MICROSTRIP PATCH ANTENNAS WITH FRACTAL BOUNDARY

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1. Introduction

Solving the Laplace equation on a set is an old mathematical problem [1]. Physically we can interpret it as a vibration of string, membrane, behavior of resonator and also a microstrip patch antenna. Problem wasn't solved analytically yet except some trivial arrangement (rectangle, circle..). To solve such equation we need to find an eigenpar which consist of a set of eigenvalues and eigenfunctions. Such eigenfunctions can represent distribution of accoustic pressure or electric field for example. Eigenvalues correspond to resonant frequency. In this paper we present solutions of Helmholz wave equation on 2D set with fractal boundary. Recent works show that fractal boundary strongly modify the behavior of resonators [2], [3] - distribution of eigenfunctions and changes of vibrational spectrum (eigenvalue distribution). Cavity model of microstrip patch antenna is considered.

2. Microstrip patch antennas with fractal boundary

2.1 Cavity model review

The Helmholz wave equation for TM modes of the microstrip patch antenna of the shape Ω lying in xy plane has the form [4]

$$\begin{aligned} (\Delta + k_d^2)E_z &= 0 \\ \frac{\partial E_z}{\partial n} &= 0 \end{aligned} \tag{1}$$

where $k_d = \omega\sqrt{\mu\varepsilon}$ being the wavenumber in dielectric and $\frac{\partial E_z}{\partial n} = 0$ is a Neumann condition on the boundary $\partial\Omega$ (perfect magnetic wall). The solution of equation (1) is so called *eigenpar* - collection of eigenvalues k_n^2 a and eigenfunctions ψ_n. Electric field is assumed to be only in the z-direction, $E_z = \psi_n$. Index n means the mode number. Resonance condition is

$$k_n^2 = k_d^2 \tag{2}$$

and resonant frequencies

$$f_n = \frac{k_n}{2\pi\sqrt{\mu\varepsilon}} \tag{3}$$

2.2 Koch microstrip patch antenna

This antenna is based on the Koch-snowflake fractal (fractal dimension of the boundary is $D_{bound} \doteq 1.26$) obtained by recursive algorithm. Equation (1) is solved using finite element method for iterations 1-5 (see Fig. 1), d was choosen to be 100 mm. The spectrum[1] (eigenvalue distribution) of this structures is shown on Fig. 2, parameter in this graph is a fractal iteration.

Patch	Koch0	Koch1	Koch2	Koch3	Koch4	Koch5	Cir1	Cir2
Area[cm^2]	43.3	57.7	64.1	67	68.3	68.8	68.3	100.7

Tab. 1 The areas of discussed microstrip patch antennas (d=100 mm)

[1] Calculated spectrum is discrete but in figures is shown as continuos for lucidity

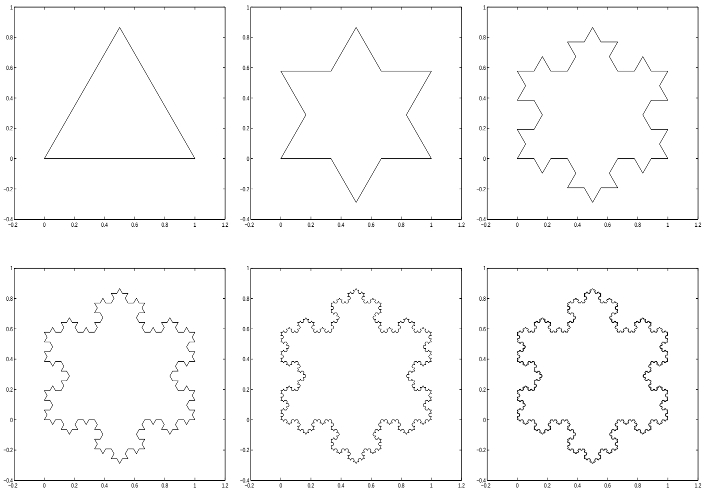

Fig. 1 Koch snowflake iterations (Koch0-Koch5), fractal boundary dimension $D_{bound} \doteq 1.26$

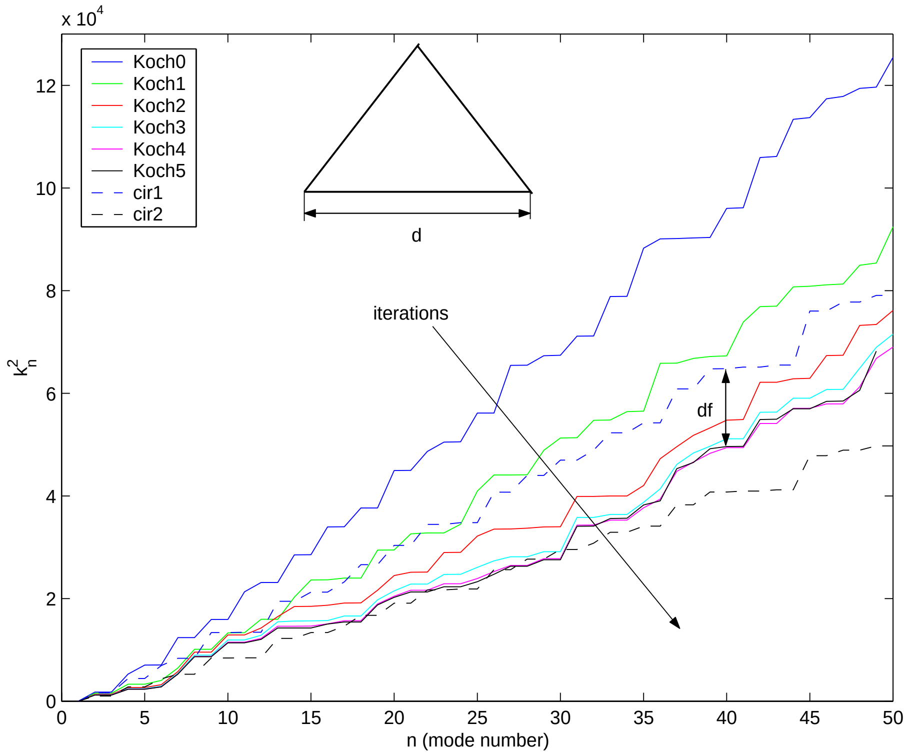

Fig. 2 Spectrum of Koch snowflake patch resonators (iterations 0-5)

Another circular-shaped patches were added into graph for confrontation. Cir1 patch has the same area as Koch4 and Cir2 patch has the radius given by cimcurscribed circle around snowflake (the area is hence necessarily bigger than any iteration). We can compare behavior of patch antennas with the same area but with different boundary (euclidean vs. fractal). Looking into Fig. 2 we can observe that although Koch4 and Cir1 have the same area, Koch4 resonates on lower frequencies (see difference df in Fig. 2). Such phenomenon is rather observed when comparing higher modes, because lower modal functions doesn't "see" boundary irregularities, its scale is too rough. Influence of fractal geometry manifests on the spectrum character also (frequencies are compressed to lower ones).

2.3 "Drum" microstrip patch antenna

Another interesting structure is created aplying generator from Fig. 3 to rectangle. The fractal dimension

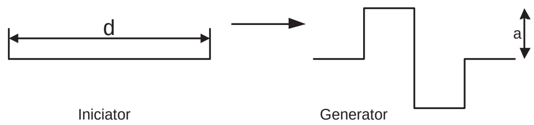

Fig. 3 Fractal drum generator

can be controlled ($D_{bound} = 1 \div 1.5$) by varying parameter a using equation $D_{bound} = \frac{\log(4+4a)}{\log\ 4}$.

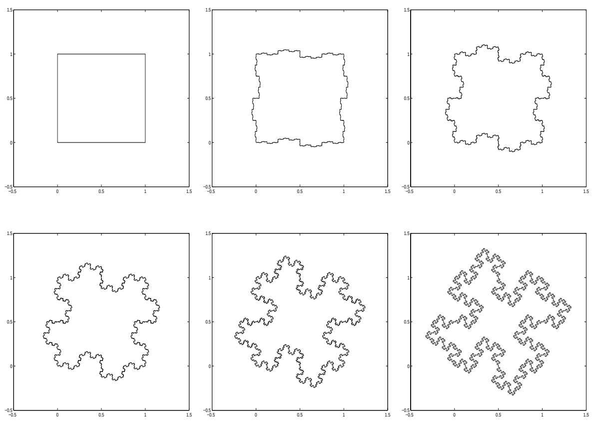

Fig. 4 Fractal drum with $D_{bound} = 1.0 - 1.5$ (step 0.1), iteration=3

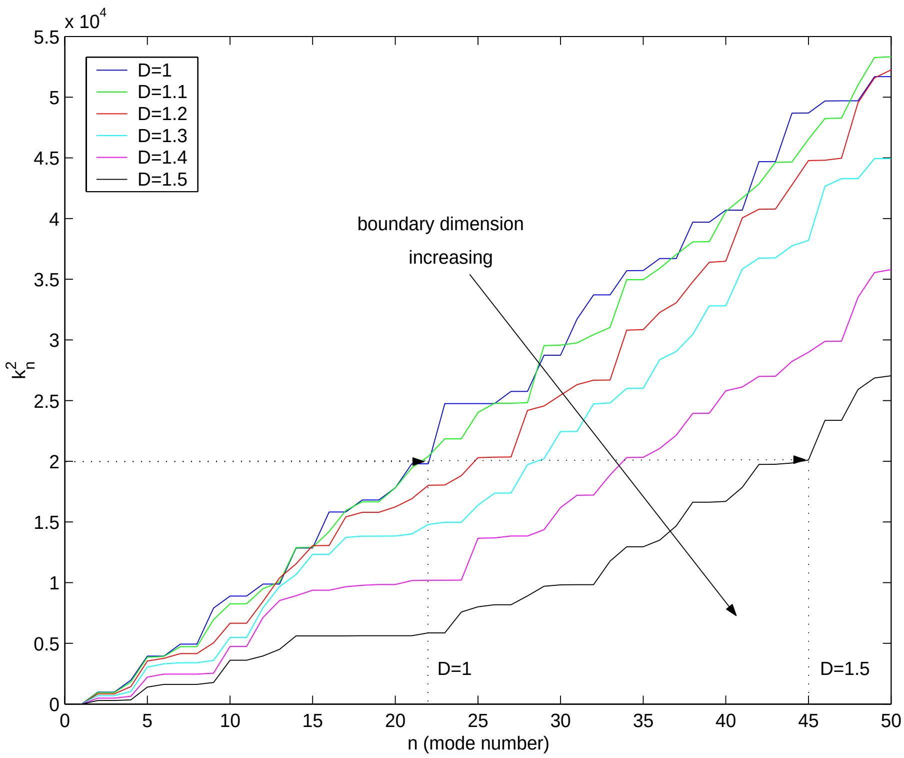

Fig. 5 Spectrum of fractal drum patch antennas with different boundary dimension and constant iteration=3

Again, we can see influence of the fractal boundary to eigenvalue distribution. In this case fractal iteration is constant (=3) and the variable is the fractal dimension of the boundary D_{bound}. From the Fig. 5 follows that with the increasing fractal dimension of the boundary the resonant frequencies decreases and spectrum behaves more "strange". Of course the density of states (modes) also increases (until a given eigenvalue k_n^2 there is more and more excited modes, see dotted arrows in Fig. 5).

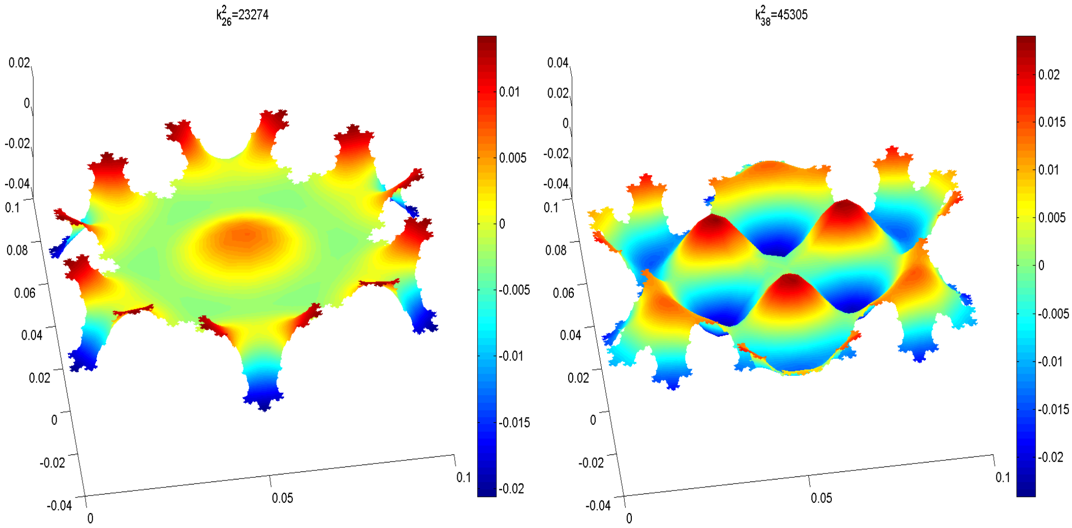

Fig. 6 Localized modes ψ_n for Koch5 patch, mode numbers n=26 resp. n=38

3. Conclusion

Behavior of resonators with fractal boundary was introduced computing the vibrating spectrum, a set of eigenvalues, which are related to the resonant frequencies. Using this method and considering the cavity model, main parameters of microstrip patch antennas with fractal boundary can be studied (resonant frequencies, input impedance, radiating pattern). Our partial results show that fractal boundary strongly modify the behavior of resonators, primarily there is a frequency shift when comparing the higher modes. Resonant frequencies are shifted both in two presented cases: creating a fractal boundary by iterations and increasing fractal dimension of resonator's boundary. Another interesting fact is the localization of certain high order modes (two interesting cases are shown at Fig. 6 for information). All these properties can be useful for design a novel types of microstrip patch antenna structures.

4. References

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5. Acknowledgement

This work has been supported by grant "Novell antenna structures", GACR 102/01/1358 and by the research program J04/98:212300014