B-025

A Note on Software Reliability Modeling Based on Hawkes Processes with Time-Dependent Base Intensity

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1. Introduction

Software reliability quantifies the likelihood that a software system operates correctly without experiencing failures within a specified time frame. While conventional Non-Homogeneous Poisson Process (NHPP)-based Software Reliability Models (SRMs) can represent time-dependent failure rates, they often assume that fault occurrences are mutually independent increments, which may not hold in the context of modern, interdependent fault behaviors.

To better capture the temporal clustering of failures, Hawkes introduced a self-exciting point process, known as the Hawkes process[1], in which each observed event increases the probability of subsequent ones. Iwamoto et al.[5] were among the first to apply the classical Hawkes process (HKP) framework to model software fault detection patterns.

In contrast to earlier HKP-based SRMs, our formulation introduces a time-dependent baseline intensity function that evolves over time, allowing the model to more accurately characterize the shifting landscape of software reliability. Furthermore, we incorporate kernel structures from ten established NHPP-based SRMs available in the Software Reliability Assessment Tool on Spreadsheet (SRATS)[3, 4]. This hybrid approach enables the model to simultaneously capture both spontaneous failure tendencies and historical excitation effects.

We evaluate the performance of our model using several diverse software fault detection time data sets, focusing on both goodness-of-fit and prediction accuracy. For the prediction evaluation, we employ Monte Carlo simulation to generate the mean function.

2. Hawkes Process-based SRMs

The Hawkes process belongs to a class of self-exciting point processes where each observed event increases the probability of future occurrences within a short time interval.

2.1. Classical Hawkes Process

In its conventional form, the Hawkes process employs a constant baseline intensity, implying a uniform background event rate. The conditional intensity function $\lambda(t)$, given the history $\mathcal{H}_t$, is typically formulated as

$$\lambda(t) = \mu + \sum_{i:T_i<t} \phi(t - t_i), \tag{1}$$

where μ represents the background intensity unaffected by prior events, and $\phi(t-t_i)$ is the excitation kernel that characterizes how much influence an event at time t_i exerts on the occurrence rate at time t.

2.2. Hawkes Process with Time-Dependent Base Intensity

In real-world scenarios such as software testing, the underlying event rate often changes as the system evolves. To better reflect such dynamics, we extend the original Hawkes process by adopting a time-dependent baseline intensity $\mu(t)$, modeled via the exponential distribution and Weibull distribution. The revised conditional intensity functions become

$$\lambda(t) = \mu(t) + \sum_{t_i<t} \phi(t - t_i), \tag{2}$$

where $\mu(t)$ denotes the **baseline intensity function**. The exponential-type(exp) baseline intensity $\mu(t)$ is defined as

$$\mu(t) = \nu\, b\, e^{-bt}, \tag{3}$$

and Weibull-type baseline intensity $\mu(t)$ is given by

$$\mu(t) = \nu \frac{b}{c}\left(\frac{t}{c}\right)^{b-1} e^{-\left(\frac{t}{c}\right)^b}, \tag{4}$$

where ν is a scaling factor.

The kernel function $\phi(t)$ remains central to modeling the self-excitation effect by quantifying how past events amplify the current failure rate. In this study, we develop ten distinct Hawkes-based SRMs by integrating excitation kernels derived from ten well-established time-to-failure distributions. Each model represents a unique instantiation of the Hawkes process tailored to different reliability patterns. The complete set of kernel functions used is summarized in Table 1.

3. Prediction Method

Accurately forecasting future occurrences in a Hawkes process involves estimating the expected cumulative number of events, denoted as $E[N(t)]$. To achieve this, we adopt a simulation-based approach grounded in Ogata ' s thinning algorithm for point processes[2]. This method allows for the sequential generation of events by updating the historical timeline after each simulated occurrence. To enhance its applicability in scenarios where future event timings are not predetermined, we adapt the original algorithm, enabling it to dynamically simulate self-exciting event sequences with greater precision.

表 1: Representative kernels of HKP-based SRMs.

Kernels	$\phi(t;\boldsymbol{\pi})$, $\boldsymbol{\pi}=(b,c)$
Exponential dist. (exp)	be^{-bt}
Gamma dist. (gamma)	$\frac{c^b t^{b-1} e^{-ct}}{\Gamma(b)}$
Pareto dist. (pareto)	$\frac{bc\left(\frac{c}{c+t}\right)^{b-1}}{(c+t)^2}$
Log-normal dist. (lnorm)	$\frac{e^{-\frac{(c-\log t)^2}{2b^2}}}{b\sqrt{2\pi}t}$
Truncated logistic dist. (tlogist)	$\frac{e^{-\frac{-c+t}{b}}}{b\left(1+e^{-\frac{-c+t}{b}}\right)^2\left(1-\frac{1}{1+e^{\frac{c}{b}}}\right)}$
Log logistic dist. (llogist)	$\frac{e^{-\frac{-c+\log t}{b}}}{b\left(1+e^{-\frac{-c+\log t}{b}}\right)^2 t}$
Truncated extreme-value max dist. (txvmax)	$\frac{e^{-e^{-\frac{-c+t}{b}}-\frac{-c+t}{b}}}{b\left(1-e^{-e^{\frac{c}{b}}}\right)}$
Log-extreme-value max dist. (lxvmax)	$\frac{e^{-e^{\frac{c-\log t}{b}}+\frac{c-\log t}{b}}}{bt}$
Truncated extreme-value min dist. (txvmin)	$\frac{e^{e^{\frac{c}{b}}-e^{-\frac{-c-t}{b}}-\frac{-c-t}{b}}}{b}$
Weibull dist. (weibull)	$\frac{b}{c}\left(\frac{t}{c}\right)^{b-1} e^{-\left(\frac{t}{c}\right)^b}$

* $\Gamma(\cdot)$: standard gamma function.

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