A FAST ANALYSIS OF SCATTERING FROM ARBITRARILY SHAPED CONDUCTING BODIES COATED WITH LOSSY MATERIALS

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1. Introduction

The calculation of electromagnetic scattering from arbitrarily shaped three-dimensional conducting objects coated with lossy dielectric materials has been of considerable interest owing to the wide application of dielectric coating in a variety of radar targets. When the coating materials are homogeneous or piecewise homogeneous, a preferred analysis method is to use the surface integral equation in conjunction with the method of moment (MoM). This method avoids the discretization of the dielectric volumes and thus reduces the number of unknowns. Despite this, the computational requirement for the solution of this kind of problems is still very high because the MoM analysis requires $O(N^3)$ complexity and $O(N^2)$ memory. This requirement can quickly exceed computational limitations when modeling electrically large and/or complicated material structures. In this paper, an algorithm using the precorrected-FFT method is developed to deal with this problem.

The precorrected-FFT method is a fast method associated with $O(N^{1.5})\log N$ or less complexity. It was originally proposed by Philips *etc.* [1] to solve the electrostatic integral equation and was recently extended by the authors to the analysis of metallic scatterers in free space [2] and large-scale microstrip structures [3]. In this paper, it is further extended to the analysis of scattering from conducting objects coated with lossy materials. The problem is formulated by an EFIE-PMCHW formulation, which employs an electric field integral equation (EFIE) for the conducting surface and the PMCHW formulation [4] for the dielectric surface. The RWG basis functions [5] are used to expand both the equivalent electric and magnetic currents and also used as the testing functions to convert the integral equations into a matrix equation. Finally, the matrix equation is solved iteratively and the precorrected-FFT method is used to speed up the matrix–vector products in iterations as well as reduce the memory requirement. Numerical examples are presented to validate the implementation.

2. Formulation

A general example of conducting bodies immersed in a dielectric material is shown in Fig.1 Let S_{ci}, $i = 1,2,\cdots N_c$ represent the surfaces of arbitrarily shaped conducting objects, S_d represent the interface between the dielectric material coating and the external medium which is usually free space. The composite structure is illuminated by an incident plane wave ($\boldsymbol{E}^{inc}, \boldsymbol{H}^{inc}$). Introduce equivalent electric currents $\boldsymbol{J}_{ci}$, $i = 1,2,\cdots N_c$ on the conducting surfaces and equivalent electric and magnetic currents ($\boldsymbol{J}_d, \boldsymbol{M}_d$) and ($-\boldsymbol{J}_d, -\boldsymbol{M}_d$) on opposite sides of the dielectric surface. By enforcing the boundary condition of continuity of the tangential fields across each interface, a set of coupled integral equations can be obtained [6]:

$$[\sum_{i=1}^{N_c} \boldsymbol{E}_c^{s2}(\boldsymbol{J}_{ci}) + \boldsymbol{E}_c^{s2}(-\boldsymbol{J}_d) + \boldsymbol{E}_c^{s2}(-\boldsymbol{M}_d)]_{\tan} = 0 \tag{1}$$

$$[\sum_{i=1}^{N_c} \boldsymbol{E}_d^{s2}(\boldsymbol{J}_{ci}) + \boldsymbol{E}_d^{s1}(-\boldsymbol{J}_d) + \boldsymbol{E}_d^{s2}(-\boldsymbol{J}_d) + \boldsymbol{E}_d^{s1}(-\boldsymbol{M}_d) + \boldsymbol{E}_{\boldsymbol{d}}^{\boldsymbol{s2}}(-\boldsymbol{M}_d)]_{\tan} = \boldsymbol{E}_{\boldsymbol{d}}^{inc}|_{\tan} \tag{2}$$

$$[\sum_{i=1}^{N_c} \boldsymbol{H}_d^{s2}(\boldsymbol{J}_{ci}) + \boldsymbol{H}_d^{s1}(-\boldsymbol{J}_d) + \boldsymbol{H}_d^{s2}(-\boldsymbol{J}_d) + \boldsymbol{H}_d^{s1}(-\boldsymbol{M}_d) + \boldsymbol{H}_{\boldsymbol{d}}^{\boldsymbol{s2}}(-\boldsymbol{M}_d)]_{\tan} = \boldsymbol{H}_{\boldsymbol{d}}^{inc}|_{\tan} \tag{3}$$

where the superscript represents the medium in which the scattered fields are evaluated and the subscript represents the surface on which the equations are enforced. The EFIE (1) and PMCHW formulation (2) and (3) constitute the necessary integral equations in this work.

To solve for the equivalent currents, we first divide each surface into planar triangular patches and expand the equivalent electric and magnetic currents by the RWG basis functions $\boldsymbol{f}_n(\boldsymbol{r}')$ [5],

$$\boldsymbol{J}_c(\boldsymbol{r}') = \sum_{n=1}^{N_t} I_{cn} \boldsymbol{f}_n(\boldsymbol{r}'), \quad \boldsymbol{J}_d(\boldsymbol{r}') = \sum_{n=1}^{N_d} I_{dn} \boldsymbol{f}_n(\boldsymbol{r}'), \quad \boldsymbol{M}_d(\boldsymbol{r}') = \eta_0 \sum_{n=1}^{N_d} M_{dn} \boldsymbol{f}_n(\boldsymbol{r}') \tag{4}$$

where N_d and N_t denote the number of edges on the dielectric surface and all N_c conducting surfaces, respectively, I_{cn}, I_{dn} and M_{dn} are the unknown coefficients. Testing (1) and (2) with $\boldsymbol{f}_m$ and (3) with $\eta_0 \boldsymbol{f}_m$ result in a $N \times N$ ($N = N_t + 2N_d$) partitioned matrix equation given by

$$\begin{bmatrix} [Z_{mn}^{J_cJ_c}] & [Z_{mn}^{J_cJ_d}] & [Z_{mn}^{J_cM_d}] \\ [Z_{mn}^{J_dJ_c}] & [Z_{mn}^{J_dJ_d}] & [Z_{mn}^{J_dM_d}] \\ [Z_{mn}^{M_dJ_c}] & [Z_{mn}^{M_dJ_d}] & [Z_{mn}^{M_dM_d}] \end{bmatrix} \begin{bmatrix} [I_{cn}] \\ [I_{dn}] \\ [M_{dn}] \end{bmatrix} = \begin{bmatrix} [0] \\ [V_{dm}] \\ [H_{dm}] \end{bmatrix} \tag{5}$$

Elements of the above submatrices can be readily obtained from (1)-(3) considering the expressions of the electric and magnetic fields in terms of the electric and magnetic currents:

$$Z_{mn}^{J_cJ_c} = -j\omega \int_{T_m} \boldsymbol{f}_m(\boldsymbol{r}) \cdot \boldsymbol{A}_{2mn} d\boldsymbol{r} + \int_{T_m} \nabla_s \cdot \boldsymbol{f}_m(\boldsymbol{r}) \Phi_{2mn} d\boldsymbol{r} \tag{6a}$$

$$Z_{mn}^{J_cJ_d} = j\omega \int_{T_m} \boldsymbol{f}_m(\boldsymbol{r}) \cdot \boldsymbol{A}_{2mn} d\boldsymbol{r} - \int_{T_m} \nabla_s \cdot \boldsymbol{f}_m(\boldsymbol{r}) \Phi_{2mn} d\boldsymbol{r} \tag{6b}$$

$$Z_{mn}^{J_cM_d} = \int_{T_m} \eta_0 \boldsymbol{f}_m(\boldsymbol{r}) \cdot \frac{\nabla \times \boldsymbol{A}_{2mn}}{\mu_2} d\boldsymbol{r} \tag{6c}$$

$$Z_{mn}^{J_dJ_c} = -j\omega \int_{T_m} \boldsymbol{f}_m(\boldsymbol{r}) \cdot \boldsymbol{A}_{2mn} d\boldsymbol{r} + \int_{T_m} \nabla_s \cdot \boldsymbol{f}_m(\boldsymbol{r}) \Phi_{2mn} d\boldsymbol{r} \tag{6d}$$

$$Z_{mn}^{J_dJ_d} = j\omega \int_{T_m} \boldsymbol{f}_m(\boldsymbol{r}) \cdot [\boldsymbol{A}_{1mn} + \boldsymbol{A}_{2mn}] d\boldsymbol{r} - \int_{T_m} \nabla_s \cdot \boldsymbol{f}_m(\boldsymbol{r}) [\Phi_{1mn} + \Phi_{2mn}] d\boldsymbol{r} \tag{6e}$$

$$Z_{mn}^{J_dM_d} = \int_{T_m} \eta_0 \boldsymbol{f}_m(\boldsymbol{r}) \cdot \nabla \times \left[\frac{\boldsymbol{A}_{1mn}}{\mu_1} + \frac{\boldsymbol{A}_{2mn}}{\mu_2} \right] d\boldsymbol{r} \tag{6f}$$

$$Z_{mn}^{M_dJ_c} = \int_{T_m} \eta_0 \boldsymbol{f}_m(\boldsymbol{r}) \cdot \frac{\nabla \times \boldsymbol{A}_{2mn}}{\mu_2} d\boldsymbol{r} \tag{6g}$$

$$Z_{mn}^{M_dJ_d} = -Z_{mn}^{J_dM_d} \tag{6h}$$

$$Z_{mn}^{M_dM_d} = j\omega \int_{T_m} \boldsymbol{f}_m(\boldsymbol{r}) \cdot \left[\frac{\eta_0^2 \boldsymbol{A}_{1mn}}{\eta_1^2} + \frac{\eta_0^2 \boldsymbol{A}_{2mn}}{\eta_2^2} \right] d\boldsymbol{r} - \int_{T_m} \nabla_{\boldsymbol{s}} \cdot \boldsymbol{f}_m(\boldsymbol{r}) \cdot \left[\frac{\eta_0^2 \Phi_{1mn}}{\eta_1^2} + \frac{\eta_0^2 \Phi_{2mn}}{\eta_2^2} \right] d\boldsymbol{r} \tag{6i}$$

where

$$\boldsymbol{A}_{imn} = \frac{\mu_i}{4\pi} \int_{T_n} \boldsymbol{f}_n(\boldsymbol{r}') G_i(\boldsymbol{r}_{\boldsymbol{m}}, \boldsymbol{r}') d\boldsymbol{r}' \qquad i = 1,2 \tag{7a}$$

$$\Phi_{imn} = -\frac{1}{4\pi j\omega\varepsilon_i} \int_{T_n} \nabla'_s \cdot \boldsymbol{f}_n(\boldsymbol{r}') G_i(\boldsymbol{r}_{\boldsymbol{m}}, \boldsymbol{r}') d\boldsymbol{r}' \qquad i = 1,2\ . \tag{7b}$$

In the above equations, $\boldsymbol{f}_m$ and $\boldsymbol{f}_n$ represent the testing and basis functions, respectively, while T_m and T_n denote their supports. $\eta_i = \sqrt{\mu_i / \varepsilon_i}$ stands for the characteristic impedance in medium i. In (6), we've already taken into account the similarities between the definitions of the vector potentials $\boldsymbol{A}_i(\boldsymbol{r})$ and $\boldsymbol{F}_i(\boldsymbol{r})$, the scalar potentials $\Phi_i(\boldsymbol{r})$ and $U_i(\boldsymbol{r})$, and rewrote all matrix elements in terms of only one vector potential $\boldsymbol{A}_{imn}$ and one scalar potential Φ_{imn}, which enables easier description and implementation of the followed precorrected-FFT approach.

3. Precorrected-FFT Approach

Implementation of the precorrected-FFT method requires the entire object enclosed in a uniform rectangular grid. Then the matrix–vector product can be computed in a four-step procedure: (1) project the element singularity distributions to point singularities on the uniform grid, (2) compute the vector and scalar potentials at the grid points due to the grid sources by FFT-accelerated convolutions, (3) interpolate the grid point potentials onto the elements, and (4) add the precorrected direct nearby interactions. The implementation of steps (2) to (4) is similar to that proposed in [2] although there are more terms to be computed and the procedure becomes much more complicated. Here we just explain the projection step briefly.

As can be seen from (7), there are three kinds of operators need to be projected onto the uniform grid, *i.e.*, the basis function $\boldsymbol{f}_n$, which is corresponding to patch currents (either electric or magnetic), the divergence operator $\nabla \cdot \boldsymbol{f}_n$ which is corresponding to patch charges (either electric or magnetic) and the curl operator $\nabla \times \boldsymbol{f}_n$, which is corresponding to $\nabla \times \boldsymbol{A}_{imn}$. The projection operators for the currents ($\boldsymbol{f}_n$) /charges ($\nabla \cdot \boldsymbol{f}_n$) are constructed by matching the vector/scalar potentials produced by the currents/charges at the grid points surrounding the triangular patch with that produced by the original current/charge distributions on that triangular patch at given test points. As for $\nabla \times \boldsymbol{f}_n$, we avoid the projection of this operator by replacing the partial derivatives of the vector potential $\boldsymbol{A}_{imn}$ with the corresponding differences, which can be computed through the vector potentials at several vicinal points of the observation point. Knowledge of these potentials can be readily obtained through interpolation once the vector potentials at grid points have been computed by the FFTs. This simplified approach has been demonstrated to be of good accuracy. Additionally, since the Green's functions will change based upon the constitutive properties for different mediums, the projection must be performed for each medium separately, namely, when the vector or scalar potentials in medium i are to be computed, the projection operators corresponding to medium i are constructed and then used to project the patch currents and charges unto the uniform grid. The detailed handling of the precorrected-FFT approach will be discussed at the conference.

4. Numerical Results

Two examples are presented to validate the precorrected-FFT solution of the EFIE-PMCHW formulation for conducting objects coated with dielectric materials. The actual memory reduction and speed-up attained by this fast method has already been demonstrated in [2,3], so it is omitted here.

Fig. 2 shows the bistatic RCS of a coated sphere at $f = 300MHz$. The conducting core has a radius of 0.3m and the coating is uniform around the sphere with a thickness of 0.05m. The dielectric permittivity of the coating material is $\varepsilon_r = 4.0 - j0.1$. For the precorrected-FFT method, we set grid order $p = 3$ and the near field threshold distance to be $0.2\lambda_0$. A grid spacing of $0.08\lambda_0$ is used in this example. It is observed that the precorrected-FFT solution agrees very well with the MoM solution and the exact solution given in [7], validating the precorrected-FFT implementation.

The second example is a conducting disk embedded in a dielectric cylinder. The radius of the disk and the cylinder is $0.4\lambda_0$ and $0.45\lambda_0$ respectively, and the height of the cylinder is $1.6\lambda_0$. The triangular patch model of the disk contains 179 edges and the dielectric cylinder contains 1914 edges, resulting in 4007 unknowns. The bistatic RCS obtained from the precorrected-FFT method and the MoM is given in Fig.3. Again perfect agreement is observed.

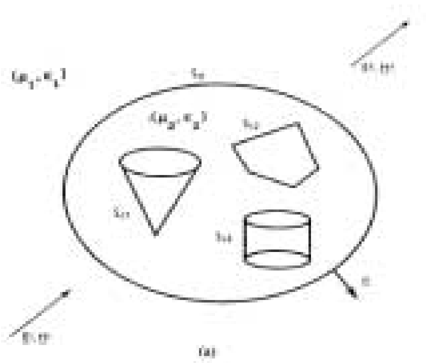

Fig.1. Conducting bodies immersed in a dielectric material

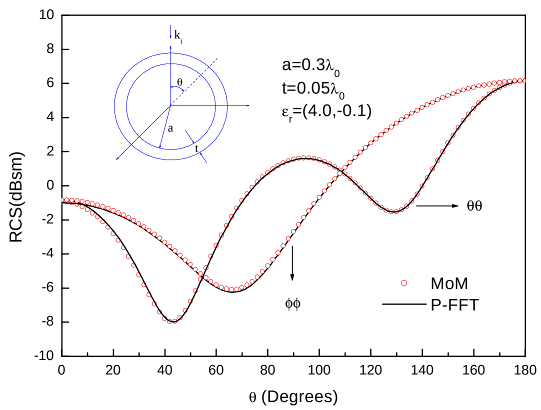

Fig. 2. Bistatic RCS of a coated sphere

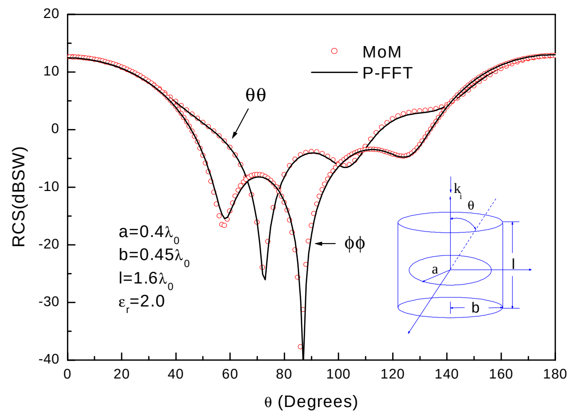

Fig. 3. RCS of a disk imbedded in a dielectric material

5. References

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