SPATIAL CORRELATION OF RAINFALL RATE AT LOW PERCENTAGES OF TIME AND ITS REPRESENTATION BY BIVARIATE GAMMA DISTRIBUTIONS FOR PERFORMANCE EVALUATION OF WIRELESS SYSTEMS

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1. Introduction

Broadband fixed wireless access (BFWA) systems have been considered a promising approach for high-speed access networks. BFWA systems operating at millimeter-wave bands are suitable for the high-density deployment of high capacity wireless access infrastructure, although they suffer from the degradation of radio link quality due to rain attenuation especially in tropical or subtropical zones. One of the authors has already proposed system architecture of multi-hop mesh wireless networks that exploits a route diversity scheme [1]. In order to accurately evaluate the radio link quality and end-to-end communications performances of the mesh wireless system with a route diversity scheme, it is essential to understand the spatial correlation of rainfall and its effect on radio links operating in millimeter-wave bands.

Conventionally, the statistical characteristics of the rainfall rate at low percentages of time have been approximated well by gamma distribution and the spatial correlation of the rainfall rate has been given as $\rho = \exp\left(-0.3\sqrt{d}\right)$, where d is the horizontal distance in kilometers between two observation points [2]. This paper first analyzes rainfall data recently acquired at five locations in the Tokyo metropolitan area and then examines the applicability of bivariate gamma distribution functions, specifically concentrating the discussion on the representation of spatial correlations at low percentages of time.

2. Rainfall Measurements

The measurement of rainfall has been conducted as summarized in Table 1. Rain gauges were installed at five locations, shown in Fig. 1, among which distances ranged from 1 km to 31.6 km. As far as pairs of locations are concerned, there are 10 pairs in total (i.e. 2 out of 5). The data available for the paper were from 1 June, 2001, to 31 March, 2002 (10 months). The rainfall rate data are obtained by a type of smoothing processing that counts the tipping of a bucket per minute.

3. Statistical Characteristics of Rainfall Rate

First, the cumulative distribution functions (CDF) of the rainfall rate at a single-site were analyzed. Figure 2 shows, for example, the distribution of $P_1\{R_1 > r_1\}$ in *Shinjuku*, where P_1 is the probability of the rainfall rate R_1 exceeding a certain value r_1. The smooth line without a symbol in Fig. 2 represents the best-fit curve of a univariate gamma distribution, where the rainfall rate range from 20 mm/h to 80 mm/h is specified for the curve fitting. It was verified that the rainfall rate data could be approximated by a gamma distribution function. The correlation coefficients of the rainfall rates for each pair of locations are computed and plotted in Fig. 3, where the measured data exhibit slightly different spatial characteristics from the conventional curve of $\rho = \exp\left(-0.3\sqrt{d}\right)$ but can be approximated better by $\rho = \exp\left(-0.27 d^{0.4}\right)$.

Next, the joint probability $P_2\{R_1 > r_{12} \,\&\, R_2 > r_{12}\}$ of the rainfall rates R_1 at location 1 and R_2 at location 2 exceeding a certain value r_{12} simultaneously was analyzed and one typical result is shown in Fig. 4 for the location pair of ***Shinjuku*** and ***Meguro*** (d = 5.0 km). In Fig. 4, other data are also plotted representing 'united, single-site probability' $P_1(R_{12} > r_1)$ for which the rainfall rate samples for location 1 and location 2 are united to be a union set. According to [3], the ratio defined by $\rho_s = r_{12}/r_1$ for the case when $P_1 = P_2$ is represented, as a simple function of distance d but independent of the rainfall rate, by the following empirical formula: $\rho_s = (1+d)^{-0.27}$. Figure 5 shows the ratio $\rho_s = r_{12}/r_1$ as a function of distance d verifying the empirical relation, although the obtained values from the measured data slightly scatter around the curve and P= 0.03% is almost a perfect fit.

4. Representation of Spatial Correlation by Bivariate Gamma Distributions

Suppose that the $\rho_s = r_{12}/r_1$ for $P_1 = P_2$ is represented by the empirical formula of $\rho_s = (1+d)^{-0.27}$ irrespective of the value of $P_1 = P_2$, the probability $P_2\{R_1 > r_{12} \,\&\, R_2 > r_{12}\}$ can be obtained for a pair of locations with any distance directly from a single-site probability $P_1\{R_1 > r_1\}$ as depicted in Fig. 6 (a), where the above-mentioned empirical formula is assumed to be true. However, as shown in Fig. 6 (b), a bivariate gamma distribution with a certain correlation coefficient gives a different curve from Fig. 6 (a). This implies that the constant correlation coefficient independent of the rainfall rate may not be assumed when the spatial correlation characteristics of the rainfall rates are approximated by a bivariate gamma distribution function. Figure 7 shows the relationship between an adjusted correlation coefficient ρ_g for a bivariate gamma distribution and a distance d with the threshold rainfall rate r_{12} as a parameter. In Fig. 7, the dashed line represents a conventional curve of $\rho = \exp\left(-0.3\sqrt{d}\right)$, which roughly lies between the curves of the rainfall rate at 20 mm/h and 30 mm/h and suggests that a heavy rainfall of more than 20-30 mm/h should have a smaller correlation coefficient than the conventional formula and vice versa. Figure 8 shows the same relationship in a different form, where the adjusted correlation coefficient ρ_g is given as a function of probability for a specified distance.

In order to investigate the spatial correlation structure of the rainfall rate data more precisely, a scatter diagram was generated as shown in Fig. 9 and, then, a bivariate cumulative distribution function was represented by a contour plot as shown in Fig. 10, both of which are for the location pair of ***Shinjuku*** and ***Meguro*** (d = 5.0 km) for example. In Fig. 10, smooth lines are also shown, which represent the approximated contours by a best-fit bivariate gamma distribution. Specifically, each of the dashed lines is obtained by directly searching the optimum correlation coefficient ρ_g with which a bivariate gamma distribution best reproduces the relevant contour. The objective function to be minimized in the optimization process is the difference of the areas surrounded by the contour and the x- and y-axes. The optimization is conducted for each contour at a certain probability (or a percentage of time) for all pairs of the five locations. Finally, Fig. 11 shows the relationship between the optimum correlation coefficient and the probability, which can be directly compared with Fig. 8. Although the quantitative agreement is not perfect, the plots in Fig. 11 resemble the curves in Fig. 8. The discrepancy is large for d = 1.0 km.

5. Conclusions

Spatial correlation of the rainfall rate was studied with experiments especially focusing on the properties at low percentages of time. It was verified that both single-site and joint CDFs of the rainfall rate at low percentages of time could be approximated by a gamma distribution. However, when applying a bivariate gamma distribution for the evaluation of statistics, the correlation coefficient needs to be adjusted depending on the prescribed percentage value of interest.

Acknowledgement

The authors would like to thank Prof. M. Yamada, Tokyo University of Technology, and Prof. Y. Karasawa, The University of Electro-Communications, for their suggestions. Thanks also go to Dr. T. Asami, Dr. Matsushima and Dr. T. Mizuike of KDDI R&D Labs. for their continuous encouragement. A part of this work is supported by the Telecommunications Advancement Organization of Japan (TAO).

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Table 1. Rainfall rate measurement campaign.

Period	June 2001 ～ March 2002 (10 months)
Locations	*Shinjuku/Ohtemachi/Meguro/Nihonbashi /Kamifukuoka* (5 locations)
Distances	1.0, 5.0, 6.3, 7.3, 7.4, 8.0, 25.6, 30.8, 31.1, 31.6 km (10 pairs)
Rain Gauge Type	Tipping Bucket (Unit=1.0 mm)
Rainfall Rate Conversion Period	1 minutes

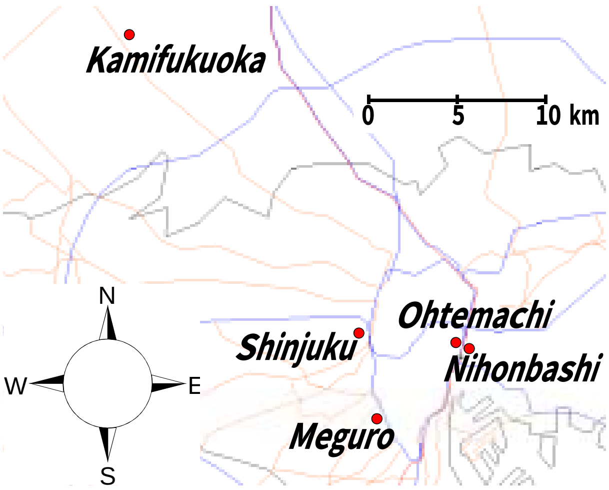


Fig. 1. Map of rain measurement locations in the Tokyo metropolitan area.

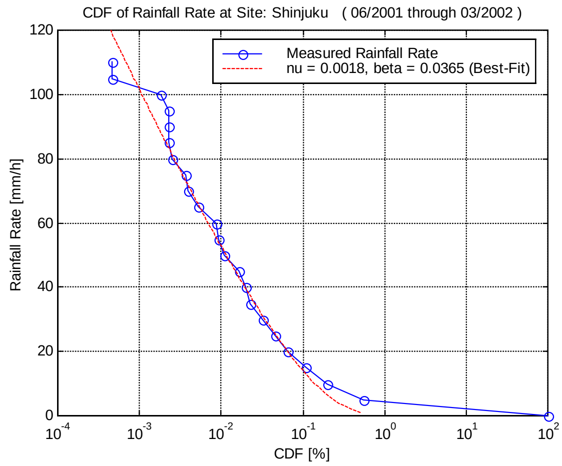


Fig. 2. Cumulative distribution function of the rainfall rate at the "*Shinjuku*" site and its approximation by a gamma function.

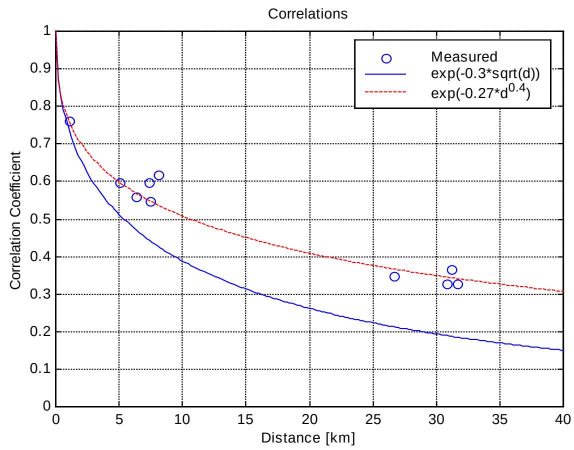


Fig. 3. Correlation of rainfall rate as a function of distance between a pair of sites.

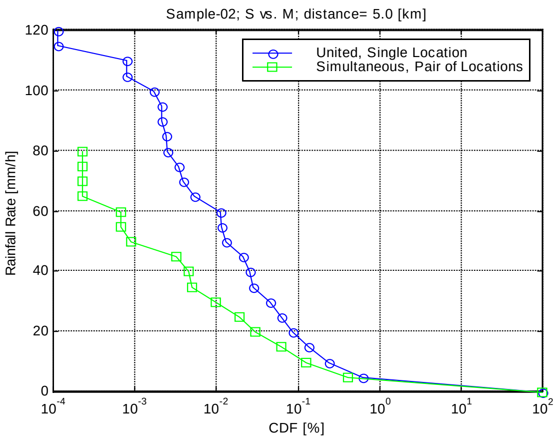


Fig. 4. Joint cumulative distribution function compared to single-site CDF.

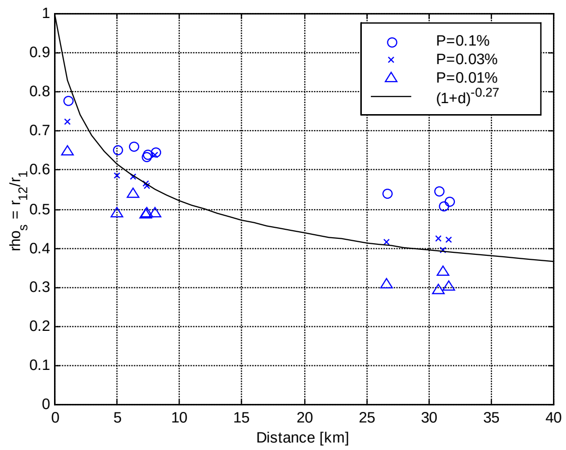
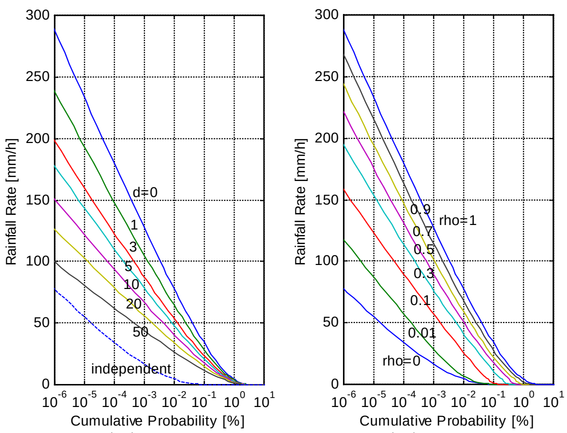
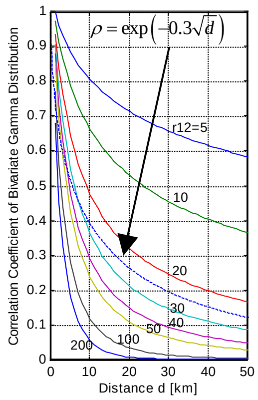


Fig. 5. Spatial factor $\rho_s = r_{12}/r_1$ obtained from single-site and joint CDFs.



(a) (b)

Fig. 6. Two comparative joint CDF representations (theory): (a) Scaling from a single-site CDF using an empirical law $\rho_s = (1+d)^{-0.27}$, (b) Bivariate gamma distributions with a specified correlation coefficient.



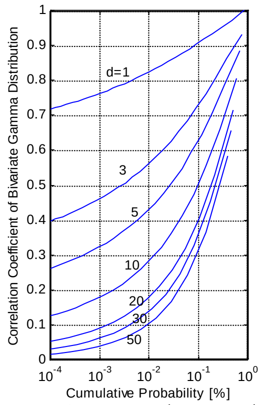


Fig. 7 (left). Correlation vs. distance (theory).
Fig. 8(right). Correlation vs. probability (theory).

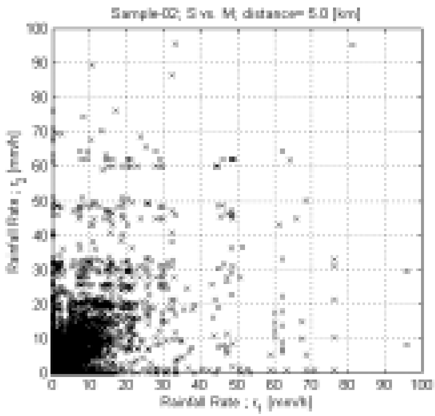


Fig. 9. Scatter diagram.

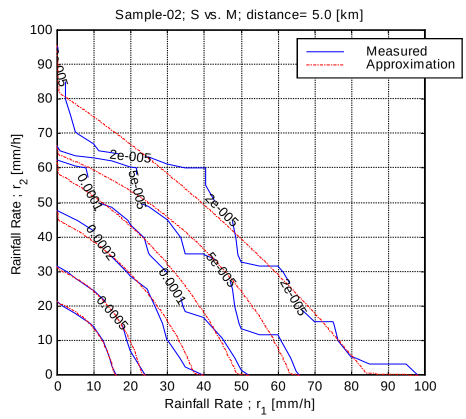


Fig. 10. Contour representation of joint CDF.

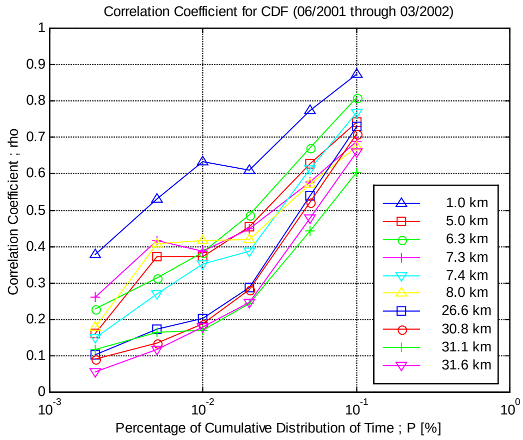


Fig. 11. Correlation vs. probability (measured).