A MODEL FOR THE PREDICTION OF DIFFERENTIAL RAIN ATTENUATION STATISTICS ON TWO CONVERGING POINT-TO-POINT TERRESTRIAL LINKS LOCATED IN A TROPICAL REGION

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1.Introduction

Rain attenuation is considered to be the main cause of the performance impairments of communication systems operating in frequencies above 10 GHz [1]. The subject of the paper is the development of an analytical procedure for the prediction of differential rain attenuation statistics on two converging point-to-point terrestrial links located in a tropical region. It is based on a convective raincell model with a correlation coefficient appropriate for these regions, an assumption that the point rainfall statistics follows a gamma form and finally a tropical raindrop size distribution. Our motivation for proposing a reliable physical model, comes from the recent use of millimeter wave radio frequencies to provide wireless access to broadband services, embodied in a service generally termed Local Multipoint Distributed System (LMDS) and its coexistence other frequencies in the same frequency bands. The suggested model is compared with a set of available experimental for the differential rain attenuation over pairs of terrestrial links in Brazil.

2. Statistical Analysis

Converging links are usual in the radio terrestrial networks. This results to interference due to rainfall inhomogeneity [2]. The problem is more critical for low percentages of time where high values of differential rain attenuation can be observed, indicating that significant reductions of carrier-to-interference ratio may occur under rainy conditions.

The configuration of the problem is shown in Figure 1 where the receivers of two converging links are collocated in *N*. The angular separation between the two converging links is symbolized as φ. For the following analysis two values of attenuation are important, the rain attenuation A_1 of the signal from the link TL_1 and the rain attenuation A_2 which is induced in the link TL_2. Our objective is to calculate the following exceedance probability:

$$P_{DRA} = P\left[A_1 - A_2 \geq r\right] \tag{1}$$

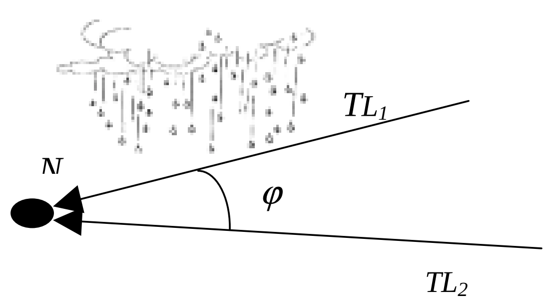

Figure1: Configuration of the problem

where r is a level of differential rain attenuation, and its values are positive. When this difference in attenuation becomes sufficiently large, then the signal from the terrestrial link TL_2 can cause interference to the point-to-point TL_1.

The assumptions below, which are required for the analysis, are taken into account:

a) The gamma form is adopted for both the unconditional point rainfall rate R and the attenuation A distribution [3]. The term unconditional point rainfall rate is meant to include both raining and non-raining time.

b) The constants a and b of the specific attenuation $A_o = a \cdot R^b \,(dB/km)$ [4] depend upon frequency, incident polarization, temperature and raindrop size distribution. In this paper, lognormal distribution for the raindrop size distribution, appropriate for these regions as suggested by Ajayi and Olsen [5] is assumed.

c) As far as the horizontal variation of the rainfall rate structure is concerned, it is adopted the convective raincell model proposed by Morita-Higuti [3] which is valid for tropical regions. According to this model, the spatial correlation coefficient ρ_o of attenuation gradient between two points of the rain medium is given by:

$$\rho_o = \exp\left(-a_S \cdot \sqrt{d}\right) \tag{2}$$

where a_S is a characteristic parameter depending on the spatial structure of the rainfall medium for the specific location and d is the distance between two points of the rainfall medium.

Following the previous considerations the exceedance probability of differential rain attenuation can be calculated as:

$$P_{DRA} = \int_r^{\infty} \int_0^{x_1 - r} f_{A_1 A_2}(x_1, x_2) \cdot dx_1 \cdot dx_2 \tag{3}$$

where $f_{A_1 A_2}(x_1, x_2)$ is the two-dimensional gamma distribution [6]. Finally after straightforward algebra we have:

$$P_{DRA} = \frac{(1-\rho)^{\nu}}{\Gamma(\nu)} \cdot \sum_{i=0}^{\infty} \frac{\rho^i}{i!} \cdot \frac{\beta_1^{\nu+i}}{\Gamma(\nu+i)} \cdot \int_r^{\infty} x_1^{\nu+i-1} \cdot e^{-\beta_1' x} \cdot \gamma\left(\nu+i, \beta_2' \cdot g(x_1, r)\right) \cdot dx_1 \tag{4}$$

$$g(x_1, r) = x_1 - r \tag{5}$$

In the above expressions $\Gamma(\ )$ and $\gamma(\ )$ are the well-known gamma and incomplete gamma function respectively [7]. The statistical parameters ν_1, β_1, ν_2, β_2, ν, β_1' and β_2' can be expressed as [6]:

$$\left.\begin{array}{ll} \nu_1 = \dfrac{L_1^2 \nu_0}{H_{11}} & \beta_1 = \dfrac{L_1 \nu_0}{H_{11} \mu_0} \\ \nu_2 = \dfrac{L_2^2 \nu_0}{H_{12}} & \beta_2 = \dfrac{L_2 \nu_0}{H_{12} \mu_0} \\ \nu = \sqrt{\nu_1 \nu_2} \approx \nu_1 \approx \nu_2 & \\ \mu_0 = \dfrac{\nu_0}{\beta_0} & \end{array}\right\} \tag{6}$$

and

$$\nu_0 = \frac{\left\{\frac{\Gamma(\nu_R+b)}{\Gamma(\nu_R)}\right\}^2}{\left\{\left\{\frac{\Gamma(\nu_R+2b)}{\Gamma(\nu_R)}\right\} - \left\{\frac{\Gamma(\nu_R+b)}{\Gamma(\nu_R)}\right\}^2\right\}} \tag{7}$$

$$\mu_0 = \frac{\left\{\frac{\Gamma(\nu_R+b)}{\Gamma(\nu_R)}\right\}\beta_R^b}{\left\{\left(\left\{\frac{\Gamma(\nu_R+2b)}{\Gamma(\nu_R)}\right\} - \left\{\frac{\Gamma(\nu_R+b)}{\Gamma(\nu_R)}\right\}^2\right)a\right\}} \tag{8}$$

$$\left.\beta_1' = \frac{\beta_1}{1-\rho}, \beta_2' = \frac{\beta_2}{1-\rho}\right\} \tag{9}$$

in terms of L_i the lengths of the terrestrial links and of the parameters ν_R, β_R of the gamma distribution for the point rainfall statistics. These parameters can be obtained from appropriate regression fitting analysis on local experimental data for the rainfall rate or the recently suggested ITU-R rainmaps [8]. Finally, the path correlation coefficient ρ can be expressed as:

$$\rho = \frac{H_2}{\sqrt{H_{11}H_{12}}} \tag{10}$$

where the factors H_2, H_{11} and H_{12} are defined and presented analytically elsewhere [6].

3. Numerical Results and Discussion

The proposed procedure for calculating the differential rain attenuation statistics on two converging links is applied to available experimental data in Brazil [9] at 14.5 GHz where the receivers are collocated in a building in Rua dos Ingleses (RIS) in Sao Paulo. The geometrical characteristics of the system under consideration can be found elsewhere [10]. Using the recent proposed rainmaps [8] and after application of an appropriate regression fitting analysis the gamma statistical parameters of point rainfall rate distribution have been calculated. In Table 1, the appropriate numerical values for the system parameters concerning the two converging links Brasdesco-RIS (TL_1) and Scania-RIS are presented. In Figure 2, the differential rain attenuation exceedance probability from both the proposed model and the experimental results is drawn.

f	14.5 GHz
a	0.038949
b	1.23456
$L_1(TL_1)$	12.79 km
$L_1(TL_2)$	18.38 km
a_S	0.25
φ	123.6^o
Polarization	Vertical
ν_R	0.0238
β_R	0.05 49

Table 1:Parameters of the communication systems under consideration

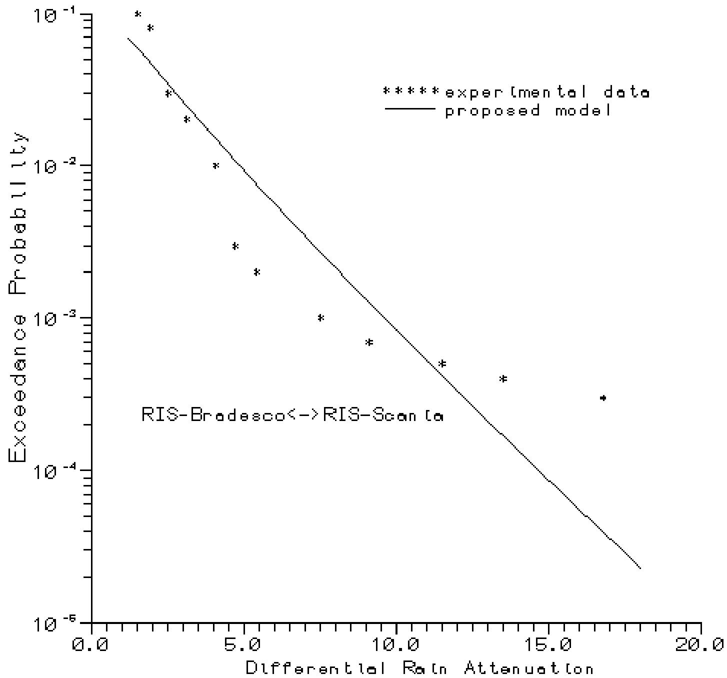

Figure 2: Differential rain attenuation exceedance probability concerning a pair of converging terrestrial links.

We can see that the agreement has been found encouraging. The impact of differential rain attenuation on carrier-to-interference ratio statistics of one of the two point-to-point links, is similar to the one proposed in [6]. Te proposed model is quite flexible and it is oriented to be applicable to any tropical region of the world where the previous assumptions are satisfied.

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