A-034

ユウゴウパズルの計算複雑さ
Computational Complexity of Yugo Puzzle

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1 Introduction

In recent years, there has been active research on the computational complexity of games. We investigate a computational complexity of "Yugo puzzle". Yugo puzzle is a side non-scrolling game. In this game, we are given colored jellies in a stage consisting of floors and walls. A player manipulates the jellies under gravity. The goal of this game is to make all the jellies of the same color stick together.

Our contribution is to prove PSPACE-completeness of Yugo puzzle. Our proof uses the nondeterministic constraint logic model by Hearn and Demaine [2].

2 Preliminary

we explain the terminologies required for the proof.

In this paper, we use the nondeterministic constraint logic (NCL) model of computation introduced in [2]. An NCL graph is an undirected graph such that non-negative integers are assigned to edges and integers are assigned to vertices. We call the integers assigned to edges and vertices *weights* and *minimum inflow constraints*, respectively. Let $G = (V, E)$ be an NCL graph. We denote by $w(e)$ the weight of an edge e and by $c(v)$ the minimum inflow constraint of a vertex v. Then, a *configuration* of G is an orientation of E such that $\sum_{e \in I^+(v)} w(e) > c(v)$ for every vertex, where $I^+(v)$ is the set of the entering edges of v in the orientation. A *flip* is to reverse the direction of an edge in a configuration. For a configuration of G, a flip is *valid* if the flip produces a different configuration of G. A vertex v of an NCL graph G is an *AND vertex* if the degree of v is 3, v is incident to 2 edges with weight 1, v is incident to an edge of weight 2, and the minimum inflow constraint of v is 2. A vertex v of an NCL graph G is an *OR vertex* if the degree of v is 3, v is incident to 3 edges with weight 2, and the minimum inflow constraint of v is 2.

Both an AND and OR vertices are degree 3.An AND/OR constratint graph is a NCL graph such that each vertex is either an AND or OR vertex.

Now, we define the problem CONFIGURATION-TO-EDGE as the decision problem.

CONFIGURATION-TO-EDGE
INSTANCE: An NCL graph G, a edge e of G and configuration of G
QUESTION: Can we flip e starting from the given a configuration of G by a series of valid flips?

The following theorem is shown by Hearn and Erik.

Theorem 1 ([2]) CONFIGURATION-TO-EDGE is PSPACE-complete, even when the input NCL graph is restricted to an AND/OR graph.

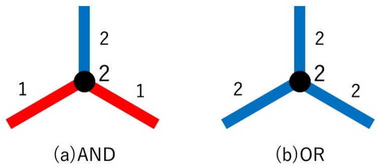

Figure 1 (a) An AND vertex and (b) OR vertex.

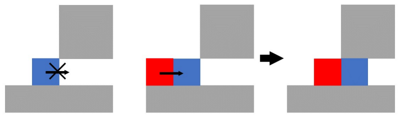

Figure 2 Jelly being placed in a space at the same height as itself.

2.1 Yugo Puzzle

The rules of the Yugo puzzle are as follows.

A player is given colored jellies and walls in a stage, which forms a rectangle, and the goal is to stick together all jellies of the same color. Players can move any jelly left or right. Once two jelly are adjacent, it never disconnects again. When you move the jelly left or right, the jelly above and below the moved jelly does not move to the left or right. Let J_1, J_2 be two jellies and suppose that J_1 is on J_2. When J_1 is moved, J_2 does not move. Similarly when J_2 is moved, J_1 does not move. When a jelly moved, The jelly on top falls by gravity only when there is no objects, which are floors or jellies to support the jelly. In addition, if there is jellies at the destination, the jelly can be pressed.

The jelly cannot enter a space at the same height as itself. However, if it is pushed by another jelly, it can enter that space (Figure 2). The jelly can also climb a step with the height of a half of unit-length.

Now, we define YUGO PUZZLE as the decision problem.

YUGO PUZZLE
INSTANCE: A stage with colored jellies
QUESTION: Is there a series of movement of jellies in the stage to stick together all jellies of the same color?

3 PSPACE-completeness of Yugo Puzzle

Let G be an instance of CONFIGURATION-TO-EDGE problem. (G, C, e) be an instance of CONFIGURATION-TO-EDGE problem. Now, we explain our gadgets below.

Lemma 2 The construction in Figure 3 satisfies the same constraints as an OR vertex.

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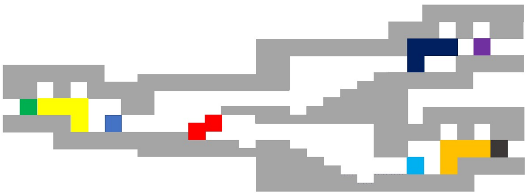

Figure 3 OR gadget for YUGO PUZZLE.

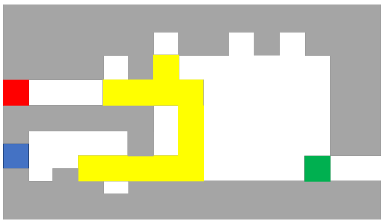

Figure 4 AND gadgets for YUGO PUZZLE.

Proof. We need to show that the purple jelly in Figure 3 can be moved if and only if the green or black jelly is entered into the middle space, in which the red jelly exists.

Initially, the yellow, dark blue, purple, light blue, and orange jellies are unable to move other jellies due to the constraints in Figure 2. Two 1×1 jellies is responsible for propagating the signal. These jellies can push out any two of the jellies at the three entry/exit place.

For example, consider the case when a green jelly enters into this gadget as illustrated in Figure 3. The green jelly can push the yellow jelly to the right. Note that, after that, the green jelly cannot get back to the left. The blue jelly can be pushed out by the yellow jelly and can move from the middle space to the upper right. The jelly can then move the dark blue jelly to the right. As a result, the purple jelly can leave the gadget.

Considering inputs from other sources as well, it is clear that the OR constraint is satisfied. □

Lemma 3 The construction in Figure 4 satisfies the same constraints as an AND vertex of [2].

Proof. We need to show that one can move the green jelly when and only if the red and blue jellies are entered.

Initially, the yellow and green jellies are unable to move other jellies due to the constraints in Figure 2. The red, blue, and green jellies in the upper left, lower left, and lower right are responsible for propagating the signal. First, When the red jelly is entered, the yellow jelly will be able to move three squares to the right. If a blue jelly is entered in this state, the yellow jelly can move one additional square to the right. This movement pushes the green jelly outward.

Consider also the reverse direction. When the green jelly is entered, the yellow jelly can move 4 squares to the left. This allows the red and blue jellies to be pushed out and output. if and only when the two left jellies are entered, the right jelly can pass through the right passage. □

Theorem 4 Yugo Puzzle is PSPACE-complete when there is no restriction on the number of colors.

Proof. The membership of NPSPACE is clear from the definition of YUGO-PUZZLE. From Savitch's theorem, the problem is in PSPACE.

The proof of PSPACE-hardness uses a reduction from CONFIGURATION-TO-EDGE. Let (G, C, e) be an instance of YUGO-PUZZLE. Since G is a 3-regular planar graph, it has a grid embedding and the embedding is computed in polynomial time [4].

The embedded graph can be transformed as a stage of YUGO PUZZLE by connecting each vertex with a passage of height 1 or by successfully connecting stairs formed by connecting steps each of which has a half of unit height and by transforming each vertex as either an AND or OR vertex. The target edge e is transformed into the gadget in Figure 5. It can be observed that the instance (G, C, e) is an yes-instance if and only if the constructed state is an yes-instance. □

References

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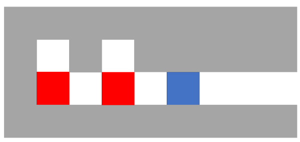

Figure 5 The gadget of the goal.