AN ANALYSIS OF EXCITATION OF MAGNETOSTATIC SURFACE WAVES IN AN IN-PLANE MAGNETIZED YIG FILM

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1. Introduction

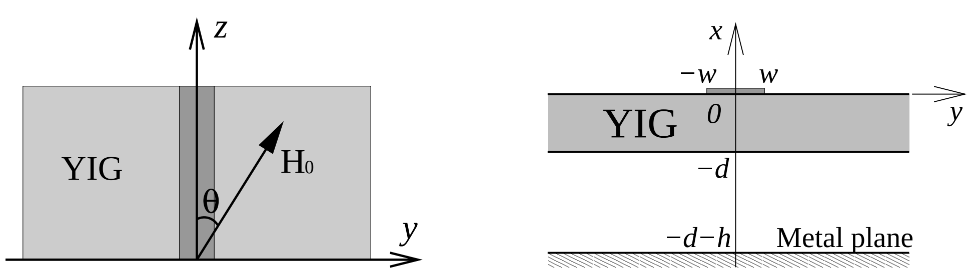
In this paper, we will analyze the excitation by a microstrip transducer on the in-plane and obliquely magnetized YIG by letting the current density be an unknown function and numerically solving it by means of the integral kernel expansion method which has been developed by our group[1] and is appropriate in solving the mixed boundary value problems.

Magnetostatic waves (MSWs), which propagate in a magnetized YIG film, have potential possibilities of application to signal processing in microwave band[2]. In a magnetized YIG film in the plane, if the bias magnetic field is perpendicular to the wave vector of MSW, then the Magnetostatic surface wave(MSSW) propagates, while if the magnetic field is parallel to the wave vector, the Magnetostatic backward volume wave(MSBVW) does. For the case that the magnetic field is neither perpendicular nor parallel, in other words, in an obliquely magnetized YIG film, both MSSW and MSBVW modes propagate, which is called MSSW/MSBVW mode, and the characteristics of such a MSSW/MSBVW mode continuously changes depending on the angle of magnetization[3].

In order to verify the validity of the present method, we will compare our numerical results with the experimental ones.

2. Theoretical Analysis

We consider the geometrical configuration of the present problem as shown in Fig.1, the left panel of which is the top view and the right one is the cross-sectional view. The metal strip having the width $2w$ and the infinitesimal thickness is constructed on a YIG film with thickness d. The metal plane exists at $y = -d - h$ as the ground conductor of the microstrip transducer. The layer-configuration goes infinitely along y- and z-directions, and the field is assumed to be independent of z, or $\partial/\partial z = 0$. The external direct magnetic field is applied in the direction making an angle θ with z-axis, and then the MSW propagates in $\pm y$-direction. In this case, the following MSW modes can propagate depending on the θ: (1) MSSW propagates for the magnetization perpendicular to the wave vector of MSW($\theta = 0°$); (2) MSBVW, for the magnetization parallel to the wave vector($\theta = 90°$); (3) MSSW/MSBVW, which propagates for the oblique magnetization of the YIG film ($0° < \theta < 90°$).



(a) Top view of the configuration

(b) Cross-sectional view of the configuration

Figure 1: Configuration for analysis

From the configuration a pair of the Fourier transform between the space and the wave number domains is defined as follows:

$$f(y) = \int_{-\infty}^{\infty} \tilde{f}(k) e^{-jky} dk, \quad \tilde{f}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(y) e^{jky} dy. \tag{1}$$

It is assumed that magnetostatic approximation, $\nabla \times \boldsymbol{H} = 0$ is valid and that the time factor is $\exp(j\omega t)$. The permeability tensor of in-plane magnetized YIGs is given by

$$\overline{\boldsymbol{\mu}}_r = \begin{bmatrix} \mu & -j\kappa_2 & j\kappa_3 \\ j\kappa_2 & \mu_2 & \mu_3 \\ -j\kappa_3 & \mu_3 & \mu_4 \end{bmatrix}, \tag{2}$$

where $\kappa_2 = \kappa \cos\theta,\ \kappa_3 = \kappa \sin\theta,\ \mu_2 = \mu \cos^2\theta + \sin^2\theta,\ \mu_3 = (1-\mu)\cos\theta \sin\theta,\ \mu_4 = \mu \sin^2\theta + \cos^2\theta,\ \mu = 1 - \dfrac{\Omega_H}{\Omega^2 - \Omega_H^2},\ \kappa = \dfrac{\Omega}{\Omega^2 - \Omega_H^2}, \Omega = \dfrac{\omega}{\gamma M}, \Omega_H = \dfrac{\gamma H_0}{\gamma M}$, and H_0, M, ω, and γ are the magnitude of the bias magnetic field, the saturation magnetization of the YIG, angular wave frequency, and the gyromagnetic ratio, respectively.

With the use of the usual boundary conditions that the tangential component of magnetic field and the normal component of magnetic flux density are continuous, we can obtain the relation between the current density flowing in the metal strip, $J_z(y)$, and the magnetic flux density, $B_x(y)$ on the $x = 0$ plane, in the wave number domain, $\tilde{B}_x(k) = j\mu_0 G(k) \tilde{J}_z(k)$, where

$$G(k) = -s \frac{(\mu q - \kappa_2 s)(\mu q + \kappa_2 s + \tanh|k|h) - (\mu q + \kappa_2 s)(\mu q - \kappa_2 s - \tanh|k|h) e^{-2q|k|d}}{(\mu q - \kappa_2 + 1)(\mu q + \kappa_2 + \tanh|k|h) - (\mu q + \kappa_2 - 1)(\mu q - \kappa_2 - \tanh|k|h) e^{-2q|k|d}}, \tag{3}$$

and $q = \sqrt{\dfrac{\mu_2}{\mu}}, s = \dfrac{k}{|k|}$. It is noted that the function $G(k)$ has a pole in $k > 0$ and $k < 0$, respectively.

Taking the inverse Fourier transform and considering that $B_x(y) = 0$ in the metal strip region ($-w \leqq y \leqq w$) lead to the following equation:

$$\int_{-\infty}^{\infty} G(k) \tilde{J}_z(k) e^{-jky} dk = 0, \quad -w \leqq y \leqq w. \tag{4}$$

This Fourier integral contains the unknown current density $\tilde{J}_z(k)$, and we can solve Eq.(4) by using the integral kernel expansion method.

First step of this method is to expand the integral kernel of Eq.(4), e^{-jky} into a series of the Legendre polynomials over the interval $-w \leqq y \leqq w$:

$$e^{-jky} = \sum_{m=0}^{\infty} (-j)^m (2m+1)\, j_m(kw) P_m\left(\frac{y}{w}\right). \tag{5}$$

Substituting Eq.(5) into Eq.(4) and invoking orthogonality of the Legendre polynomials, we can obtain

$$\int_{-\infty}^{\infty} G(k) \tilde{J}_z(k) j_m(kw) dk = 0, \quad m = 0, 1, \cdots. \tag{6}$$

Next, we expand the unknown function $J_z(y)$ into a series of orthogonal polynomials with unknown coefficients.

$$J_z(y) = \left\{1 - \left(\frac{y}{w}\right)^2\right\}^{-\frac{1}{2}} \sum_{n=0}^{\infty} a_n T_n\left(\frac{y}{w}\right) \Pi\left(\frac{y}{w}\right), \quad \text{where } \Pi(x) = \begin{cases} 1, & |x| \leqq 1 \\ 0, & |x| > 1 \end{cases}, \tag{7}$$

and $T_n(z)$ is the Chebyshev polynomial and $a_n (n = 0, 1, \cdots)$'s are the unknown coefficients to be determined. The Fourier transform of Eq.(7) is given by

$$\tilde{J}_z(k) = \frac{w}{2} \sum_{n=0}^{\infty} a_n j^n J_n(kw). \tag{8}$$

Substituting Eq.(8) into Eq.(6) we can reduce the Fourier integral containing the unknown function into linear equations with respect to the unknown coefficients:

$$\begin{bmatrix} L_{00} & L_{01} & \cdots & L_{0n} \\ L_{10} & L_{11} & \cdots & L_{1n} \\ \vdots & \vdots & & \vdots \\ L_{m0} & L_{m1} & \cdots & L_{mn} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = 0, \quad \text{where } L_{mn} = j^n w \int_{-\infty}^{\infty} G(k) j_m(kw) J_n(kw) dk, \tag{9}$$

which can be calculated numerically, then we can determine the coefficients a_n's to obtain the current density in Eqs.(7) and (8).

Calculation of the infinite integrals of Eq.(9) is not straightforward because the integrands oscillate. We describe the techniques to overcome this difficulty. Consider the integral over the positive k. As $k \to +\infty$, the function $G(k)$ approaches exponentially to the constant, $G^+ = -\dfrac{\mu q - \kappa_2}{\mu q - \kappa_2 + 1}$. Thus, we separate the integral into two parts as follows:

$$L^+_{mn} = \int_0^\infty \left\{ G\left(\frac{z}{w}\right) - G^+ \right\} j_m(z) J_n(z) dz + G^+ \int_0^\infty j_m(z)\, J_z(z)\, dz \tag{10}$$

It is found here that the integrand of the first term of Eq.(10) vanishes exponentially and the convergence of the numerical computation can be obtained easily. The second term of Eq.(10) can be calculated analytically, and the result is given by

$$\frac{\pi\,\Gamma(\frac{m+n+1}{2})\,G^+}{2\,\Gamma(\frac{m-n+2}{2})\,\Gamma(\frac{n-m+1}{2})\,\Gamma(\frac{m+n+2}{2})}. \tag{11}$$

Note that the result is identically equal to 0 when $n - m - 2$ or $m - n - 1$ is even. The integration over the negative half-infinite interval can be done in a similar manner.

3. Numerical and experimental results

Experiments were performed to verify the validity of the present theory. Using a vector network analyzer(HP8720C), we measured the S_{21} of the transducer assuming the attenuation is due to the excitation of MSWs. The YIG used in this experiment is 20 μm thick and 8 mm wide. The metal strip which is 125 μm wide, or $w = 62.5\mu$m in Fig.1, is directly constructed by evaporating aluminum on the YIG surface. The distance between the metal strip and the ground plane is about 400 μm, for which the characteristic impedance and the effective dielectric constant of this transmission line are approximately 195Ω and 8.63, respectively, if the YIG is regarded just as a dielectric.

Figure 2 shows the experimental results of S_{21} of the transducer changing the angle of the biased magnetic field, θ, with constant magnitude, 78.0[kA/m] (= 980 Oe). The frequency band where MSSW exists for $\theta = 0°$ is from 4.60 GHz to 5.24 GHz, and for $\theta = 90°$ MSBVW exists from 2.74 GHz to 4.60 GHz. It is known that, as θ increasing, the upper limit of MSSW band decreases and the lower limit of MSBVW band increases[3], and indeed the tendency can be found in Fig.2.

Figures 3, 4 and 5 show the numerical results estimated by the present method and a conventional one which is based on the assumption that the current distribution $J_z(y)$ is constant in the strip, and the experimental results for $\theta = 0°$, $\theta = 20°$ and $\theta = 30°$, respectively. The used parameters are same as the above experiments and we truncate the expansion with respect to n in Eq.(8) by 10 terms. As you can see, the numerical results of the present method generally agree with the experimental ones. However, there is a serious discrepancy between the conventional and the experimental results, in particular, in the upper frequency of MSSW band. This tendency seems to be more prominent for larger θ angle.

The experimental data exhibit $1 \sim 2$ dB greater loss than the numerical results by using the present method. This may be due to the undesirable return loss at transition between the the coaxial and microstrip line, radiation into free-space, or conductive loss of the metal strip. The notches exist at about 4.60 GHz, which corresponds to the lower limit of MSSW band. In the magnetostatic approximation, the wavelength of MSSW tends to infinity toward the lower limit of MSSW band, and hence the approximation may not be always good near the lower limit. In the experiment, there exists a ground plane, which might affect the MSSW because the wavelength becomes longer around this lower limit.

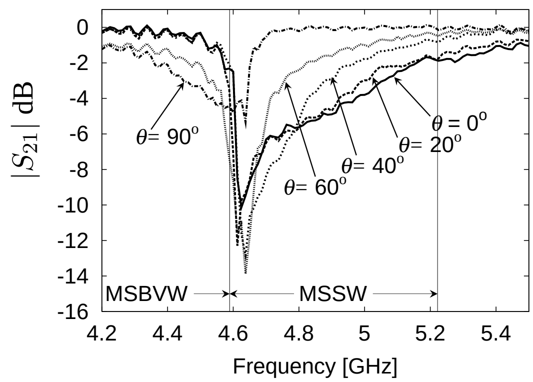


Figure 2: S_{21} of the test device changing the angle of the magnetic field

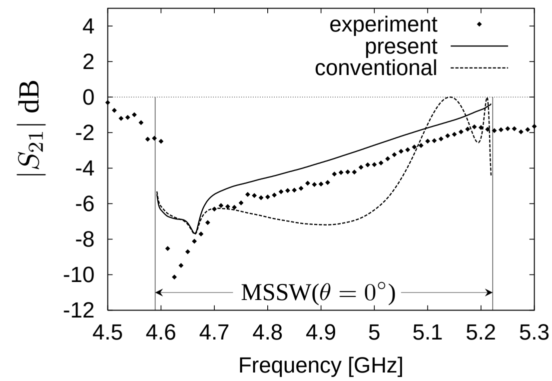


Figure 3: Numerical and experimental results for $\theta = 0°$

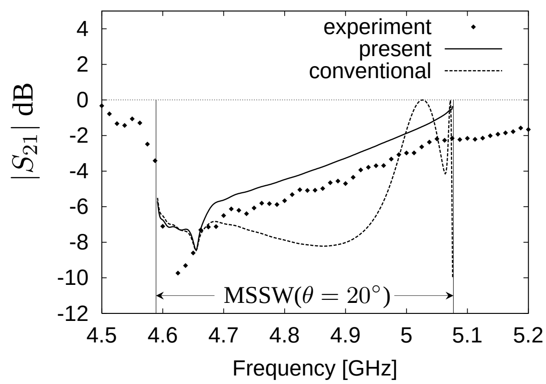


Figure 4: Numerical and experimental results for $\theta = 20°$

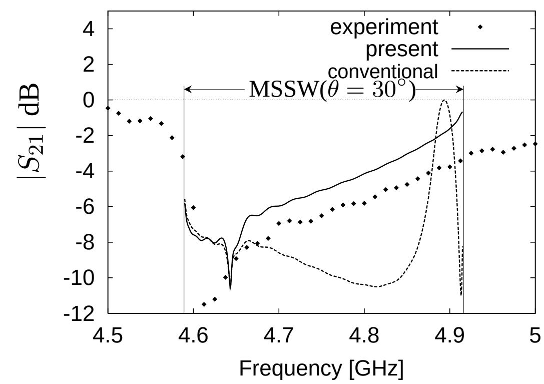


Figure 5: Numerical and experimental results for $\theta = 30°$

4. Conclusion

We have presented the integral kernel expansion method which is an integral equation solver, as the application to analysis of the MSSW excitation in the in-plane magnetized YIG film. This method allows us to formulate the present problem elegantly and solve it successively. The agreement of the numerical results by the present method with the corresponding experiment is generally good, while there is still a significant discrepancy between the conventional ones and the experiment. It has been evident that the integral kernel expansion method used in this paper is appropriate for the analyses of MSW excitation problems.

References

[1] Y. Ando, N. Guan, K. Yashiro, and S. Ohkawa, "Excitation of magnetostatic surface waves by slot line transducers," IEICE Trans. Electron., vol.E82-C, no.7, pp.1123-1128, 1999.

[2] W.S. Ishak, "Magnetostatic wave technology:A review," Proc.IEEE, vol.76, no.2, pp.171-187, 1988.

[3] M.J. Hurben and C.E. Patton, "Theory of magnetostatic waves for in-plane magnetized isotropic films," J. Magn. Magn. Mater., vol.139, pp.263-291, 1995.