Numerical Calculation of Scattering from N Dielectric Circular Cylinders by Using Greengard-Rokhlin's Fast Multipole Algorithm

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1 Introduction

The boundary element method (BEM), one of the representative methods for numerically calculating electromagnetic (EM) wave scattering, has been widely used for solving boundary integral equations. In this method, however, we finally have to solve a dense linear system of equations. The number of unknowns for the linear system becomes very large for two cases of scattering by a large body and by many bodies. As the analysis area for these two cases is very large, we can see them as scattering from large objects.

A fast multipole algorithm (FMA) [1] is one of the method which reduces the floating point operation of a matrix-vector product in iterative method for the linear system. We applied Greengard-Rokhlin's FMA (GRFMA) [2] to numerical calculation of EM scattering from a large conducting circular cylinder [3] and from many conducting ones [4]. As a result, the floating point operation of one time matrix-vector product was reduced by GRFMA from $O(L^2)$ to $O(L)$ for both cases, where L is the number of unknowns. The reduction was numerically shown for conducting bodies but not for dielectric bodies.

In this paper, we consider EM wave scattering from N dielectric circular cylinders and apply GRFMA to the part of matrix-vector product. In numerical examples, we show the efficiency and accuracy of one time matrix-vector product by GRFMA and discuss characteristics of iteration by changing the relative permittivity and number of cylinders.

2 Formulation

Let us consider the two-dimensional problem of EM wave scattering by N infinitely long dielectric cylinders in the medium of $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$. The relative permittivity and relative permiability of the ith cylinder are $\varepsilon_r^{(i)}$ and $\mu_r^{(i)}$, respectively. Each axis of the cylinders is parallel to the z-axis of the cylindrical coordinate system. When a TM incident plane wave E_z^{inc} is assumed, the electric field integral equations (EFIE) for the surface electric currents $\partial E_z/\partial n_n$ and surface magnetic currents E_z are obtained by

$$\frac{1}{2}E_z(\boldsymbol{\rho}_i) = E_z^{\mathrm{inc}}(\boldsymbol{\rho}_i) - \frac{\mathrm{j}}{4}\sum_{n=1}^{N}\left[\int_{C_n}\left\{E_z(\boldsymbol{\rho}'_n)\frac{\partial \mathrm{H}_0^{(2)}(k_0|\boldsymbol{\rho}_i-\boldsymbol{\rho}'_n|)}{\partial n'_n} - \mathrm{H}_0^{(2)}(k_0|\boldsymbol{\rho}_i-\boldsymbol{\rho}'_n|)\frac{\partial E_z}{\partial n'_n}(\boldsymbol{\rho}'_n)\right\}\mathrm{d}l'_n\right] \quad (1)$$

$$\frac{1}{2}E_z(\boldsymbol{\rho}_i) = \frac{\mathrm{j}}{4}\int_{C_i}\left\{E_z(\boldsymbol{\rho}'_i)\frac{\partial \mathrm{H}_0^{(2)}(k_i|\boldsymbol{\rho}_i-\boldsymbol{\rho}'_i|)}{\partial n'_i} - \mathrm{H}_0^{(2)}(k_i|\boldsymbol{\rho}_i-\boldsymbol{\rho}'_i|)\mu_r^{(i)}\frac{\partial E_z}{\partial n'_i}(\boldsymbol{\rho}'_i)\right\}\mathrm{d}l'_i \quad (i=1,2,\ldots,N). \quad (2)$$

Here, C_i is the boundary of the ith dielectric cylinder, $\boldsymbol{\rho}_i$ and $\boldsymbol{\rho}'_i$ are the observation and integration point on C_i, respectively. $\mathrm{H}_0^{(2)}$ is the zero order Hankel function of the second kind, and $\partial/\partial n_i$ is the outward normal derivative on C_i. k_i is the wavenumber of the ith dielectric cylinder and given by $k_i = \omega\sqrt{\varepsilon_r^{(i)}\varepsilon_0\mu_r^{(i)}\mu_0} = k_0\sqrt{\varepsilon_r^{(i)}\mu_r^{(i)}}$.

Equations (1) and (2) can be discretized through the BEM. Dividing each boundary into M boundary elements and using the point matching method, a linear system of $2NM$ equations is obtained. The square coefficient matrix for the linear system is given by

$$\begin{bmatrix} a_{11}^{\mathrm{out}} & \cdots & a_{1N}^{\mathrm{out}} & b_{11}^{\mathrm{out}} & \cdots & b_{1N}^{\mathrm{out}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{N1}^{\mathrm{out}} & \cdots & a_{NN}^{\mathrm{out}} & b_{N1}^{\mathrm{out}} & \cdots & b_{NN}^{\mathrm{out}} \\ a_{11}^{\mathrm{in}} & & \mathit{0} & b_{11}^{\mathrm{in}} & & \mathit{0} \\ & \ddots & & & \ddots & \\ \mathit{0} & & a_{NN}^{\mathrm{in}} & \mathit{0} & & b_{NN}^{\mathrm{in}} \end{bmatrix}. \tag{3}$$

Here, the notation a_{ij}^{out}, a_{ij}^{in}, b_{ij}^{out}, b_{ij}^{in} are blocks expressed by square matrices of M order. Each block has elements

$$\left[a_{ij}^{\mathrm{out}}\right]_{nm} = \frac{\mathrm{j}}{4}\int_{\Delta C_{jm}} \frac{\partial \mathrm{H}_0^{(2)}(k_0|\boldsymbol{\rho}_{in} - \boldsymbol{\rho}'_j|)}{\partial n'_j}\,\mathrm{d}l'_j + \frac{\delta_{ij}\delta_{nm}}{2} \tag{4}$$

$$\left[b_{ij}^{\mathrm{out}}\right]_{nm} = -\frac{\mathrm{j}}{4}\int_{\Delta C_{jm}} \mathrm{H}_0^{(2)}(k_0|\boldsymbol{\rho}_{in} - \boldsymbol{\rho}'_j|)\,\mathrm{d}l'_j \tag{5}$$

$$\left[a_{ii}^{\mathrm{in}}\right]_{nm} = \frac{\mathrm{j}}{4}\int_{\Delta C_{im}} \frac{\partial \mathrm{H}_0^{(2)}(k_i|\boldsymbol{\rho}_{in} - \boldsymbol{\rho}'_i|)}{\partial n'_i}\,\mathrm{d}l'_i - \frac{\delta_{nm}}{2} \tag{6}$$

$$\left[b_{ii}^{\mathrm{in}}\right]_{nm} = -\mu_r^{(i)}\frac{\mathrm{j}}{4}\int_{\Delta C_{im}} \mathrm{H}_0^{(2)}(k_i|\boldsymbol{\rho}_{in} - \boldsymbol{\rho}'_i|)\,\mathrm{d}l'_i \qquad (n,\ m = 1, 2, \ldots, M). \tag{7}$$

The notation *0* is zero matrix of M order. When the linear system of equations is solved by using an iterative method, the floating point operation is $O((NM)^2)$ per one iteration due to the product of the upper half of the coefficient matrix and a vector. GRFMA is used to expedite this calculation. As the lower half of the coefficient matrix is sparse, the product of the lower half of the coefficient matrix and a vector is calculated directly.

3 Greengard-Rokhlin's FMA (GRFMA)

GRFMA, proposed by *L. Greengard* and *V. Rokhlin*, is one of the multilevel FMA. In this algorithm, dividing the cell which encloses all scatterers, we make a hierarchical group of boundary elements and observation points. GRFMA takes two steps. The first step is called aggregation step, and the sum of the radiation fields at boundary elements of a group is expressed in the form of a multipole expansion. The second step is called disaggregation step, and the field at each observation point of a group is expressed in the form of a local expansion. The first step requires two processes: the multipole expansion for the sum of the radiation fields at boundary elements of the smallest group (MP) and the translation from a small group's multipole expansion into a large group's one (M2M). The second step requires three processes: the translation from a multipole expansion into a local expansion (M2L), the translation from a large group's local expansion into a small group's one (L2L) and the field calculation by a local expansion (LP).

Each process is followed by an infinite series. In numerical calculation, the infinite series is truncated after a finite number of terms. The finite number p is given in order that the accuracy of the field calculation by a point source may be about 10^{-4}. If p is large enough, M2M, M2L and L2L are calculated by the FFT because these processes are expressed in terms of the discrete convolution [3].

4 Numerical Examples

We have carried out GRFMA for EM scattering by N dielectric circular cylinders. In numerical calculation, the size of the cell which include all scatterers is fixed as $386k_0a$, where k_0a is the normalized radius of cylinders and assumed as $k_0a = 3.0$. The relative permittivity of all cylinders is ε_r. The relative permiability of all cylinders is fixed as 1.0. The cylinders are placed on a plane in the form of lattice whose number is 3×3 to 129×129. The number of unknowns for the linear system of equations increases from 576 to $1,065,024$ in line with N. A generalized minimal residual (GMRES) with block Jacobi preconditioning [5] is used to solve the linear system and restarted after 100 iterations. The experiments were performed on a COMPAQ Alpha 21264 (667MHz) processor.

The efficiency of GRFMA is shown in figure 1, where the computation time and the amount of used memory by the one time matrix-vector product are plotted. The computation time is about $O((NM)^{4/5})$. The amount of used memory is about $O((NM)^{3/4})$. In the case of scattering from many conducting circular cylinders, the computation time and the amount of used memory was about $O(NM)$ and $O((NM)^{2/3})$, respectively [4]. These orders are the lowest for scattering from large objects as far as we know. However the reasons for the difference in computation time and used memory for conducting and dielectric cylinders are under consideration.

The accuracy of the matrix-vector product by GRFMA is shown in table 1. In GRFMA, the finite number p was given in order that the accuracy for the field calculation by a point source may be about 10^{-4}. The accuracy of the matrix-vector product is much less than 10^{-4} for all cases.

Scattering by 9×9 and 17×17 dielectric cylinders was numerically calculated from $\varepsilon_r = 2.0$ to 64.0. The number of iteration and the net computation time is shown in table 2. It is found that as ε_r is large, the number of iteration becomes smaller for both cases. Therefore, we expect that the numerical calculation of scattering from many dielectric cylinders becomes easier as the relative permittivity is large.

Table 3 shows the number of iteration and the net computation time for numerical calculation of scattering by dielectric cylinders with $\varepsilon_r = 4.0$ and 32.0. The number of cylinders was changed from 3×3 to 33×33. It is found that as the number of cylinders is large, that of iteration becomes larger; especially, it increases rapidly in the case of more than 17×17 cylinders.

5 Concluding Remarks

EM scattering from N dielectric circular cylinders is considered, and GRFMA is applied to the part of matrix-vector product in iterative method. In numerical calculation, the efficiency and accuracy of GRFMA for one time matrix-vector product have been estimated in the range of $2NM < 1,100,000$. Althogh GRFMA is said to have the floating point operation of $O(NM)$ theoretically, our numerical results show that the floating point operation is about $O((NM)^{4/5})$. The amount of used memory is about $O((NM)^{3/4})$. The matrix-vector product by GRFMA is in good agreement with direct calculation. Therefore, GRFMA is very useful for the calculation of the matrix-vector product in iterative method.

Moreover, we have estimated the number of iterations by changing the relative permittivity and the number of cylinders. Our numerical results show that the number of iterations increase as the relative permittivity is small and as the number of cylinders is large. In order to reduce the number of iterations, we try to investigate the mechanism of an increase in iteration and apply other iterative methods and preconditioning techniques.

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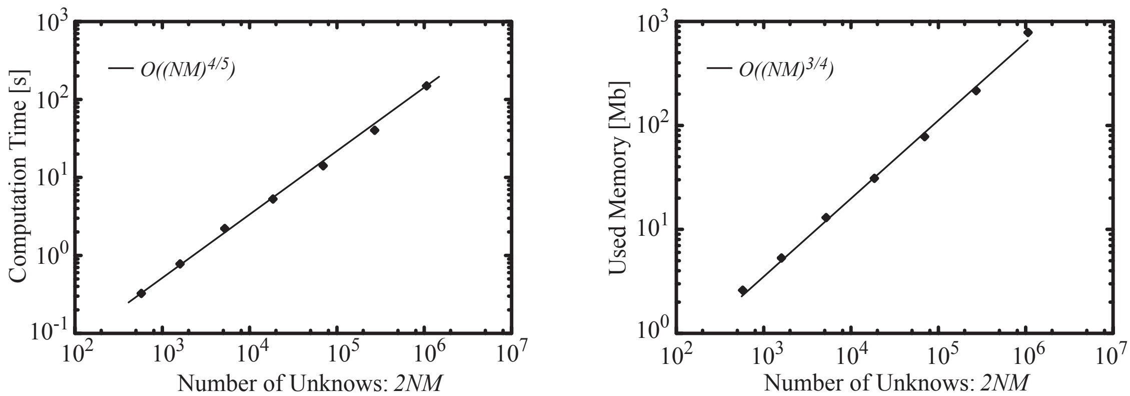

Figure 1: The computation time and the amount of used memory by GRFMA.

Table 1: The accuracy of the matrix-vector product by GRFMA.

N	3×3	5×5	9×9	15×15
ε_r=2.0	4.0871e-7	3.4519e-7	6.5614e-7	1.8169e-6
32.0	5.7745e-7	4.8330e-7	8.4279e-7	2.1848e-6

Table 2: The number of iteration and the net computation time for numerical calculation of scattering from 9×9 and 17×17 dielectric cylinders.

N		ε_r =2.0	4.0	8.0	16.0	32.0	64.0
9×9	Iteration	259	99	156	47	34	38
	Time [s]	566.8	208.8	341.4	102.8	74.4	83.2
17×17	Iteration	3457	491	2699	143	142	213
	Time [s]	19195.7	2720.2	14990.4	795.2	789.2	1185.3

Table 3: The number of iteration and the net computation time for numerical calculation of scattering from N dielectric cylinders with $\varepsilon_r = 4.0$ and 32.0.

ε_r		$N = 3 \times 3$	5×5	9×9	17×17	33×33
4.0	Iteration	41	56	99	491	5075
	Time [s]	13.9	45.4	208.8	2720.2	75105.2
32.0	Iteration	23	25	34	142	1633
	Time [s]	8.2	21.1	74.4	789.2	23899.4