GENETIC ALGORITHM APPLIED TO OPTIMAL DESIGN OF A POLARIZATION-TRANSFORMATION FILTER

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1. Introduction

A chiral medium has been of great interest and practical importance due to its applications in a variety of electromagnetic devices [1]-[5]. The authors showed that a stratified chiral slab acts as a polarization-transformation filter that passes only a cross-polarized component of the transmitted wave at some frequency band [2]. Since the transmission and reflection characteristics of the stratified chiral slab strongly depend on the small change of the material parameters and thickness of each layer, it is not easy to design a polarization-transformation filter.

A genetic algorithm (GA) is a stochastic search method modeled on the Darwinian concepts of natural selection and evolution [6]. The GA works well in an optimization problem without any gradient information of a functional to be optimized [7]-[9].

The purpose of this paper is to consider the optimal design of a polarization-transformation transmission filter, which is composed of a stratified chiral slab. The two-stage procedure for designing the filter based on the GA is proposed. A chain-matrix formulation is used to derive the transmitted and the reflected electric fields. Then one can obtain the cross- and co-polarized powers carried by the transmitted and reflected waves. We define a functional according to the design conditions of a high-pass filter at each stage. Applying the GA to the maximization of the functionals, one can search the optimal material parameters of the stratified chiral slab. Numerical results are presented to confirm the effectiveness of our design procedure. It is seen from the results that the stratified chiral slab acts as a polarization-transformation high-pass filter for the transmitted wave.

2. Transmitted and reflected powers

Let us consider a plane electromagnetic wave with arbitrary polarization incident from free space upon a stratified chiral slab, which is located on a dielectric substrate with material parameters ε_s and μ_s. The slab is composed of M chiral layers with different material parameters ε_m, μ_m, and ξ_{cm}, and thicknesses d_m, where $m = 1,\ 2,\ \ldots,\ M$. The quantities ε, μ, and ξ_c denote the permittivity, the permeability, and the chiral admittance.

Assuming $\exp(-j\omega t)$ time dependence, the constitutive relations for the m-th chiral layer are expressed as [10]

$$\mathbf{D}_m = \varepsilon_m \mathbf{E}_m + j\xi_{cm}\mathbf{B}_m, \tag{1}$$

$$\mathbf{H}_m = j\xi_{cm}\mathbf{E}_m + \mathbf{B}_m/\mu_m, \tag{2}$$

where $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, and $\mathbf{B}$ are the electric field, the magnetic field, the electric flux density, and the magnetic flux density vectors, respectively.

Imposing the boundary conditions at the interface of two adjacent chiral layers and using a chain-matrix formulation, we can obtain the parallel and the perpendicular components of the transmitted and reflected electric fields. The transmitted and reflected powers carried by these components are represented as

$$P_{t\|} = \frac{\eta_0 \cos\theta_s}{\eta_s \cos\theta_0} \frac{|E_{t\|}|^2}{|E_{i\|}|^2 + |E_{i\perp}|^2}, \tag{3}$$

$$P_{t\perp} = \frac{\eta_0 \cos\theta_s}{\eta_s \cos\theta_0} \frac{|E_{t\perp}|^2}{|E_{i||}|^2 + |E_{i\perp}|^2}, \tag{4}$$

$$P_{r||} = \frac{|E_{r||}|^2}{|E_{i||}|^2 + |E_{i\perp}|^2}, \tag{5}$$

$$P_{r\perp} = \frac{|E_{r\perp}|^2}{|E_{i||}|^2 + |E_{i\perp}|^2}, \tag{6}$$

where the subscripts i, r, and t refer to the incident, the reflected, and the transmitted fields, and the subscripts $||$ and $\perp$ to the parallel and perpendicular components of an electric field vector. The parameters $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ and $\eta_s = \sqrt{\mu_s/\varepsilon_s}$ are the intrinsic impedances of free space and the dielectric substrate, respectively. The parameters θ_0 and θ_s are the angle of incidence and the refracted angle of the transmitted wave.

3. Two-stage procedure for designing a polarization-transformation transmission filter

The design procedure for a polarization-transformation transmission filter consists of two stages: (1) designing an ordinary high-pass filter constructed from a stratified *non-chiral* slab, and (2) designing a polarization-transformation high-pass filter, which is composed of a stratified *chiral* slab. Therefore, the design variables at the first and the second stages are the refractive indices of a stratified non-chiral slab and the chiral admittances of a stratified chiral slab, respectively. It should be noted that each layer of the polarization-transformation filter has the same permittivity as that obtained at the first stage.

To find the optimal refractive indices of the at the first stage, we define a functional,

$$g_A(\mathbf{X}_A) = \sum_{i=1}^{2} P_r(f_i; \mathbf{X}_A) + \sum_{i=3}^{4} P_t(f_i; \mathbf{X}_A), \tag{7}$$

where $\mathbf{X}_A$ is a M-dimensional row vector constructed from the refractive indices of the stratified non-chiral slab. The parameters f_1, f_3 and f_2, f_4 are the lowest and highest frequencies for the stop- and the pass-bands, respectively. Putting $\xi_{cm} = 0$ for $m = 1, 2, \ldots, M$, $P_{t||}(P_{t\perp})$ and $P_{r||}(P_{r\perp})$, respectively, become P_t and P_r with $P_{t\perp}(P_{t||}) = P_{r\perp}(P_{r||}) = 0$ in the case of $E_{i\perp}(E_{i||}) = 0$.

At the second stage, the chiral admittances are found to design a polarization-transformation high-pass filter for the transmitted wave. The transmission characteristics of the filter satisfies the following requirements: (i) The pass-band of the filter is the same as that obtained at the first stage, (ii) Only the cross-polarized transmitted wave exists at the pass-band. Then we define the following functional:

$$g_B(\mathbf{X}_B) = \sum_{i=3}^{4} \left[P_{t,\,cr}(f_i; \mathbf{X}_B) - P_{t,\,co}(f_i; \mathbf{X}_B)\right], \tag{8}$$

where $\mathbf{X}_B$ is a M-dimensional row vector composed of the chiral admittances of the stratified chiral slab. Furthermore, $P_{t,\,cr}$ and $P_{t,\,co}$ are the cross- and co-polarized powers carried by the transmitted wave, respectively.

The problem of designing each filter can be formulated as an optimization problem, which maximizes the functional given by Eq. (7) or (8). Applying a binary-coded GA [6] to the maximization of two functionals, we may obtain the optimal refractive indices and chiral admittances of the stratified chiral slab.

4. Numerical results

Numerical results are presented for the incident wave of perpendicular polarization, i.e., $E_{i||} = 0$. The incident power is now normalized to unity, and the angle of incidence is $0°$. The material parameters of the dielectric substrate are assumed to be $n_s = \sqrt{\varepsilon_s/\varepsilon_0} = 1.52$ and $\mu_s = \mu_0$. We apply the two-stage procedure to the structure of a 45-layered chiral slab [11],

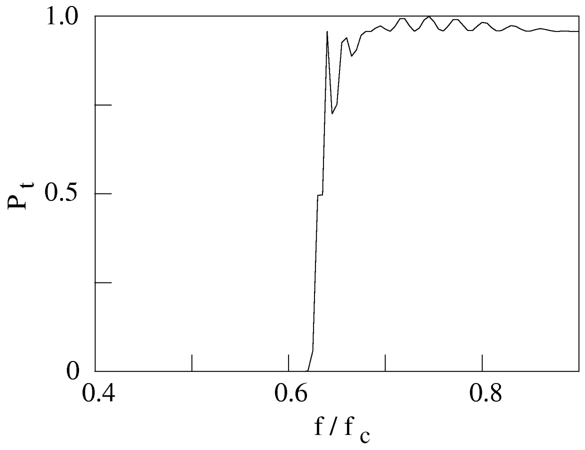

Fig. 1 Transmitted power for the 45-layered dielectric slab with $n_{\rm I} = 2.33$ and $n_{\rm II} = 5.18$.

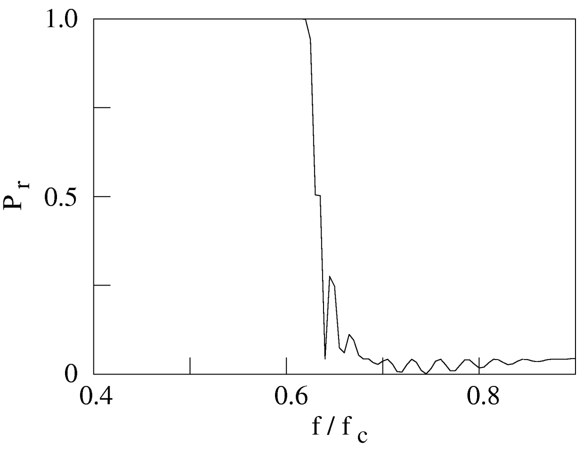

Fig. 2 Reflected power for the 45-layered dielectric slab with $n_{\rm I} = 2.33$ and $n_{\rm II} = 5.18$.

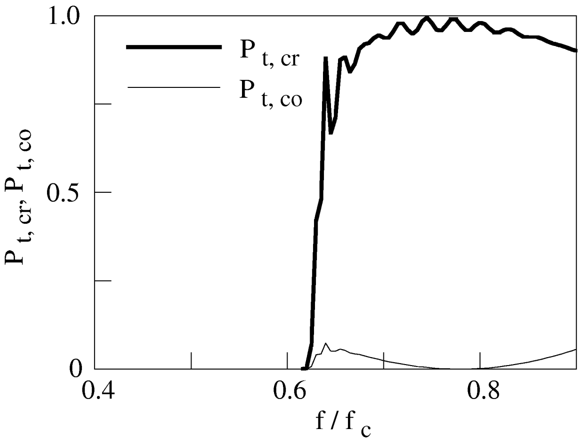

Fig. 3 Cross- and co-polarized powers carried by transmitted wave for the 45-layered chiral slab with $\xi_{c\rm I} = 2.047 \times 10^{-5}$(S) and $\xi_{c\rm II} = 1.333 \times 10^{-5}$(S).

Table 1 Parameters used in the GA at the first and the second stages.

Parameters	First	Second
Number of gene per chromosome	2	2
Number of bits per gene	10	24
Population size	30	30
Crossover probability	0.9	0.9
Mutation probability	0.1	0.25
Number of sampling frequencies	98	53

$$\text{free space} \;\Big|\; (\,\mathrm{I}\;\mathrm{II}\;\mathrm{I}\,)^{15} \;\Big|\; \text{substrate},$$

where the superscript 15 denotes the number of periods. We employ the parameters: $n_{\rm I}d_{\rm I} = \lambda_c/2$, $n_{\rm II}d_{\rm II} = \lambda_c/4$, and $\mu_{\rm I} = \mu_{\rm II} = \mu_0$. Here n_i and d_i (i = I, II) are the refractive indices and the thicknesses, and λ_c is the wavelength in free space at the central frequency f_c. The parameters used in the GA at the first and the second stages are summarized in Table 1. The fitness functions at the two stages are given by Eqs. (7) and (8).

The parameters used at the first stage are as follows: $f_1 = 0.4f_c$, $f_2 = 0.62f_c$, $f_3 = 0.64f_c$, and $f_4 = 0.9f_c$. Applying the GA to the maximization of $g_A(\mathbf{X}_A)$, we can obtain $n_{\rm I} = \sqrt{\varepsilon_{\rm I}/\varepsilon_0} = 2.33$ and $n_{\rm II} = \sqrt{\varepsilon_{\rm II}/\varepsilon_0} = 5.18$ for the non-chiral slab at the 11-th generation. Figures 1 and 2, respectively, illustrate the transmitted and reflected powers obtained for these refractive indices. It can be seen from Figs. 1 and 2 that the non-chiral slab shows the transmission characteristics of a high-pass filter.

At the second stage, we use the refractive indices obtained at the first stage. The other material parameters of the slab are the same as those selected at the first stage. In getting a binary coded form of the chiral admittance of each layer, $\xi_{c\mathrm{I}}$ and $\xi_{c\mathrm{II}}$ are restricted in the range,

$$0 \leq \xi_{c\beta} \leq \sqrt{\frac{\varepsilon_{\mathrm{I}}}{\mu_0}} \quad (\beta = \mathrm{I},\ \mathrm{II}). \tag{9}$$

Application of the GA to the maximization of $g_B(\mathbf{X}_B)$ gives $\xi_{c\mathrm{I}} = 2.047 \times 10^{-4}$ (S) and $\xi_{c\mathrm{II}} = 1.333 \times 10^{-4}$ (S) at the 12-th generation. Figure 3 presents the cross- and co-polarized powers carried by the transmitted wave for these chiral admittances. Note that the cross-polarized reflected power $P_{r\,||}$ is zero since the polarization state of the reflected wave is the same as that of the incident wave at normal incidence. From Fig. 3, we can see that the two-stage procedure proposed is very effective in the optimal design of a polarization-transformation high-pass filter for the transmitted wave.

5. Conclusion

The GA has been applied to the optimal design of a polarization-transformation transmission filter, which is composed of a stratified chiral slab. Using a chain-matrix formulation, one can obtain the cross- and co-polarized powers carried by the transmitted and reflected waves. The two-stage procedure for designing the polarization-transformation filter has been proposed based on the GA. We define two functionals according to the design conditions of the ordinary and the polarization-transformation high-pass filters. The optimal material parameters of a stratified chiral slab are found by maximizing the functionals with the GA. Numerical results are presented to demonstrate the effectiveness of our design procedure. It is seen from the numerical results that the stratified chiral slab acts as a polarization-transformation transmission high-pass filter for the transmitted wave.

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