown the polarization characteristics of diffracted waves with the design parameters.

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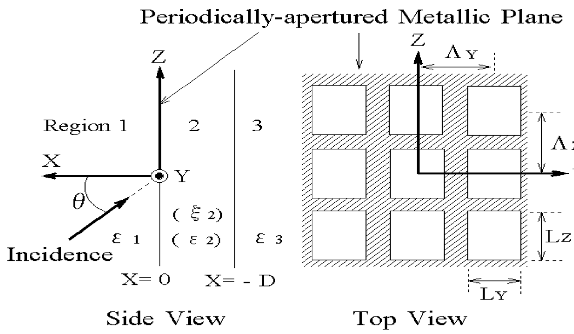


Figure 1: Geometry of the problem

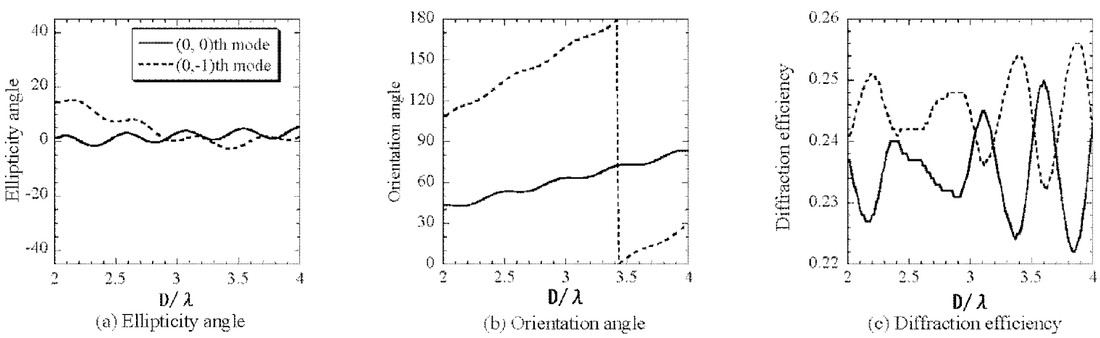


Figure 2: Polarization characteristics of tramsmitted waves