Improvement of Accuracy for FDTD Analysis of Antennas on Dielectric Substrate

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1 Introduction

Modern and future wireless applications require portable or compact antennas such as patch, slots, or both, on dielectric substrates. The most antenna designers use some CAD tools on the market or their own computer program. These tools are generally based on three fundamental computational methods, that is, the method of moments (MoM)[1], the finite element method or the finite difference time domain (FDTD) method[2],[3]. The FDTD method is a lately established electromagnetic computational method, however it is widely recognized that the FDTD gives an accurate result within a practical level. However, quite smaller cell requires to obtain an extremely accurate result for impedance characteristics of antennas on dielectric substrates. One reason may be considered as follows. The electromagnetic field indicates a singularity at the edge of conductor. Furthermore the antenna conductor is attached just on the dielectric surface. Therefore the electromagnetic field changes very rapidly near the edge of antenna conductor on the interface. Small but finite cell may be insufficient for modeling the above mentioned field distribution. In order to overcome this difficulties, two techniques has been proposed. One of which is a subgridding method[4] in which the fine cell is locally used for the region that the field distribution is expected to change seriously. Other technique is a so-called local cell technique in which the field behavior which has to be theoretically expected in advance, is incorporated to the FDTD local cell by using integral form of the Faraday's or Ampere's low.

The propose of this paper is to analyze the impedance characteristics of the patch antennas accurately by using the FDTD method. It has been reported that the resonance frequency of the patch antenna does not converge to the carefully measured resonance frequency ever if the extremely fine cell is used. In this calculation the cell size was shrunk to some hundred per wavelength. Therefore other useful method should be investigated. In this paper the so-called local cell method is tested. In this method, how physically-correct field distribution is introduced to the FDTD cell is essential. In this paper a quasi-static field distribution is utilized because this field is dominant in the region near the conductor edge. Then, for the strip lines on the substrate, the electromagnetic fields in the cross-section is well approximated by electrostatic and magnetostatic field, that is, it becomes transverse electromagnetic (TEM) mode. This paper applies this approximation to the field in the cross-section of the patch antenna.

This paper consists of two parts. First half describes the TEM approximation. The quasi-static field expressions are derived by assuming an appropriate current distribution on the conductor. In the second half, the FDTD formulation incorporating with the TEM approximation is introduced. The validity and effectiveness of the method are confirmed numerically and experimentally.

2 TEM approximation

This section describes the TEM approximation of the field on the antenna cross-section. If the antenna conductor has relatively long structure, the TEM mode may be dominant on its cross-section. It is known that the electromagnetic field on the cross-section of the stripline are expressed by an electrostatic potential or an magnetostatic potential. The expressions of these fields are formulated in this section.

Geometry of the problem is shown in Fig. 1. The $2a$-wide strip conductor is attached on the dielectric substrate. The conductor is assumed as infinitely long along x-axis. In this case, the electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ on $y-z$ plane are expressed by the electrostatic potential ϕ as

$$\left.\begin{aligned} \boldsymbol{E}(y,z) &= -\nabla\phi = -\frac{\partial\phi}{\partial y}\hat{y} - \frac{\partial\phi}{\partial z}\hat{z} \\ \boldsymbol{H}(y,z) &= \sqrt{\frac{\varepsilon}{\mu}}\hat{x} \times \boldsymbol{E}(y,z) \end{aligned}\right\} \qquad (1)$$

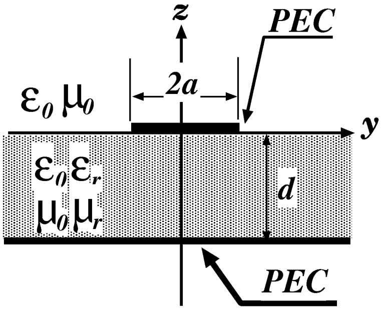

Fig. 1: Antenna on dielectric substrate

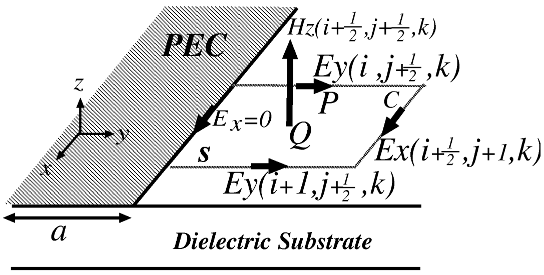

Fig. 2: The yee cell on x-y plane

These equations are convenient when a charge distribution on the conductor is known in advance. Other expressions of the fields are also possible by using a vector potential A_x, because the current on the conductor flow into x direction only. These expressions are easily derived as

$$\left.\begin{array}{rcl} \boldsymbol{H}(y,z) & = & \frac{1}{\mu}\left(\frac{\partial A_x}{\partial z}\hat{y} - \frac{\partial A_x}{\partial y}\hat{z}\right) \\ \boldsymbol{E}(y,z) & = & \sqrt{\frac{\mu}{\varepsilon}}\boldsymbol{H}(y,z)\times\hat{x} \end{array}\right\} \tag{2}$$

Eq.(1) and eq.(2) give the same electromagnetic field. However in general, the permeability of the dielectric substrate is considered as $\mu_r \simeq 1$. Furthermore, it is easily for us to consider the antenna characteristics basing on the current distribution. Therefore, eq.(2) is more convenient for our purpose. The current distribution of the strip conductor shown in Fig.1 is easily expected, and may be approximated as

$$J_x\left(y\right) = \frac{I_0}{\sqrt{y^2 - a^2}} \tag{3}$$

where I_0 is constant[6]. Then, the vector potential A_x is easily calculated as

$$A_x\left(y,z\right) = \frac{\mu_0 I_s}{2}\left[\log\left\{\frac{u_0 + w_0}{2a}\frac{Q_0 - z}{Q_0}\right\} - \log\left\{\frac{u+w}{2v}\left(1 - \frac{t}{\sqrt{1-k^2}}\right)\right\}\right] \tag{4}$$

where

$$\begin{array}{rcl} u_0 & = & \sqrt{(y-a)^2 + z^2} \quad, w_0 = \sqrt{(y+a)^2 + z^2} \\ Q_0 & = & \sqrt{\left(\frac{u_0 + w_0}{2a}\right)^2 - y^2} \quad, u = \sqrt{(y-a)^2 + (z+2d)^2} \\ w & = & \sqrt{(y+a)^2 + (z+2d)^2} \quad, v = \sqrt{a^2 + (2d)^2} - 2d \\ t & = & \frac{2(z+2d)}{u+w}, k = \frac{2y}{u+w} \end{array}$$

The electromagnetic fields can be calculated by substituting eq.(4) into eq.(2).

3 Incorporation of TEM approximation

This section describes how the TEM approximation is incorporated into FDTD cells. Fig. 2 shows Yee cell placed near the antenna edge on $x-y$ plane. In this region , the quasi-static field is considered dominant. Therefore, the electric field and magnetic field are approximated by as

$$E_y(y,z,t) \simeq E_y(P,z,t)\frac{1}{B_P}\frac{\partial A_x}{\partial y} \tag{5}$$

$$H_z(y,z,t) \simeq H_z(Q,z,t)\frac{1}{B_Q}\frac{\partial A_x}{\partial y} \tag{6}$$

where

$$B_P = \left.\frac{\partial A_x}{\partial y}\right|_P \quad , \quad B_Q = \left.\frac{\partial A_x}{\partial y}\right|_Q$$

Applying the Faraday's law

$$\begin{aligned}\oint_C \boldsymbol{E}\cdot d\boldsymbol{l} &= -\frac{\partial}{\partial t}\int_S \mu\boldsymbol{H}\cdot d\boldsymbol{S} \\ &= -\frac{\partial}{\partial t}\int_S \mu H_z\, dS \end{aligned} \tag{7}$$

to the contour C shown in Fig.2, we obtain

$$\begin{aligned}\oint_C \boldsymbol{E}\cdot d\boldsymbol{l} &= -E_x\left(i+\frac{1}{2}, j+1, k, t\right)\Delta x \\ &- E_y\left(i, j+\frac{1}{2}, k, t\right)\frac{1}{B_P}\left\{A_x\left(j+1,k\right)-A_x\left(a,k\right)\right\} \\ &+ E_y\left(i+1, j+\frac{1}{2}, k, t\right)\frac{1}{B_P}\left\{A_x\left(j+1,k\right)-A_x\left(a,k\right)\right\}+0\end{aligned} \tag{8}$$

and

$$\begin{aligned}\frac{\partial}{\partial t}\int_S \mu_0 H_z\, dS &= \frac{\Delta x\mu_0}{B_Q}\int_a^{(j+1)\Delta y}\frac{\partial H_z}{\partial t}\frac{\partial A_x}{\partial y}\,dy \\ &= \frac{\Delta x\mu_0}{A_0}\frac{\partial H_z\left(i+\frac{1}{2}, j+\frac{1}{2}, k, t\right)}{\partial t}\left\{A_x\left(j+1,k\right)-A_x\left(a,k\right)\right\}\end{aligned} \tag{9}$$

The FDTD update equation of the magnetic field can be obtained by substituting $t = n\Delta t$ into eq.(9) and by using eq.(7) and eq.(8). If we apply the integral form of Ampere's law to other yee cell, the update equation of the electric field is obtained by the similar fashion.

4 Numerical examples

In order to confirm the validity of the above mentioned method, we analyzed the planer band pass filter that is made on dielectric substrate as shown fig. 3. The thickness of substrate is $1.272mm$ and its relative permittivity $\varepsilon_r = 10.0$. The cell sizes are $\Delta x = \Delta y = \Delta z = 0.212mm$ for original FDTD and also the proposed method. In this calculation the TEM approximations are incorporated around the conductor's edge in length direction. Calculated results are shown in Fig.4 . The broken line shows the reflection coefficient calculated by the original FDTD. The solid line is the one obtained by the proposed method. It is found that the significant improvement of the calculation accuracy is achieved although the same cell size is used.

Next, we analyzed two patch antennas as shown fig. 5 and fig. 7. In the analysis of pach antenna of fig. 5, the following parameters are used $l_a = 16.0mm, 2b = 12.45mm, s = 1.95mm, 2a = 2.33mm$. The thickness of substrate is $0.795mm$ and its relative permittivity $\varepsilon_r = 2.2$. The cell sizes are $\Delta x = 0.4mm$, $\Delta y = 0.389mm$, $\Delta z = 0.265mm$ for original FDTD and also the proposed method. In the analysis of pach antenna of fig. 7, the thickness of substrate is $0.827mm$ and its relative permittivity $\varepsilon_r = 3.274$. The cell size is set as $\Delta x = 0.251mm$, $\Delta y = 0.2mm$, $\Delta z = 0.2757mm$.

In these case, the proposed method is also applied near the bold line shown in fig.5 and fig.7 . The calculated reflection coefficients are shown in fig.7 and fig.9, respectively. Improvement of the accuracy is also significant.

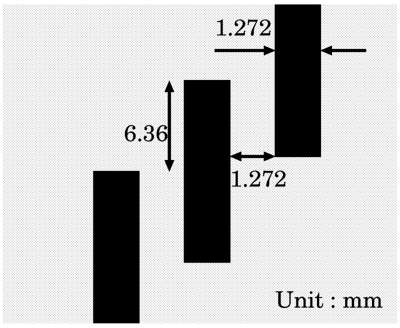

Fig. 3: Band pass filter

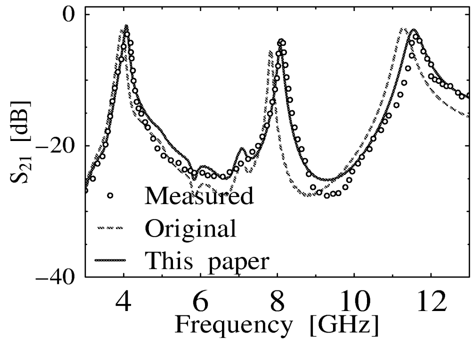

Fig. 4: S_{21} of band pass filter

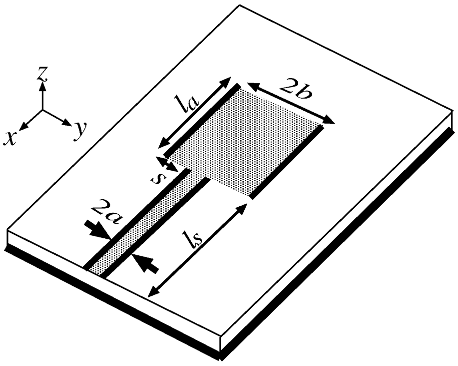

Fig. 5: Patch antenna

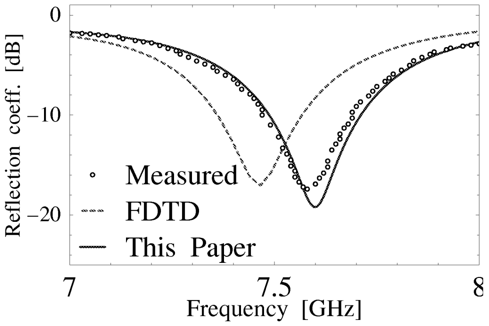

Fig. 6: Reflection coefficient of fig.5

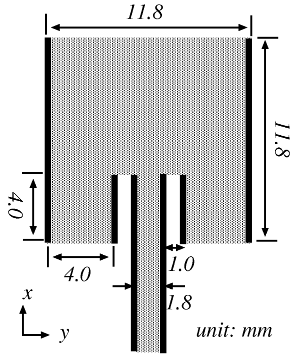

Fig. 7: Rectangular patch antenna

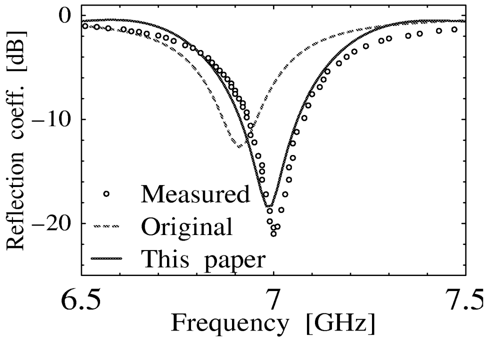

Fig. 8: Reflection coefficient of fig.7

5 Conclusions

The improvement method of FDTD calculation accuracy for antennas on substrate such as patch antennas is proposed. The TEM approximation of the field was incorporated into the FDTD update equation by using the integral form of the Faraday's low.Three examples were analyzed by using the proposed method, then it has been indicated that the significant improvement of calculation accuracy is achieved even if the same cell size as original FDTD were used.

References

[1] R. F. Harrington, "*Field Computation by Moment Method,*" FL,Kirieger, 1982.

[2] K. S. Kunz and R. J. Luebbers, "*The Finite Difference Time Domain Method for Electromagnetics,*" Boca Raton, FL, CRC Press, 1993.

[3] A. Taflove, "*Computational Electrodynamics: The Finite-Difference Time-Domain Method,*" Norwood, MA, Artech House, 1995.

[4] M. W. Chevelier, R. J. Luebbers, and V. P. Cable, "*FDTD Local Grid with Material Traverse,*" IEEE Trans. Antennas Propagat. vol.43, no.3, pp.411-421, March 1997

[5] D. B. Shorthouse and C. J. Railton, "*The Incorporation of Static Field Solutions Into the Finite Difference Time Domain Algorithm,*" IEEE Trans. Microwave Thory Tech., vol.40, pp.986-994, May 1992.

[6] R. E. Collin, "*Field Theory of Guided Wave,*" 2-nd ed. chap. 3., IEEE Press., 1991

[7] D. M. Sheen, S. M. Ali, M. D. abouzahra and J. A. Kong, "*Application of the three-dimensional Finite-Difference Time-Domain method to the analysis of planar microstrip circuits* ," IEEE Trans. Microwave Thory Tech., vol.38, pp.849-857, 1990.