

## ノード間距離に基づく空間ネットワークの統計分析法 Statistical analysis method for spatial network based on inter-node link distance

アリフ マウラナ<sup>†</sup>      齊藤 和巳<sup>†</sup>      池田 哲夫<sup>†</sup>      湯瀬 裕昭<sup>†</sup>  
Arief Maulana      Saito Kazumi      Ikeda Tetsuo      Yuze Hiroaki

### 1. Introduction

Studies of the structure and functions of large complex networks have attracted a great deal of attention in many different fields such as sociology, biology, physics and computer science [1]. As a particular class, we focus on spatial networks embedded in the real space, like urban streets, whose nodes occupy a precise position in two or three-dimensional Euclidean space, and whose links are real physical connections with some inter-node distances [2].

In this paper, by incorporating such link distances with the conventional notions of degree distributions and degree correlations [1], we propose a statistical analysis method for analyzing spatial networks, which consists of link distance distributions, link distance correlations, and link distance and degree correlations. In our experiments using spatial networks obtained from OSM (OpenStreetMap) data sets, we show that our method can find some intrinsic properties of these networks such that their link distance distributions can be reasonably approximated by a power law distribution.

### 2. Proposed Method

Let  $G = (\mathcal{V}, \mathcal{E})$  be a spatial network where  $\mathcal{V}$  and  $\mathcal{E}$  are sets of nodes and links, respectively. Also, let  $k(u)$  and  $d(u, v)$  be the degree of node  $u \in \mathcal{V}$  and the inter-node link distance of link  $(u, v) \in \mathcal{E}$ . Here, we divide the set of links  $\mathcal{E}$  into

$$\mathcal{E}_m = \{(u, v) \in \mathcal{E} \mid m - 1 < d(u, v) \leq m\}$$

according to the unit distance  $m \in \{1, 2, \dots\}$ . Then, as a notion inspired from degree distributions [1], we propose the following link distance distribution  $f_1(m)$ :

$$f_1(m) = |\mathcal{E}_m|$$

where  $|\mathcal{A}|$  means the number of elements in  $\mathcal{A}$ .

Next, as notions inspired from degree correlations [1], we also propose the following link distance correlation  $f_2(m)$  and link distance and degree correlation  $f_3(m)$ :

$$f_2(m) = \frac{1}{|\mathcal{E}_m|} \sum_{(u,v) \in \mathcal{E}_m} \frac{1}{|\mathcal{A}(u,v)|} \sum_{(x,y) \in \mathcal{A}(u,v)} d(x,y)$$

$$f_3(m) = \frac{1}{|\mathcal{E}_m|} \sum_{(u,v) \in \mathcal{E}_m} \frac{k(u) + k(v)}{2}$$

where  $\mathcal{A}(u, v)$  denotes the set of links adjacent to the link  $(u, v)$  defined by

$$\mathcal{A}(u, v) = \{(t, u) \mid (t, u) \in \mathcal{E}\} \cup \{(v, w) \mid (v, w) \in \mathcal{E}\}.$$

Namely, by using  $f_2(m)$  and  $f_3(m)$ , we can analyze some types of mixing patterns such a property whether the long-distance links in a network are likely to connect to long-distance links or not.

### 3. Evaluation by Experiments

We used OSM (OpenStreetMap) data of 17 cities studied in [3], where the numbers of nodes and links in these urban street networks are from 120,068 to 9,543,619 and from 266,540 to 19,445,402, respectively. In our experiments, by setting the unit distance  $m$  to 1 meter, we analyzed these networks by our proposed method based on the formulae  $f_1(m)$ ,  $f_2(m)$  and  $f_3(m)$ .

From our experimental results, as shown in Figures 1, 2 and 3 by using the 4 sample cities, London, Paris, Los Angeles, and New York, we observed that all the networks can be assumed to have the scale-free property of link distance distributions (Fig. 1), like degree distributions of most complex networks, clearly positive link distance correlations (Fig. 2), like degree correlations of most social networks, and slightly positive link distance and degree correlations (Fig. 3). We consider that these are the intrinsic properties of spatial networks constructed from large urban street networks.

By more closely examining our experimental results, we could classify these 17 cities roughly into two groups according to the condition whether each plotted curve contains some peaks or not. For instance, as to the 4 sample cities shown in Figs. 1, 2 and 3, we can classify London and Paris into the group without peaks, while Los Angeles and New York into the group with peaks. Here we should note that such peaks are typically formed around 80, 100, 200 and 400 meters link distance points in each horizontal axis.

In our experimental results, among these 17 cities, historically new cities typically in USA such as Washington D.C., San Francisco, and Richmond were classified into the group with peaks. Here, we may naturally explain the reason why these peaks are formed by assuming that some regular lattice types of urban streets were constructed in main area of cities under their large scale city planning, where such regular lattices are generally characterized by relatively larger

<sup>†</sup>静岡県立大学大学院経営情報イノベーション研究科

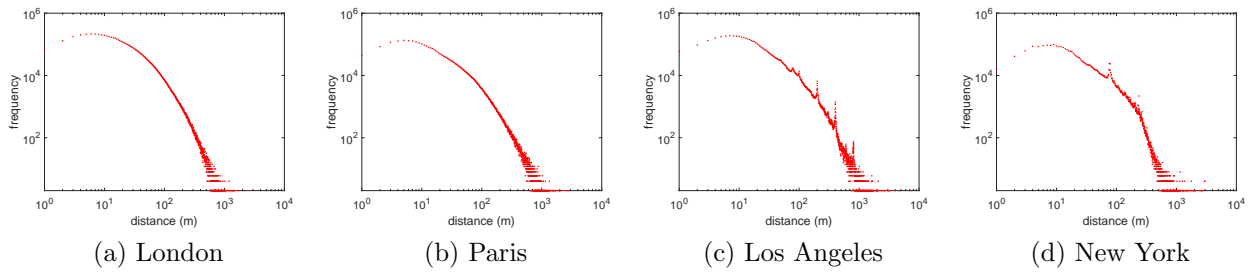


Figure 1: Link distance distribution

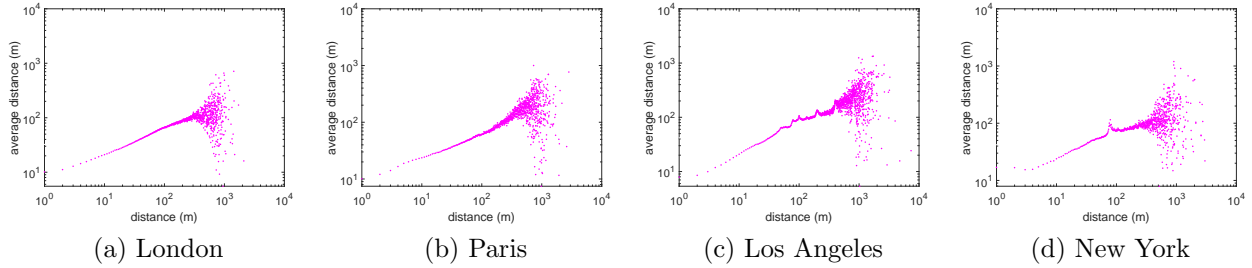


Figure 2: Link distance correlations

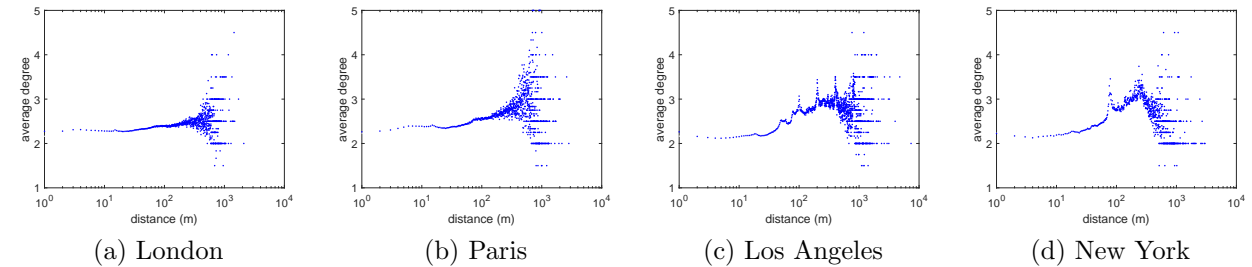


Figure 3: Link distance and degree correlations

number of links with the same link distances and their connections to nodes with degree 4. These experimental results suggest that our proposed method can have a capability to uncover some intrinsic properties of spatial networks.

#### 4. Conclusion

By incorporating inter-node link distances with the conventional notions of degree distributions and degree correlations, we proposed a statistical analysis method for analyzing spatial networks using link distance distributions, link distance correlations, and link distance and degree correlations. In our experiments using spatial networks obtained from OSM (OpenStreetMap) data sets, we showed that our method can find some intrinsic properties of these networks. In future, we plan to evaluate our method using various spatial networks.

**Acknowledgments** This work was supported by JSPS Grant-in-Aid for Scientific Research (No.15K00311).

#### References

- [1] Newman, M. E. J.: The structure and function of complex networks, *SIAM Review*, Vol. 45, pp. 167–256 (2003).
- [2] Crucitti, P., Latora, V. and Porta, S.: Centrality measures in spatial networks of urban streets, *Physical Review E*, Vol. 73, No. 3, pp. 036125+ (2006).
- [3] Maulana, A., Saito, K., Ikeda, T., Yuze, H., Watanabe, T., Okubo, S. and Muto, N.: Analyzing Similarity Structure of Spatial Networks Based on Degree Mixing Patterns, *Advanced Information Networking and Applications*, Vol. 30, pp. 1–8 (2016).