A WORLDWIDE RAIN ATTENUATION PREDICTION METHOD WHICH USES SIMPLIFIED MOUPFOUMA DISTRIBUTION AND REGIONAL CLIMATIC PARAMETERS

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1. Introduction

The most serious transmission impairment is caused by the rain attenuation in radio systems using frequencies above about 10 GHz. So, in the system design, it is required to predict the rain attenuation distributions on radio links. Although rain attenuation distributions are estimated by using the rain rate distributions that can be obtained in the world, the standardized view has not yet been obtained what statistical distribution should be used for approximation of rain rate and rain attenuation distributions.

In Japan, the Gamma distribution is used on terrestrial links and lognormal distribution is used on satellite links. But both distributions have large errors if wide cumulative probability range is considered. For rain attenuation distribution prediction, it is necessary to use a statistical distribution, that gives good approximation for rain rate and attenuation distributions in wide cumulative probability range, and for that the relations between distribution parameters and mean and standard deviation of the distribution can be easily calculated. One of the distributions which satisfy this requirement is the simplified Moupfouma distribution[1] (termed as M distribution[2]).

In this paper, a new prediction method for the worldwide rain attenuation distribution on terrestrial link is proposed. This new method uses the M distribution and the rain rate spatial correlation, in which regional climatic parameters such as the thunderstorm ratio are considered. This new method was compared with Rec. ITU-R P.530-8 method[3] that is currently used as the worldwide standard prediction method. From the results, it is shown that significant improvements to existing methods are obtained by this newly proposed prediction method.

2. Worldwide databank of rain attenuation on terrestrial links (KIT data bank)

For analysis in this study, a databank of rain attenuation on terrestrial links was newly constructed from many literatures. It contains 82 data sets in 22 countries. Those are : 1.Kjeller(NO), 2.Stockholm(SE), 3.Copenhagen(DK), 4.Dubuna(RU), 5.Warsaw(PL), 6.Mendlesham(GB), 7.Chilbolton(GB), 8.Leidschendam(NL), 9.Kolberg(DE), 10.Darmstadt(DE), 11.Praha(CZ), 12.Paris(FR), 13.Dijon(FR), 14.Graz(AT), 15.Torino(IT), 16.Ghilardona(IT), 17.Fucino(IT), 18.Rome(IT), 19.Lisbon(PT), 20.Athens(GR), 21.Korhogo(CI), 22.Brazzaville(CG), 23.Merrimack Valley(US), 24.Holmdel(US), 25.Palmetto(US), 26.Fortaleza(BR), 27.Governador(BR), 28.Rio de Janeiro(BR), 29.Porto Alegre(BR), 30.Koganei(JP), 31.Shakujii(JP), 32.Musashisakai(JP), 33.Yokosuka(JP), 34.Nagoya(JP), 35.Owase(JP), 36.Xinxiang(CN), 37.New Delhi(IN), 38.Guwahati(IN), 39.Calicut(IN), 40.Riyadh(SA).
This databank also includes the average annual total rainfall, the average number of thunderstorm days, and the thunderstorm ratio, etc.

3. The prediction method using M distribution[2]

The probability distribution function F(R) of the M distribution is eqn.(1). Parameters p, u are positive values. R^* is the lower limit value of R and the solution of eqn.(2) from the condition that $F(R^*) = 1$, and an approximation equation for R^* was obtained[4]. Mean m and variance σ^2 of M distribution are given as eqns.(3) and (4), respectively. $E_1(x)$ is the integral exponential function.

From eqns.(3) and (4), the distribution parameter u is represented by eqn.(5). Substituting eqn.(4) for eqn.(5), eqn.(6) is obtained. Because R^* is obtained from the solution of eqn.(6), u is obtained by eqn.(5), and p is obtained by eqn.(3). Therefore if mean and variance of distribution are found, the distribution parameters can be calculated[4]. The specific attenuation γ_R (dB/km) is obtained from the rain rate R using the power-law relationship as eqn. (7). The parameters γ and n are given by Rec. ITU-R P.838[5].

$$F(R) = \frac{p}{R}\exp(-uR) \quad (R^* \leqq R < \infty) \quad (1), \qquad p = R^* \exp(uR^*) \quad (2), \qquad m = p\{\exp(-uR^*) + E_1(uR^*)\} \quad (3)$$

$$\sigma^2 = p(R^* + 2/u)\exp(-uR^*) - m^2 \quad (4), \qquad u = \frac{2R^*}{\sigma^2 + m^2 - R^{*2}} \quad (5), \qquad \gamma_R = \gamma R^n \quad (7)$$

$$g(t) = t\left[1 + E_1\left\{\frac{2t^2}{1 + (m/\sigma)^2 - t^2}\right\} \times \exp\left\{\frac{2t^2}{1 + (m/\sigma)^2 - t^2}\right\}\right] - m/\sigma = 0 \quad \text{where} \quad t \equiv R^*/\sigma \quad (6)$$

In the first place, if the random variable X_1 is R^n, the probability density function of X_1, $f_1(X_1)$ is represented by eqn.(8) using the parameters p and u of rain rate distribution, where $X_1^* = R^{*n}$. Mean m_1 and variance σ_1^2 of this distribution are given as eqns.(9) and (10), respectively. $\Gamma(\nu, x)$ is the incomplete gamma function of the second kind. Therefore, if the random variable X is γR^n, when X^* is equal to γR^{*n}, mean m_x and variance σ_x^2 of this distribution are given as eqns.(11) and (12), respectively.

$$f_1(X_1) = \frac{p}{nX_1}\left(u + X_1^{-1/n}\right)\exp(-uX_1^{1/n}) \quad (X_1^* \leqq X_1 < \infty) \quad (8), \qquad m_X = \gamma m_1 \quad (11)$$

$$m_1 = \frac{pn}{u^{n-1}}\left\{\frac{\Gamma(n, uR^*) - (uR^*)^{n-1}\exp(-uR^*)}{n-1}\right\} + R^{*n} \quad (9), \qquad \sigma_X^2 = \gamma^2 \sigma_1^2 \quad (12)$$

$$\sigma_1^2 = pu^{1-2n}\{\Gamma(2n, uR^*) + \Gamma(2n-1, uR^*)\} - m_1^2 \quad (10)$$

In the second place, it is assumed that the statistical characteristics for R in each locations is the same, mean m_z and variance σ_z^2 of the distribution of rain attenuation Z(dB) for a propagation path of length D(km) can be calculated from eqns.(13) and (14), respectively. $f(\alpha, D)$ is the function of the rain rate spatial correlation function $\rho(\alpha, d)$, where d(km) is the distance and α is the spatial correlation parameter.

$$m_Z = Dm_X \quad (13), \qquad \sigma_Z^2 = \frac{D^2}{f(\alpha, D)}\sigma_X^2 \quad (14)$$

In Japan, it is known that the spatial correlation of rain rate can be approximated by eqn.(15)[6]. Manabe has pointed out that, in Europe, eqn.(16) has larger regional dependence and larger correlation with CCIR rain climatic zones than eqn.(15)[7]. If these spatial correlation functions are used, $f(\alpha, D)$ are given by eqns.(17) and (18).

$$\rho(\alpha_{1/2}, d) = \exp(-\alpha_{1/2}\sqrt{d}) \quad (15), \qquad \rho(\alpha_1, d) = \exp(-\alpha_1 d) \quad (16)$$

$$f(\alpha_{1/2}, D) \equiv \frac{\alpha_{1/2}^2 D}{4\left\{1 - \frac{6}{\alpha_{1/2}^2 D} + \exp(-\alpha_{1/2}\sqrt{D})\left(2 + \frac{6}{\alpha_{1/2}\sqrt{D}} + \frac{6}{\alpha_{1/2}^2 D}\right)\right\}} \quad (17)$$

$$f(\alpha_1, D) \equiv \frac{(\alpha_1 D)^2}{2\{\alpha_1 D + \exp(-\alpha_1 D) - 1\}} \quad (18)$$

Therefore Z^*, p_z and u_z can be calculated with m_z and σ_z^2, and the rain attenuation distributions can be estimated by using the rain rate distributions.

Because the structure of rain is influenced by regional climate, it is expected that α values are different from one region to another. Actually, from above data bank, we calculated inversely appropriate α in each location using M distribution and eqns.(15) or (16). It was found that α has strong regional dependence. Therefore, it is necessary to estimate appropriate α in each location for the establishment of the propagation prediction method.

4. Examination of spatial correlation parameter α for 1-min. rain rate

To establish the physically correct prediction method, we excluded 3 data sets without one-minute rain rate, 2 data sets whose measurement duration is different between rain rate and rain attenuation observations, and 13 data sets with $\alpha=0$ (this means that rain does not change with distance, and this is not realistic in practical situations).

If the propagation distance is too short, the spatial correlation characteristics does not affect attenuation distribution. This means that the error may be increased when we calculate inversely appropriate α. So, from rain rate measurements in Japan[6] and Italy[8], $\alpha_{1/2}$ and α_1 values are calculated and are shown in Fig.1 as ●. In this figure, ○ is α for $d \geqq 5$km and × is α for d<5km calculated inversely from KIT data bank. From Fig.1 it was found that the tendency of ○ correspond with that of ●. Therefore we decided to use the data sets only for $d \geqq 5$km for estimation of α.

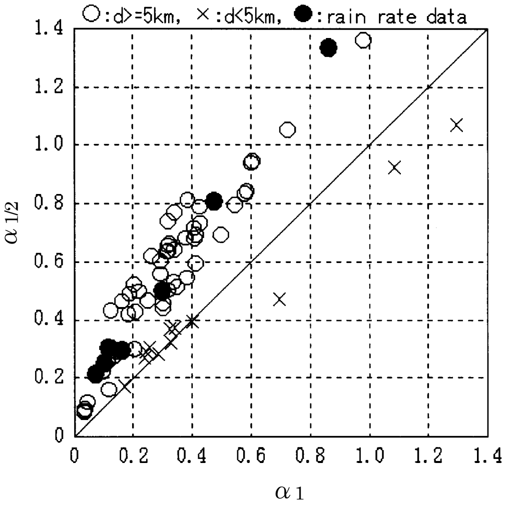

Fig 1. Relationship between $\alpha_{1/2}$ and α_1.

5. Examination of the relation between α and the regional climatic parameters

The range of the calculated $\alpha_{1/2}$ and α_1 in the above were from 0.083 to 1.362, and from 0.033 to 1.282 respectively. We carried out multiple regression analysis by using 10 regional climatic parameters that seem to influence α. As the result, a prediction equation for α was obtained as eqns.(19) and (20).

$$\alpha_{1/2} = -0.0143\ln(p) + 0.3126\ln(u) - 0.0087f + 0.0019|\phi| - 7.271\times10^{-5}M - 0.0052D_{th} - 2.646\times10^{-4}M_m + 0.0094R_{0.01} + 2.2\times10^{-4}R_{0.001} - 0.1543\ln(\beta) + 1.1839 \quad (19)$$

$$\alpha_1 = -0.0997\ln(p) + 0.4535\ln(u) - 0.0050f + 0.0022|\phi| - 5.353\times10^{-5}M - 0.0062D_{th} - 1.862\times10^{-4}M_m + 0.0109R_{0.01} + 0.0022R_{0.001} - 0.0855\ln(\beta) + 0.8576 \quad (20)$$

f(GHz) is the frequency. $|\phi|$(°) is the absolute value of latitude. M(mm) is the average annual total rain[9]. D_{th}(day) is the average number of thunderstorm days[10]. M_m(mm) is the highest monthly precipitation observed over 30 consecutive years[10]. $R_{0.01}$(mm/h) and $R_{0.001}$(mm/h) are 1-min. rain rates for 0.01% and 0.001% of a year, respectively. β is the thunderstorm ratio of Dutton[11]. The thunderstorm ratio β is defined as $\beta = M_t/M$, where M_t(mm) is the average annual total rain arising from thunderstorm. Table 1 shows partial correlation coefficients between α and regional climatic parameters. It is shown that correlations between α and $R_{0.01}$, D_{th} and β are comparatively large.

6. Comparison of measured rain attenuation with predicted rain attenuation

Fig.2 and Fig.3 show the comparisons of measured rain attenuation with values predicted by eqns.(19) and (20) respectively. Fig.4 shows the similar result for Rec. ITU-R P.530-8 prediction method[3] that is widely recognized to have the best accuracy at present. The probability range shown in these figures is from 0.001% to 1%, because the employed range for Rec. ITU-R P.530-8 method is from 0.001% to 1%. These figures show all the data including for d<5km. From these figures, it is found that the new prediction methods proposed in this paper have the same accuracy for two spatial correlation functions, and have the better accuracy than existing method.

7. Conclusion

From theoretical analysis and databank, it is found that good prediction accuracy can be obtained by using M distribution and the suitable spatial correlation parameter α for each location. Moreover, the α has regional dependence, and can be estimated by regional climatic parameters. By using estimated α and M distribution, it is found that the prediction error for the rain attenuation is smaller than the existing method.

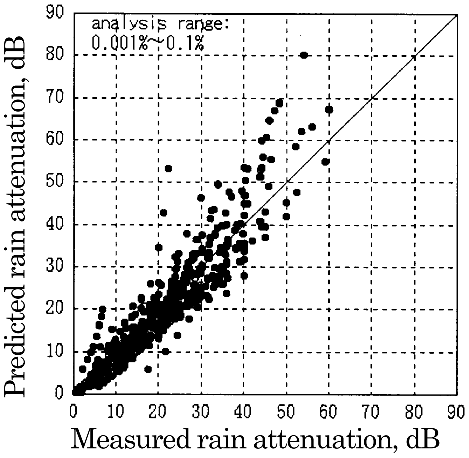

Fig.2 Comparison of measured and predicted rain attenuation by eqn.(19) and the model in section 3.

Table 1. Partial correlation coefficients between α and regional climatic parameters.

Climatic Parameters	partial correlation coefficients	
	appropriate $\alpha_{1/2}$	appropriate α_1
ln(p)	-0.0125	-0.1224
ln(u)	0.1678	0.3304
f (GHz)	-0.2870	-0.2390
$\mid\phi\mid$ (°)	0.0605	0.0983
M (mm)	-0.0620	-0.0647
D_{th} (day)	-0.1977	-0.3200
M_m (mm)	-0.1276	-0.1273
$R_{0.01}$ (mm/h)	0.1434	0.2320
$R_{0.001}$ (mm/h)	0.0071	0.0991
ln(β)	-0.1973	-0.1561

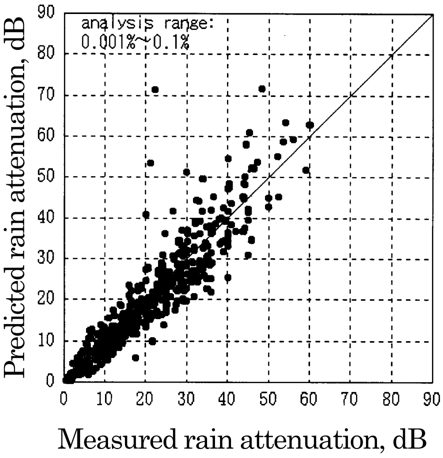

Fig.3 Comparison of measured and predicted rain attenuation by eqn.(20) and the model in section 3.

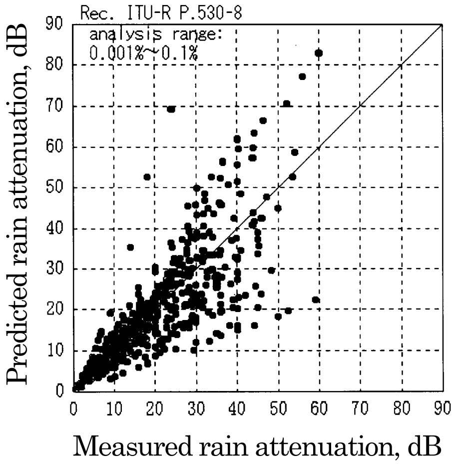

Fig.4 Comparison of measured and predicted rain attenuation by Rec. ITU-R P.530-8 method.

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