ON ANTENNA ARRAY CALIBRATION WITH MINIMUM KNOWN SOURCES

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1 Introduction

Application of high-resolution direction-of-arrival (DOA) estimation (*e.g.* MUSIC[1], ESPRIT, *etc*) has been widely studied in the field of communication and sensing. However, estimation precision of the method is sensitive to quality of measured data due to high-resolution capability of the method. Performance of the high-resolution method is degraded for imperfect data.

When we use an antenna array for DOA estimation in radio wave propagation, two factors mainly affect DOA estimation performance. One is channel imbalance (gain and phase error of the elements), and the other is mutual coupling among array elements. Many calibration techniques have been proposed for the high-resolution array system.

The authors have proposed a calibration technique based on signal/noise subspace property[2]. The technique can estimate calibration parameters corresponding above two error factors separately. Some sets of known DOA data (calibration data set) are required in the technique. This calibration-parameter estimation problem becomes nonlinear minimization problem. In [2], we proposed to use additional data set to linearize the problem so that the calibration technique becomes stable. However, it often becomes difficult to get additional calibration data set.

In this report, we propose a modified calibration scheme using iterative estimation. This modification makes theoretically [3] the number of required calibration data set smaller than that in [2]. Numerical and experimental results are provided to show the validity of the proposed scheme.

2 Problem Formulation

We assume that d uncorrelated waves impinge on a uniform L-element circular antenna array of radius R. The measured value of the l-th element$(l = 1, 2, \cdots, L)$ is given by

$$r_l = \sum_{k=1}^{d} s_k \mathrm{e}^{j\frac{2\pi}{\lambda} R \cos\{\theta_k - \frac{2\pi}{L}(l-1)\}} + n_l, \tag{1}$$

where λ, s_k and θ_k denote wave length, the complex amplitude and the DOA of the k-th incident wave, respectively. n_l is a noise component at the l-th element. We can express $r_l(l = 1, 2, \cdots, L)$ in vector form as follows:

$$\boldsymbol{r} = [r_1, r_2, \cdots, r_L]^T = \sum_{k=1}^{d} \boldsymbol{a}(\theta_k) s_k + \boldsymbol{n}, \tag{2}$$

$$\boldsymbol{a}(\theta_k) = [\mathrm{e}^{j\frac{2\pi}{\lambda} R \cos\{\theta_k\}}, \mathrm{e}^{j\frac{2\pi}{\lambda} R \cos\{\theta_k - \frac{2\pi}{L}\}}, \cdots, \mathrm{e}^{j\frac{2\pi}{\lambda} R \cos\{\theta_k - \frac{2\pi}{L}(L-1)\}}]^T, \tag{3}$$

$$\boldsymbol{n} = [n_1, n_2, \cdots, n_L]^T, \tag{4}$$

where ^{T} denotes transpose.

In ideal case, the received data can be written by Eq.(2), however there exist channel imbalance and mutual coupling effect in real system. Introducing a diagonal matrix $\boldsymbol{\Gamma}$ as a channel imbalance matrix and a circulant matrix $\boldsymbol{C}$ as a mutual coupling matrix, these matrices can be expressed by

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_1 & & & & \mathbf{0} \\ & \gamma_2 & & & \\ & & \gamma_3 & & \\ & & & \ddots & \\ \mathbf{0} & & & & \gamma_L \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} c_0 & c_1 & \cdots & c_{\xi-1} & c_\xi & c_{\xi-1} & \cdots & c_1 \\ c_1 & c_0 & c_1 & & & & & \\ \vdots & c_1 & c_0 & & & & \ddots & \vdots \\ c_{\xi-1} & & & & & & & \\ c_\xi & & \ddots & \ddots & \ddots & \ddots & & c_\xi \\ c_{\xi-1} & & & & & & & \\ \vdots & & & \ddots & & & c_0 & c_1 \\ c_1 & & & & c_\xi & & c_1 & c_0 \end{bmatrix}, \tag{5}$$

where, self coupling factor c_0 in $\boldsymbol{C}$ is 1, and, $\xi = \lfloor L/2 \rfloor$.

Using $\boldsymbol{C}$ and $\boldsymbol{\Gamma}$, actual received data vector $\tilde{\boldsymbol{r}}$ can be written by

$$\tilde{\boldsymbol{r}} = \sum_{k=1}^{d} \boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{a}(\theta_k)s_k + \boldsymbol{n}. \tag{6}$$

3 Calibration Technique

Using signal/noise subspace property, we can obtain the following relation for noise eigenvector of measurement data $\tilde{\boldsymbol{r}}$.

$$\boldsymbol{e}_j^{(m)H}\boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{a}(\theta_k^{(m)}) = 0, \ (m = 1, \cdots, M, \ k = 1, \cdots, d) \tag{7}$$

where ^{H} denotes complex conjugate transpose. M is the number of calibration data sets. $\boldsymbol{e}_j^{(m)}$ is the j-th noise eigenvector in m-th calibration data sets. For simplify, we demonstrate the calibration procedure for $L = 4$. In the estimation, the needed number of known sources can decrease to the number of unknowns (c_i and γ_l). The proposed iterative calibration can be summarized as follows.

Step.0

Generally, c_is have the relation of $1 = c_0 >> c_1, \cdots, c_\xi$. Therefore, we set $\hat{\boldsymbol{C}} = \hat{\boldsymbol{\Gamma}} = \boldsymbol{I}$ as an initial value.

Step.1

We can calculate γ_ls by the simultaneous equation in Eq.(8) with all sets of k and m. Note $\gamma_1 = 1$ as a reference channel.

$$\boldsymbol{e}_j^{(m)H}\hat{\boldsymbol{C}} \begin{bmatrix} a_{k,1}^{(m)} & 0 & 0 & 0 \\ 0 & a_{k,2}^{(m)} & 0 & 0 \\ 0 & 0 & a_{k,3}^{(m)} & 0 \\ 0 & 0 & 0 & a_{k,4}^{(m)} \end{bmatrix} \begin{bmatrix} \hat{\gamma_1} \\ \hat{\gamma_2} \\ \hat{\gamma_3} \\ \hat{\gamma_4} \end{bmatrix} = 0, \tag{8}$$

where $\boldsymbol{a}(\theta_k^{(m)}) = [a_{k,1}^{(m)}, \cdots, a_{k,L}^{(m)}]^T$. The $\hat{\boldsymbol{\Gamma}}$ can be constructed by the $\hat{\gamma_l}$s in the above least square estimation.

Step.2

We can calculate $\hat{c_i}$s by the simultaneous equation in Eq.(9) with all sets of k and m.

$$\boldsymbol{e}_j^{(m)H}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\hat{\boldsymbol{\Gamma}}\boldsymbol{a}(\theta_k^{(m)})\hat{c_0} + \boldsymbol{e}_j^{(m)H}\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}\hat{\boldsymbol{\Gamma}}\boldsymbol{a}(\theta_k^{(m)})\hat{c_1} + \boldsymbol{e}_j^{(m)H}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\hat{\boldsymbol{\Gamma}}\boldsymbol{a}(\theta_k^{(m)})\hat{c_2}$$

$$= 0 \tag{9}$$

$\hat{\boldsymbol{C}}$ can be updated from the estimated values of $\hat{c}_i$.

Step.3

Continue the estimations in step 1 and 2, and update $\hat{\boldsymbol{\Gamma}}$ and $\hat{\boldsymbol{C}}$ until the unknowns in these matrices converged.

The number of equations should be greater than or equal to the number of unknowns. This is the necessary condition. Therefore, we must satisfy the next inequality.

$$(L-1) + \left\lfloor \frac{L}{2} \right\rfloor \leq d(L-d)M \tag{10}$$

In Eq.(10), the left terms show the number of unknowns, and the right term shows the number of equations. In this inequality, we assume that number of incident wave is d in each calibration data set. In addition, all DOAs are different.

4 Numerical and Experimental Result

First, we show numerical results by NEC2[4]. Consider a model with uniform circular array using eight dipoles ($L = 8$). Radius of the wire is 0.5 [mm] and length is 58.0 [mm] for the dipole. The wire was divided by 11 segments and added 100 [Ω] to the feed point. Frequency of incident wave is 2.5 [GHz]. Element spacing is half-wavelength. We use 3 calibration data sets ($M = 3$). Each data set has one incident wave ($d = 1$). DOA of these waves are 0, 20 and 40 [deg.], respectively.

Furthermore, we obtained measured data of an antenna array using HP8720C. The antenna elements were monopoles having a wire of radius 0.5 [mm] and length 29.0 [mm] on ground plane. This experiment is physically equivalent to the NEC2 simulation.

MUSIC spectrum using estimated $\boldsymbol{C\Gamma}$ is evaluated by the following expression.

$$P_{music}(\theta) = \frac{(\boldsymbol{C\Gamma a}(\theta))^H(\boldsymbol{C\Gamma a}(\theta))}{\sum_{h=d+1}^{L} |\boldsymbol{C\Gamma a}(\theta)^H \boldsymbol{e}_h|^2} \tag{11}$$

Fig.1 and Fig.2 are the DOA estimation results after 100 iterations of the proposed technique. The calibrated MUSIC spectra have sharp peaks in comparison with the uncalibrated MUSIC spectra ($\boldsymbol{C\Gamma} = \boldsymbol{I}$) in both estimation.

Fig.3 shows convergence property. Fast convergence property is realized by both the simulation and the experiment.

5 Conclusions

In this report, we proposed a modified parametric calibration technique. Performance of the technique was verified both numerically and experimentally.

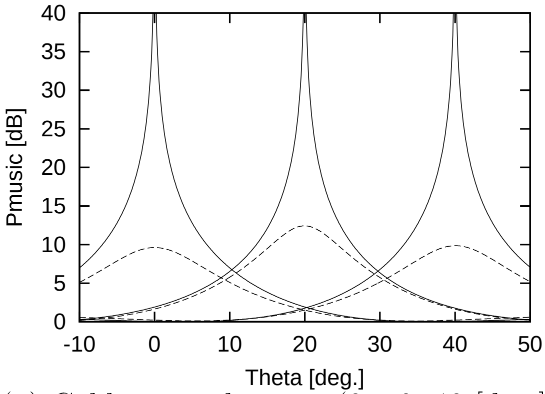
References

[1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas and Propagat.*, vol.AP–34(no.3), pp.276–280, Mar. 1986.

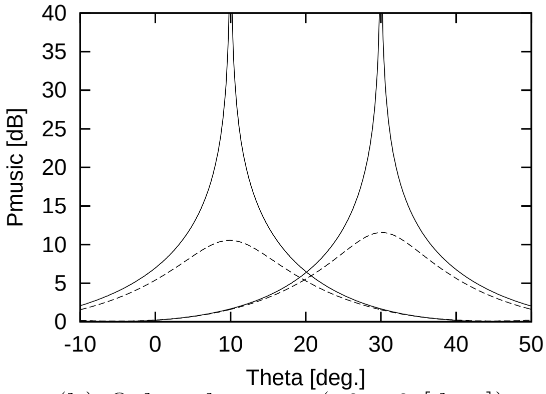
[2] T. Arai, K. Chiba, H. Yamada, Y. Yamaguchi, "Antenna Array Calibration Using Known Sources(1)," *Technical Report of IEICE*, AP2001–152, pp.53–60, Nov. 2001.

[3] R. Hara, T. Arai, H. Yamada, Y. Yamaguchi, "Antenna Array Calibration Using Known Sources(2)," *Technical Report of IEICE*, AP2001–152, pp.61–68, Nov. 2001.

[4] NEC WIN Professional TM, "Antenna Analysis Software Version 1.1," *Nattany Scientific Inc.*, California, 1997.

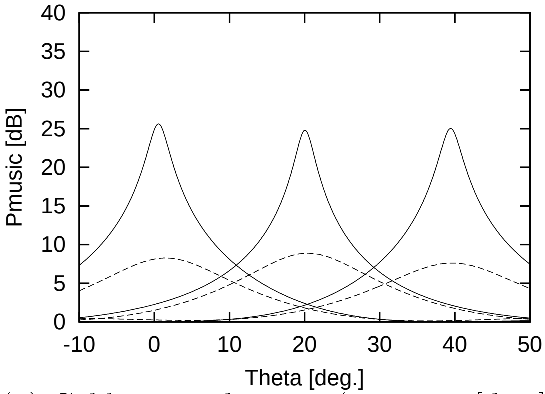


(a) Calibration data set (0, 20, 40 [deg.])

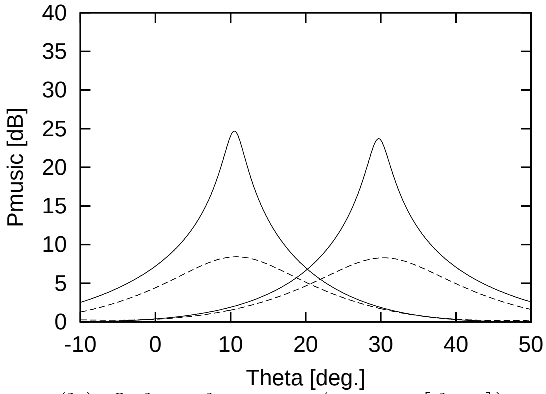


(b) Other data set (10, 30 [deg.])

Figure 1: DOA estimation results of NEC2 data (Uncalibrated MUSIC(dashed curve), Calibrated MUSIC(continuous line))

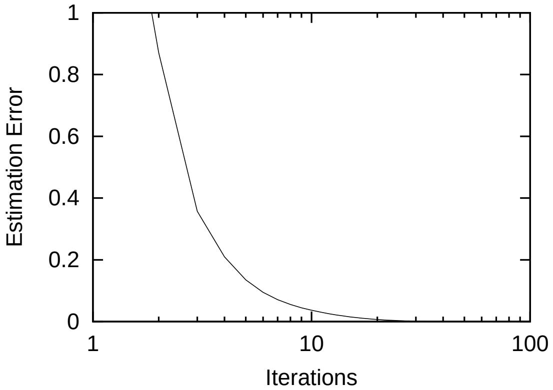


(a) Calibration data set (0, 20, 40 [deg.])

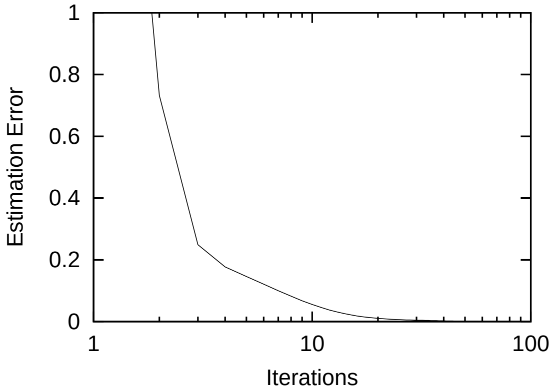


(b) Other data set (10, 30 [deg.])

Figure 2: DOA estimation results of experiment data (Uncalibrated MUSIC(dashed curve), Calibrated MUSIC(continuous line))



(a) NEC2 data



(b) Experiment data

Figure 3: Convergence Property