EXPERIMENTAL STUDY ON PERFORMANCE OF CALIBRATION TECHNIQUES FOR SUPERRESOLUTION ARRAY

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1 Introduction

Recently, superresolution techniques such as a MUSIC method[1] attract attention, as the high resolution Direction-of-Arrival(DOA) estimation using an array antenna. However, gain/phase errors of the antenna and mutual coupling among elements cause array manifold errors in MUSIC spectrum. It is necessary to calibrate array system because the MUSIC algorithm is sensitive to these errors.

The authors proposed parametric calibration technique which utilize orthogonal property between signal and noise subspace[2]. The method can estimate gain/phase error and mutual coupling matrices directly. This type of the method is required when we use the root-MUSIC, the MODE, the ESPRIT in some array configurations, and also the Spectral-MUSIC for coherent signal detection with spatial smoothing preprocessing. On the other hand, calibration technique which utilize measured array manifold is one of the well known method for the Spectral-MUSIC algorithm. This kind of method is applicable even when error model is not known. However, many sets of calibration data are often required.

In this report, we verify performance above two types of calibration technique. Experimental results show that good calibration performance can be also obtained by the parametric method for well-modeled arrays.

2 Calibration methods

2.1 Calibration method I (Parametric Calibration Technique[2])

Array manifold error is caused by the gain/phase imbalance of each element and mutual coupling. The mutual coupling is the function of antenna element position. The gain/phase of each channel may change by temperature, but we assume that it does not change during the data acquisition period. Suppose that we have several calibration data sets of a known DOA signal. For simplicity, we also assume that each data set has only one incident wave. Under these conditions, the received data vector of the m-th calibration data set $(m = 1, 2, \cdots, M)$ is defined by

$$\boldsymbol{r}_m = \boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{a}(\theta_m)s_m + \boldsymbol{n}_m, \qquad m = 1, 2, \cdots, M, \tag{1}$$

where s_m and θ_m denote the signal amplitude and DOA of m-th calibration data set respectively, and $\boldsymbol{a}(\theta_m)$ is the ideal mode vector, and $\boldsymbol{n}_m$ is a noise vector. $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$ denote the mutual coupling matrix and gain/phase matrix, respectively. In case of 4 element linear array, it can be written by

$$\boldsymbol{C} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_1 & c_4 & c_5 & c_2 \\ c_2 & c_5 & c_4 & c_1 \\ c_3 & c_2 & c_1 & c_0 \end{bmatrix}, \boldsymbol{\Gamma} = \begin{bmatrix} \gamma_1 & & & 0 \\ & \gamma_2 & & \\ & & \gamma_3 & \\ 0 & & & \gamma_4 \end{bmatrix}.$$

Considering subspace property of the data correlation matrix estimated by Eq.(1), we can obtain the following equations.

$$\begin{aligned} &\boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{a}(\theta_m) \perp \{\boldsymbol{e}_2^{(m)}, \boldsymbol{e}_3^{(m)}, \boldsymbol{e}_4^{(m)}\} \\ &\qquad \Longrightarrow (\boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{a}(\theta_m))^H \boldsymbol{e}_j^{(m)} = 0 \\ &\qquad \Longrightarrow \boldsymbol{e}_j^{(m)H}(\boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{a}(\theta_m)) = 0, \ j = 2, 3, 4, \end{aligned} \tag{2}$$

where ^{H} denotes complex conjugate transpose.

Equation (2) shows that the estimation for $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$ becomes non-linear problem. However, introducing new unknowns, we can linearize the equation as follows:

$$\left[\boldsymbol{f}_j^{(m)}\tilde{\boldsymbol{A}}(\theta_m), \boldsymbol{g}_j^{(m)}\tilde{\boldsymbol{A}}(\theta_m), \boldsymbol{h}_j^{(m)}\tilde{\boldsymbol{A}}(\theta_m), \boldsymbol{k}_j^{(m)}\tilde{\boldsymbol{A}}(\theta_m), \boldsymbol{p}_j^{(m)}\tilde{\boldsymbol{A}}(\theta_m), \boldsymbol{q}_j^{(m)}\tilde{\boldsymbol{A}}(\theta_m)\right] \begin{bmatrix} c_0\boldsymbol{\gamma} \\ c_1\boldsymbol{\gamma} \\ c_2\boldsymbol{\gamma} \\ c_3\boldsymbol{\gamma} \\ c_4\boldsymbol{\gamma} \\ c_5\boldsymbol{\gamma} \end{bmatrix} = 0, \quad (3)$$

$$\boldsymbol{f}_j^{(m)} = [e_{j,1}^{(m)*}, 0, 0, e_{j,4}^{(m)*}],\ \boldsymbol{g}_j^{(m)} = [e_{j,2}^{(m)*}, e_{j,1}^{(m)*}, e_{j,4}^{(m)*}, e_{j,3}^{(m)*}],\ \boldsymbol{h}_j^{(m)} = [e_{j,3}^{(m)*}, e_{j,4}^{(m)*}, e_{j,1}^{(m)*}, e_{j,2}^{(m)*}],$$
$$\boldsymbol{k}_j^{(m)} = [e_{j,4}^{(m)*}, 0, 0, e_{j,1}^{(m)*}],\ \boldsymbol{p}_j^{(m)} = [0, e_{j,2}^{(m)*}, e_{j,3}^{(m)*}, 0],\ \boldsymbol{q}_j^{(m)} = [0, e_{j,3}^{(m)*}, e_{j,2}^{(m)*}, 0],$$
$$\tilde{\boldsymbol{A}}(\theta_m) = diag\{\boldsymbol{a}(\theta_m)\},$$

where * denotes the complex conjugate and $\gamma = [\gamma_1, \cdots, \gamma_4]^T$.

Assuming $c_0 = \gamma_1 = 1$, the number of unknown parameters becomes 23. Three equations can be provided by Eq.(2) for each calibration data set because of 1 incident wave on 4 element array. Therefore, we can estimate all unknowns by more than 8 calibration data sets($3M \geq 23$). When this condition is satisfied, we can solve the following equation.

$$\begin{bmatrix} \boldsymbol{f}_1^{(1)}\tilde{\boldsymbol{A}}(\theta_1) & \boldsymbol{g}_1^{(1)}\tilde{\boldsymbol{A}}(\theta_1) & \boldsymbol{h}_1^{(1)}\tilde{\boldsymbol{A}}(\theta_1) & \cdots & \cdots & \boldsymbol{q}_1^{(1)}\tilde{\boldsymbol{A}}(\theta_1) \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \boldsymbol{f}_3^{(1)}\tilde{\boldsymbol{A}}(\theta_1) & \boldsymbol{g}_3^{(1)}\tilde{\boldsymbol{A}}(\theta_1) & \boldsymbol{h}_3^{(1)}\tilde{\boldsymbol{A}}(\theta_1) & \cdots & \cdots & \boldsymbol{q}_3^{(1)}\tilde{\boldsymbol{A}}(\theta_1) \\ \hline \boldsymbol{f}_1^{(2)}\tilde{\boldsymbol{A}}(\theta_2) & \boldsymbol{g}_1^{(2)}\tilde{\boldsymbol{A}}(\theta_2) & \boldsymbol{h}_1^{(2)}\tilde{\boldsymbol{A}}(\theta_2) & \cdots & \cdots & \boldsymbol{q}_1^{(2)}\tilde{\boldsymbol{A}}(\theta_2) \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \boldsymbol{f}_3^{(2)}\tilde{\boldsymbol{A}}(\theta_2) & \boldsymbol{g}_3^{(2)}\tilde{\boldsymbol{A}}(\theta_2) & \boldsymbol{h}_3^{(2)}\tilde{\boldsymbol{A}}(\theta_2) & \cdots & \cdots & \boldsymbol{q}_3^{(2)}\tilde{\boldsymbol{A}}(\theta_2) \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{f}_4^{(M)}\tilde{\boldsymbol{A}}(\theta_M) & \boldsymbol{g}_4^{(M)}\tilde{\boldsymbol{A}}(\theta_M) & \boldsymbol{h}_4^{(M)}\tilde{\boldsymbol{A}}(\theta_M) & \cdots & \cdots & \boldsymbol{q}_4^{(M)}\tilde{\boldsymbol{A}}(\theta_M) \end{bmatrix} \times \begin{bmatrix} \boldsymbol{\gamma} \\ c_1\boldsymbol{\gamma} \\ c_2\boldsymbol{\gamma} \\ c_3\boldsymbol{\gamma} \\ c_4\boldsymbol{\gamma} \\ c_5\boldsymbol{\gamma} \end{bmatrix} = 0.$$

When $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$ are estimated, the calibrated Spectral-MUSIC spectrum can be calculated by

$$P_{music}(\theta) = \frac{\boldsymbol{a}(\theta)^H \boldsymbol{W}^H \boldsymbol{W} \boldsymbol{a}(\theta)}{\boldsymbol{a}(\theta)^H \boldsymbol{W}^H \boldsymbol{E}_N \boldsymbol{E}_N^H \boldsymbol{W} \boldsymbol{a}(\theta)}, \quad (4)$$

where $\boldsymbol{W} = \boldsymbol{C}\boldsymbol{\Gamma}$.

2.2 Calibration method II (Array manifold Interpolation Technique)

We assume one wave of known DOA impinges on a linear array of N elements. The received data vector is defined by

$$\boldsymbol{r}_m = s_m \boldsymbol{W}_m \boldsymbol{a}(\theta_m) + \boldsymbol{n}_m, \quad (5)$$

where $\boldsymbol{W}_m$ is a diagonal matrix of $N \times N$ which relates to antenna errors.

When we apply eigenanalysis to the correlation matrix of the vector in Eq.(5), following relation is obtained,

$$\boldsymbol{W}_m \boldsymbol{a}(\theta_m) \boldsymbol{a}(\theta_m)^H \boldsymbol{W}_m^H \ \propto\ \boldsymbol{e}_1^{(m)} \boldsymbol{e}_1^{(m)H} \quad (6)$$
$$\Longrightarrow \boldsymbol{W}_m \boldsymbol{a}(\theta_m) \propto \boldsymbol{e}_1^{(m)}. \quad (7)$$

From Eq.(7), we obtain,

$$\boldsymbol{W}_m = c\tilde{\boldsymbol{E}}_m \tilde{\boldsymbol{A}}_m^{-1}, \quad (8)$$

where c is complex constant, and $\tilde{\boldsymbol{E}}_m = diag\{\boldsymbol{e}_1^{(m)}\}$、$\tilde{\boldsymbol{A}}_i = diag\{\boldsymbol{a}(\theta_m)\}$. The complex constant c is determined by array profit and the basis element as follows.

$$\| \boldsymbol{W}_m \|^2 = N \Longrightarrow |c| = \sqrt{N / \sum_{j=1}^{N} \left| \frac{e_{1,j}^{(m)}}{a_{m,j}} \right|^2}, \arg(c) = -\arg\left[\tilde{\boldsymbol{E}}_m \tilde{\boldsymbol{A}}_m^{-1}\right]_1, \quad (9)$$

where $e_{1,j}^{(m)}$ and $a_{m,j}$ denote the j-th element of $\boldsymbol{e}_1^{(m)}$ and $\boldsymbol{a}(\theta_m)$, respectively.

By above-mentioned procedure, we acquire enough set of $\boldsymbol{W}_m$ for required DOA range. The set becomes $\boldsymbol{W}_1, \boldsymbol{W}_2, \cdots, \boldsymbol{W}_M$. The calibrated DOA can be estimated by Eq.(4) with Eq.(10), when the correction matrix $\boldsymbol{W}$ is assumed to be smooth enough in θ.

$$\theta = t\theta_{j+1} + (1-t)\theta_j, \quad 1 \leq j < M, 0 < t \leq 1, \quad \boldsymbol{W} = t\boldsymbol{W}_{j+1} + (1-t)\boldsymbol{W}_j. \tag{10}$$

3 Experimental study

3.1 Measurement system and configuration of the experiment

In the experiments, the FM-CW based 4-ch sounder was employed. In this system, measured signal in each channel is the beat signal between transmitted and received signal. This is the real signal. To obtain a complex (analytic) signal for MUSIC analysis, we apply software I-Q transform by a DOS/V computer. Configuration of the measurement system is shown in Figure 1.

This measurement system is composed by a DOS/V personal computer for the control and a transmitting signal formation unit(Saw Tooth Wave Generator, Sweeper), a transmission and receiving antenna and FM-CW radar circuit(Power Divider, Mixer) and a receiving signal management unit.

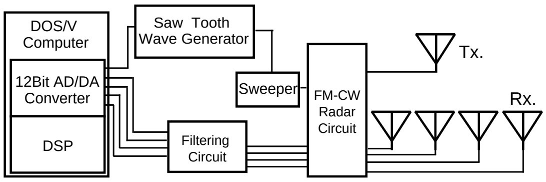

Figure 1 Measurement system configuration

In order to examine performance of above-mentioned calibration methods, DOA estimation experiment was carried out in anechoic chamber shown in Figure 2. The parameters of the experiment are listed in Table 1. The angle of the incident wave was measured every 5° with range from −90° to 90°.

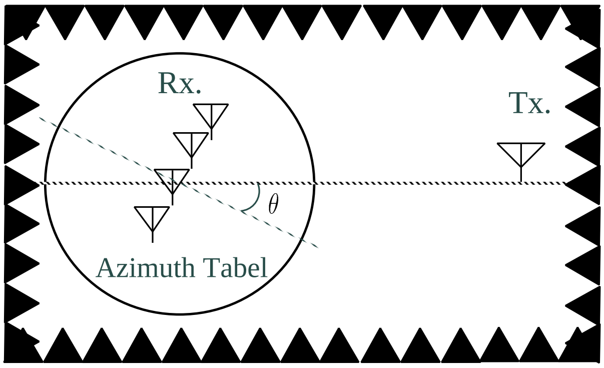

Figure 2 Experiment circumstances

Table 1 Parameters of experiment

Incident wave	1
Center Frequency	2.4 [GHz]
Sweep Frequency	2.3 [GHz] - 2.5[GHz]
Element space	$\lambda/2$
Element number	4
Snapshot	1

3.2 Experimental results

We used 13 calibration data sets ($M = 13$) for both Calibration method I and II in this experiment. DOA of used data sets are listed in Table 2. Figure 3 shows the estimation error of DOA by the MUSIC method for Calibration method I and II and without calibration.

Both Calibration methods can improve estimation precision in comparison with the results without calibration. The result of Calibration method II yields estimation performance in almost all directions. Calibration method I also shows comparable performance to method II. The characteristic of Calibration method II is good at and around the calibrated DOA. But, it needs a lot of data set for the observation area in order to get better estimation. As a result the computation process becomes very complicated and a large calibration table is required for the good DOA estimation. However, Calibration method I is different. The characteristic of Calibration method I depends only on $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$. Therefore Calibration method I does not need a table in Calibration method II.

In antenna analysis, a software called NEC2 is often used. Table 3 shows the estimated $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$ by the experiment and by the numerical simulation by NEC2. We can say that the estimated values by the experiment show a good agreement with those by NEC2.

4 Conclusions

In this report, we verified performance of two types of array antenna calibration methods for superresolution array by using known sources. In addition, we showed that the parametric calibration method has almost the same performance as the array manifold interpolation technique by the experiments. Verification of the parametric calibration method for various methods(e.g. root-MUSIC, MODE) and coherent signal detection will be reported in near future.

References

[1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas and Propagat.*, vol.AP–34, no.3, pp.276–280, March 1986.

[2] T. Arai, K. Chiba, H. Yamada, and Y. Yamaguchi "Antenna Array Calibration Using Known Sources (1)," *Technical report of IEICE*, AP2001-152,pp.53-60, Nov. 2001.

Table 2 DOA of used data set

Calibration method I	Calibration method II
−75°, −70°, −45°, −40°, −30°, −15°, 0°, 15°, 30°, 35°, 40°, 45°, 80°	−90°, −75°, −60°, −45°, −30°, −15°, 0°, 15°, 30°, 45°, 60°, 75°, 90°

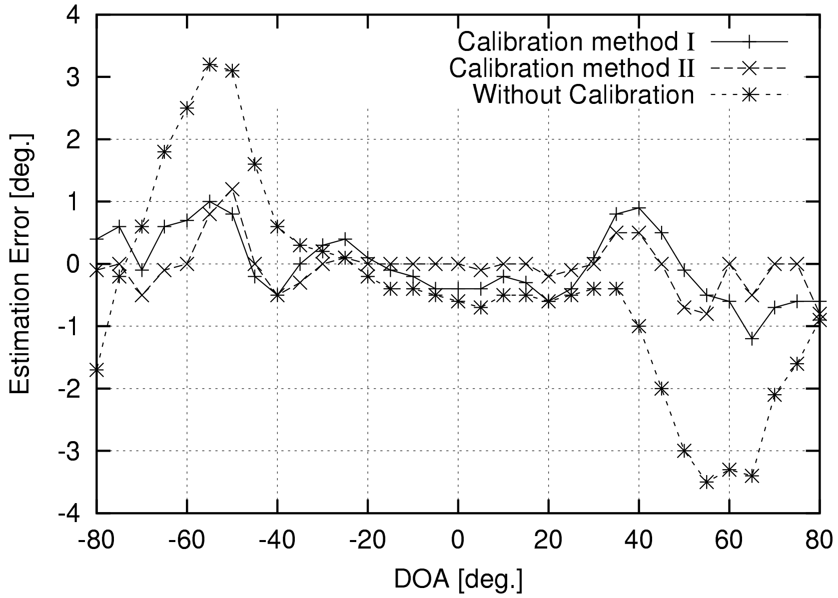

Figure 3 Estimation error of Calibration method I and II and without calibration.

Table 3 Estimation result of $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$

	Experiment Data		NEC2 Data	
	Amplitude [linear]	Phase [deg.]	Amplitude [deg.]	Phase [deg.]
γ_1	1.00	0.0	1.00	0.0
γ_2	1.30	4.7	1.00	0.0
γ_3	1.23	21.4	1.00	0.0
γ_4	0.92	3.5	1.00	0.0
c_0	1.00	0.0	1.00	0.0
c_1	0.20	54.6	0.25	66.1
c_2	0.08	−164.1	0.11	−123.3
c_3	0.06	41.4	0.07	63.0
c_4	0.62	−26.3	0.96	2.6
c_5	0.13	37.1	0.24	62.0