

# RADIATION MODES FOR EXPRESSING RADIATED FIELDS IN MULTILAYER MMIC CONFIGURATION

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**Abstract** - Radiation modes for general multilayer configuration are defined and their specific functional expressions are given. Radiated fields from arbitrary field sources can be uniquely expanded in terms of these radiation modes. Thus, once the source fields are obtained by some numerical means such as, e.g., the FD-TD method, the radiated fields can be calculated easily. Practical application of this radiation mode theory is the evaluation of radiation from multilayered microwave integrated circuit elements.

## 1. Introduction

In microwave and millimeter wave integrated circuits (MIC) or monolithic microwave integrated circuits (MMIC), radiation from circuit elements or line discontinuities is one of important concerns to MMIC designers, since it causes often inter-element coupling, field leakage, circuit function deterioration, etc. Analytical or numerical calculation of radiation from MIC or MMIC, however, is not so easy matter generally. The major reason is the existence of substrate of practically infinite extent, which is different essentially from the problem of usual antennas with finite-sized substrate.

The author presented the theory of radiation modes and radiation mode expansion of radiated fields, and proposed a method of calculating radiated fields from MIC elements in combination with the FD-TD method [1]-[3]. However, these works were all assuming a single layer substrate and so the extent of its application was somewhat limited. In this paper, the radiation modes were generalized to those for multilayer configuration; the number of layers is now arbitrary. This generalization may be straightforward in theory but it is to be noted that the accomplishment of the theoretical details and the total algorithm utilizable for practical application is not so simple.

## 2. Radiation modes and general expressions of fields

The geometry of the problem and the coordinates are shown in Fig. 1. The region  $z \geq z_N$  is the free space with dielectric constant  $\varepsilon_0$ , and the region  $0 \leq z \leq z_N$  consists of  $N$  layers of dielectric constant  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N$  and thickness  $d_1, d_2, \dots, d_N$ , respectively, with plane boundaries at  $z_1, z_2, \dots, z_N$ .

Radiation modes are divided into two types. One is space modes that are propagating into the space region and the other is substrate modes that are propagating

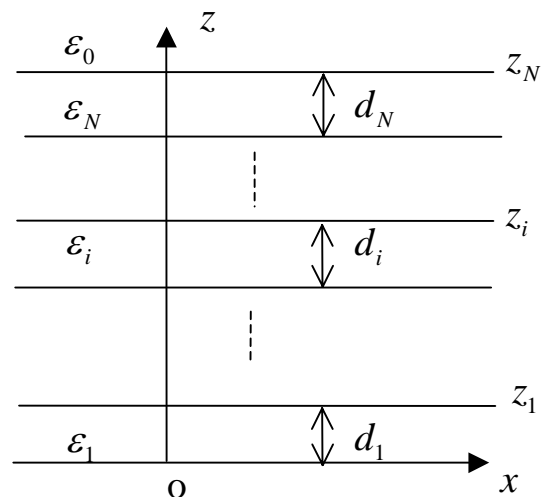


Fig. 1. Multilayer configuration.

into the multilayer region as cylindrical guided modes. The difference of these two modes is that the space modes have a standing wave form in the  $z$  direction, while the substrate modes have an evanescent wave form in the region  $z \geq z_N$ . For each type mode, we have two kinds of independent modes, the field composition of which can be chosen variously. We choose here as these modes the following two modes: one is the E type for which  $H_z = 0$ , and the other is H type for which  $E_z = 0$ . Each mode belonging to each type has its respective wavenumber spectrum.

General expressions of radiation modes with, for example, subscript  $p$  are

$$\mathbf{E}_p = \tilde{\mathbf{e}}_p e^{-j\alpha_p y - j\beta_p x}, \quad \mathbf{H}_p = \tilde{\mathbf{h}}_p e^{-j\alpha_p y - j\beta_p x} \quad (1)$$

and the expansion of electric field  $\mathbf{E}$  in terms of radiation modes is

$$\mathbf{E} = \int_{-\infty}^{\infty} d\alpha_p e^{-j\alpha_p y} \sum_{l=e,h} \left[ \sum_m a_m^l(\alpha_p) \tilde{\mathbf{e}}_m^l(\alpha_p, z) e^{-j\beta_p x} + \int_0^{\infty} b^l(\gamma_p, \alpha_p) \tilde{\mathbf{e}}_p^l(\gamma_p, \alpha_p, z) e^{-j\beta_p x} d\gamma_p \right] \quad (2)$$

Magnetic field  $\mathbf{H}$  is expressed similarly. The first term in the bracket [ ] corresponds to substrate modes with mode number  $m$ , while the second term corresponds to space modes with  $\gamma_p (= \sqrt{k_i^2 - \alpha_p^2 - \beta_p^2})$  being the spectrum in the  $z$  direction. The superscript  $l$  takes e (E type) or h (H type). The  $z$  components of  $\tilde{e}_p$  and  $\tilde{h}_p$  satisfy

$$\left( \frac{\partial^2}{\partial z^2} + k_i^2 - \alpha^2 - \beta^2 \right) \begin{Bmatrix} \tilde{e}_z \\ \tilde{h}_z \end{Bmatrix} = 0 \quad (3)$$

where subscript  $p$  is omitted for simplicity.

### 3. Functional expressions and orthonormality property

The  $z$  component of E type substrate mode is given as

$$\begin{aligned} \tilde{e}_z &= C_0^e(\alpha) e^{-\bar{\gamma}_0(z-z_N)}, & z \geq z_N \\ &= C_i^e(\alpha) \sin \gamma_i z + D_i^e(\alpha) \cos \gamma_i z & (i = 1 \sim N), z_i \geq z \geq z_{i-1} \end{aligned} \quad (4)$$

where  $i$  is the layer number and  $\bar{\gamma}_0 = \sqrt{\alpha_p^2 + \beta_p^2 - k_0^2}$ . Other components are derived from Maxwell's equations and their specific functional forms are omitted here for brevity. On the other hand, the  $z$  component of E type space mode is given as

$$\tilde{e}_{zi} = F_i^e(\alpha) \sin \gamma_i z + G_i^e(\alpha) \cos \gamma_i z \quad (i = 1 \sim N + 1) \quad (5)$$

where  $i = N+1$  stands for the region  $z \geq z_N$ . Similar expressions are employed for the  $\tilde{h}_z$  component of H type mode, which are omitted for brevity.

Another important property on mode concept is the orthonormality property, which is defined here to be

$$\frac{1}{2} \int_S (\mathbf{e}_{tm} \times \mathbf{h}_{tm}^*) \cdot \mathbf{i}_x dS = \frac{\beta_n}{|\beta_n|} \delta_{mn} \delta(\alpha_m - \alpha_n) \quad (6)$$

for the case of E type substrate modes and

$$\frac{1}{2} \int_S (\mathbf{e}_{tp} \times \mathbf{h}_{tp}^*) \cdot \mathbf{i}_x dS = \frac{\beta_q}{|\beta_q|} \delta(\alpha_p - \alpha_q) \delta(\gamma_{0p} - \gamma_{0q}) \quad (7)$$

for the case of E type space modes, where S is a  $x=\text{constant}$  plane surface,  $\mathbf{i}_x$  is the unit vector in the x direction,  $\delta_{mn}$  stands for the Kronecker delta function, and  $\delta(\cdot)$  stands for the Dirac delta function. Similar orthonormality property is defined for H type substrate modes and H type space modes, which are omitted here.

When the necessary boundary conditions at  $z = z_i$  ( $i = 0, 1, \dots, N$ ) are applied for, e.g., E type substrate modes, all coefficients  $C_i^e$  and  $D_i^e$  in eq.(4) are related to just one unknown  $D_1^e$ , which is finally determined by using orthonormality property (6), and at the same time characteristic equations are also derived, which can be solved numerically by means of simple bisection method to obtain specific z-direction spectrum corresponding to respective mode. All coefficients  $F_i^e$  and  $G_i^e$  in eq.(5) are also determined if boundary conditions at  $z = z_i$  ( $i = 0, 1, \dots, N$ ) and orthonormality property (7) are used.

#### 4. Determination of mode expansion coefficients

Once the source fields in a limited area are obtained, mode expansion coefficients  $a_m^l(\alpha)$  and  $b^l(\gamma, \alpha)$  can be calculated. One of the effective means for obtaining such source fields is to invoke the FD-TD method to calculate field distribution  $(\mathbf{E}_0, \mathbf{H}_0)$  on a closed surface  $S_0$  that encloses within it all circuit elements or discontinuities causing sources of radiated fields. Thus, from reaction integral of  $(\mathbf{E}_0, \mathbf{H}_0)$  with the corresponding radiation modes  $(\mathbf{E}_p, \mathbf{H}_p)$ , mode expansion coefficients can be calculated, for example, as follows:

$$a_m^l(\alpha) = -\frac{1}{4} \int_{S_0} (\mathbf{E}_p^{l*} \times \mathbf{H}_0 + \mathbf{E}_0 \times \mathbf{H}_p^{l*}) \cdot \mathbf{i}_n dS \quad (8)$$

where  $\mathbf{i}_n$  is the inward normal on  $S_0$ . This expression is derived by applying Lorentz reciprocity principle for the region enclosed by  $S_0$  inside and appropriate parallelepipedon surface outside. The equation (8) can be said to be very general expression in that reflection and transmission coefficients of guided modes are calculated also from the same reaction integral, if guided mode fields are used instead of  $(\mathbf{E}_p, \mathbf{H}_p)$ .

#### 5. Radiation patterns and radiated power

Expressions for calculating far fields are obtained from expression (2) by invoking the conventional method of saddle point technique. In the case of evaluation of radiation into substrate region, the technique is applied to  $\alpha_p$  integral only, while in the case of

calculating radiated fields in the space region, the saddle point technique is applied first to  $\alpha_p$  integral and next to  $\gamma_p$  integral.

Though details of derivation are omitted here, the integral expression for calculating radiated power in the space region  $P_s$ , for example, can be derived as the following:

$$P_s = \int_0^\pi \sin^2 \theta d\theta \int_0^{2\pi} |\cos \phi| d\phi k_0^2 (|b^e|^2 + |b^h|^2) \Big|_{\alpha=\alpha_0, \beta=\beta_0} \quad (9)$$

where  $\theta$  and  $\phi$  are those of spherical coordinates and  $\alpha_0 = k_0 \sin \theta \sin \phi$  and  $\beta_0 = k_0 \sin \theta \cos \phi$ .

## 6. Conclusion

Numerical technique for evaluating radiated fields (patterns and power) emitted from multilayer MMIC elements is presented. In this technique, radiation modes for multilayer configuration are defined and used to express radiated fields; the most important point is that no Green's functions are used, enabling to make numerical calculation very easy. Multilayered structure is expected to use, for instance, in lowering loss of microstrip lines and spiral inductors in MMIC[4]. Bend discontinuities of such low-loss multilayer microstrip line will be dealt with as numerical examples.

## References

- [1] N. Morita, "A new formulation for radiated fields using radiation mode expansions and its application to radiation from microstrip antennas," IEICE Trans. on Electronics, Vol.E77-C, No.11, pp.1795-1801, Nov. 1994.
- [2] N. Morita, "Radiation mode expansion theory combined with FD-TD technique for rigorous evaluation of transmission, reflection, and radiation from microstrip circuit elements," URSI XXVII General Assembly, BD1-P2, p.632, Lille, France, Aug.-Sept. 1996.
- [3] N. Morita and N. Hosoya, "Calculation of radiation from MMIC elements using radiation mode expansion formulation combined with FD-TD analysis," 2000 Int. Symp. on Antennas and Propagation, 1F3-3, Fukuoka, Aug. 2000.
- [4] I. J. Bahl, "High-Q and low-loss matching network elements for RF and microwave circuits," IEEE Microwave Magazine, Vol.1, No.3, pp.64-73, Sept.2000.