

## Application of Advance Constrained Simplex Method to an e-SYSTEM Network Design

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*Abstract*— e-Science is based on e-System, and has three factors. The first factor is optimization proceeding. Large data are categorized into function data or noise data through an optimization proceeding. The second factor is networking. Information about the categorized function data are spread all over the world though networking. The final factor is modeling, which can be designed from the 2- or 3-dimensional view of function data.

This study first describes advanced constrained simplex method (advanced complex method). Subsequently, the study shows whether this complex procedure has any problems when used for finding the maximum of a general nonlinear function of several variables within a constrained region described in wireless communication systems, especially for multiple-input multiple-out (MIMO) configuration.

Next, advanced constrained simplex method is described, presenting how to resolve the problem of MIMO as well as how to make it efficient, by comparing it with the complex method and classical simplex method using some simulations. The proposed wireless network design can be used as an e-system network design.

**Keywords**-component; Wireless design; e-system design

### I. INTRODUCTION

e-Science information consists of voice, video, and data signal in a same-area application. When multimedia is transmitted over wireless networks, video signal traffic consumes most of the bandwidth of the wireless networks, and also requires a high level of quality of service (QoS) to be guaranteed by the multimedia wireless networks. To reduce the bandwidth requirement, video signal must be compressed before transmission over the wireless networks. The video signal compression is based on removing the temporal and spatial redundancy of information in a video stream. This particular method is related to the e-science information transfer system, which is suitable to transfer data from Nanoscale and Molecular Networking exchange server for generating a great number of e-science information having high bit rate streams to a server and a client in a client-server system (conventional communication systems).

The mobile propagation modeling (MS and BS) is described in Figure 2 and the maximal ratio combination in MIMO configuration is presented in Figure 3. In MIMO

configuration, the most important aspect is how to determine the optimum weights.

## Reference Model

Goal: Each of these components to be plug and play, both in simulation and actual implementation.

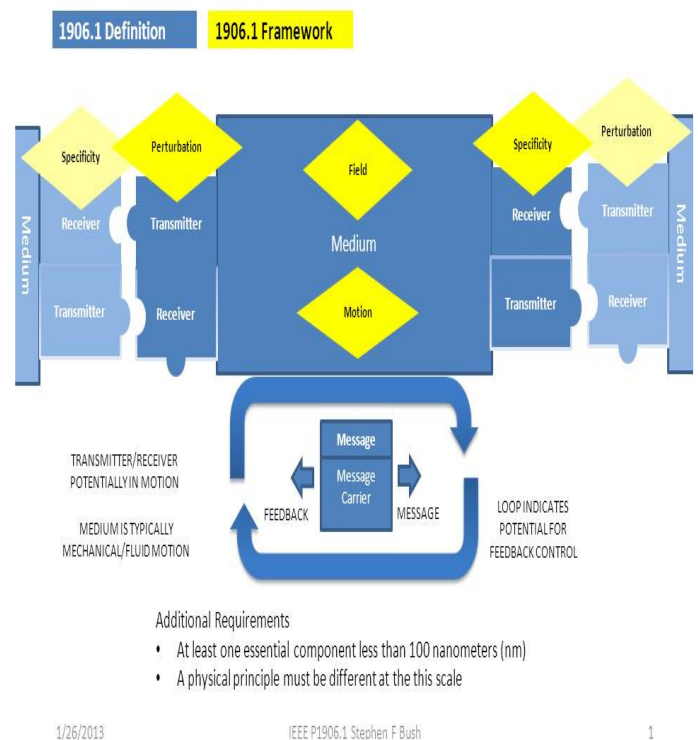


Figure 1. Nanoscale Molecular Network Reference Model (IEEE 1906.1)

Many methods have been proposed to solve nonlinear problem. However, there are many kinds of angular distribution functions in the mobile propagation modeling (MS and BS). When one angular distribution is normal distribution and the others are in the multiplicative or

additive process, then we cannot solve nonlinear problem by classical method.

Hence, we developed the advanced constrained simplex method. In many cases of solving linear problem, we make use of the simplex method. On the other hand, in the case of solving nonlinear problem, if this problem has no constrained condition, we can use the simplex method to solve it. Furthermore, constrained simplex method can be used to solve the nonlinear problem with constrained condition.

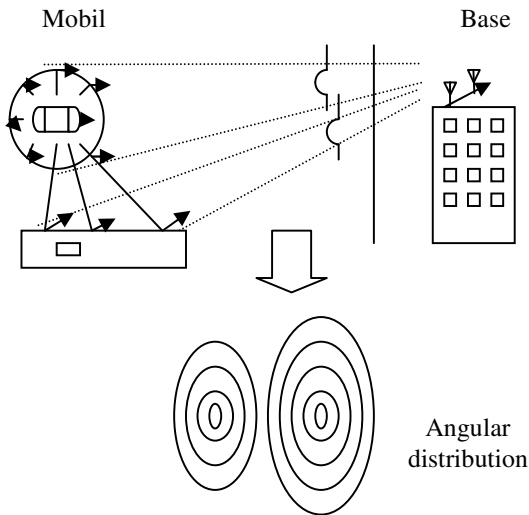


Figure 2. Mobile Propagation Modeling (MS and BS)

Let us consider the following problem as nonlinear.

Let the received signal vector of m'th terminal be:

$$r^{(m)} = A^{(m)} w_m S_m(t) + A^{(m)} \sum_{k=1, k \neq m}^K w_k S_k(t) + n^{(m)}$$

By maximizing SNR of rx (if possible), we get:

$$r^{(m)}(t) = w_m^H A^{(m)H} A^{(m)} w_m S_m(t) + \text{impairment}$$

$$= w_m^H B^{(m)} w_m S_m(t) + \text{impairment}$$

where

$$B^{(m)} = A^{(m)H} A^{(m)}$$

The weight vector for transmit MRC is:

$$w_m = e_{\max}^{(m)}$$

where

$e_{\max}^{(m)}$ : Eigenvector corresponding to the maximum eigenvalue of  $B^{(m)}$

$$w_t \quad A = [a_{mn}] \quad w_r$$

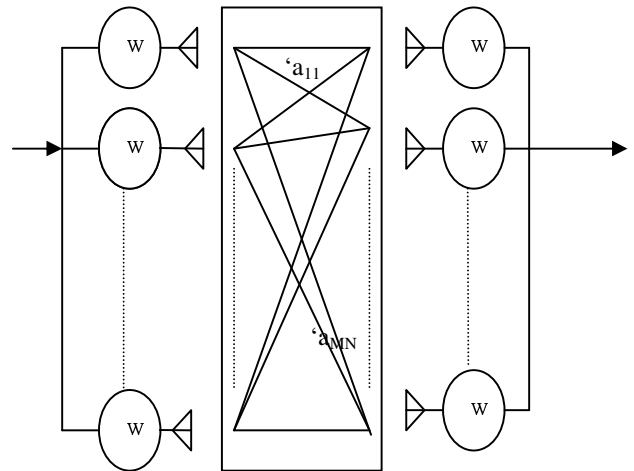


Figure 3. Maximal Ratio Combination in MIMO Configuration

## II. REFLECTION

Similar to the simplex method, the complex method can solve the problem without object function's figure. However, if there is an optimized point near the constrained concision, or if the optimized point does not satisfy the constrained condition, then the complex method cannot solve the problem depending on the initial point.

Hence, a new algorithm is required to advance the constrained simplex method. This algorithm is given as follows.

If the new point is the worst point, then a new constrained condition is added to the research direction, and the research restarts from the initial point, which is the best point for this research step.

### (1) Summary of the method

We have proposed the advanced constrained simplex method, which can solve problems with constrained condition and does not depend on the figure of object function or initial point. Figure 4 and 5 shows the flowchart of the advanced constrained simplex method.

The equality constrained nonlinear programming problem is as follows:

Minimize  $f(x)$

Subject to  $g_i(x)=1, j=1, m, (1)$

where  $x=(x_1, x_n)^T, n>m$ , and both  $f(x)$  and  $g_i(x)$  are assumed to have continuous second partial derivatives.

Step 1) The first point is determined, which can satisfy the constraint function.

Step 2) Determine n points produced by random function.

Step 3) If m (m =< n)th value is not a sufficient constraint function, from 1 through to the (m - 1)th point of the middle point of the m'th point, move half distance to the middle point of 1 through to the (m - 1)th point.

- Step 4) Points above  $n + 1$  points are estimated by object function.
- Step 5) Point A is the middle point of the  $n$  points without the best value point. Point B is the middle point of the  $n$  points without the worst value point.
- Step 6) Point C is at the opposite side of point A and point B. Furthermore, the distance between point C and point B is 2 times longer than that between point A and point B.
- Step 7) If point C does not satisfy the constrain function, then point C is the middle of point C and point B. If the distance between point C and point B is less than 0.000001, then point B is point C.
- Step 8) A method that has been the subject of much recent research for the solution of (1) is the sequential quadratic programming (SQP) technique (e.g., see Han [9], Powell [11], or Tapia [16]). A current estimate  $x^{(k)}$  to the minimized  $x^*$  is known, and a search direction  $s^{(c)}$  is generated to solve the quadratic programming problem

$$\text{Minimize } \frac{1}{2} s^{(k)T} B^{(k)} s^{(k)} + s^{(k)T} \nabla f(x^{(k)}) \quad (2)$$

$$\text{Subject to } \nabla g(x^{(k)})^T s^{(k)} + g(x^{(k)}) = 0,$$

where  $\nabla g(x^{(k)}) = \{\nabla g_1(x^{(k)}), \dots, \nabla g_m(x^{(k)})\}$  is the matrix of the constraint normal evaluated at  $x^{(k)}$ ,  $g^{(k)}$  is the vector  $(g_1(x), \dots, g_m(x))^T$ , and  $B^{(k)}$  is a positive definite approximation to the Hessian of the Lagrange with respect to  $x$ . Here, the Lagrange is

$$l(x, \lambda) = f(x) - \lambda^T g(x), \quad (3)$$

where  $\lambda = (\lambda_1, \dots, \lambda_m)^T$  is the vector of Lagrange multipliers and  $B^{(k)}$  is an estimate to  $l_{xx}(x^{(k)}, \lambda^{(k)})$  for an estimate  $\lambda^{(k)}$  to the optimal Lagrange multipliers  $\lambda^*$ .

- Step 9) In step 8, we find a new point that is estimated as an object function. If this new point is not better than old point C and  $n+1$  points are not within 0.000001, then we add a new constraint function that it is point B to point C line function. Now, the best point is decided to be the original point. Go to step 2.
- If this new point is better than old point C, then this new point is the new point C. Go to step 5.

If this new point is not good more than old C point, and  $n+1$  points are within 0.000001, then this research has inished. We continue the iterative procedure a new estimate to the minimized.

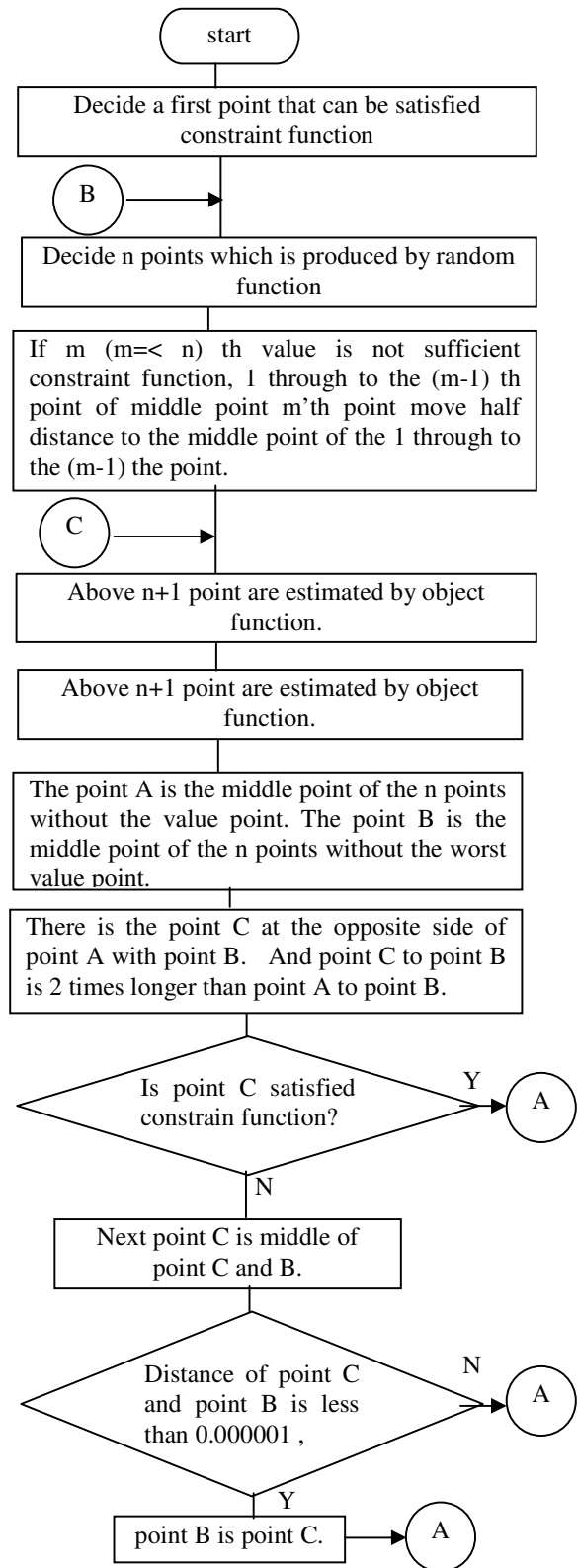


Figure 4. Flowchart of the complex method 1

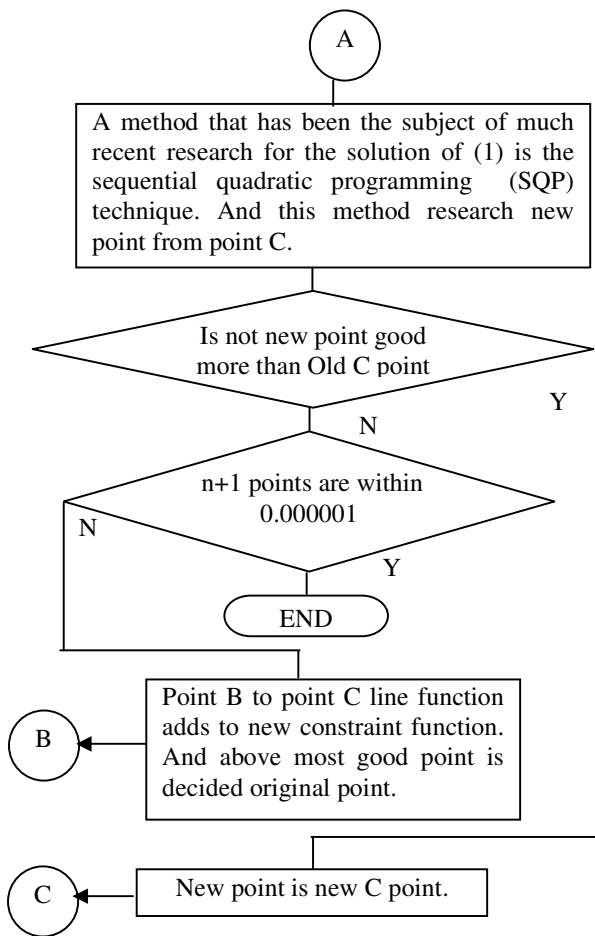


Figure 5. Flowchart of the complex method 2

### III. EVALUATION

We evaluated three cases based on the simulation.

#### A. Case 1

Object function and constraint function are normal, which can be solved by the existing method.

#### B. Case 2

The figure of object function is very narrow, and the existing method cannot solve it without the complex method.

#### C. Case 3

There are two object functions in the research field. In this case, the complex method cannot solve this problem.

Figures 6, 7, and 8 show the proceeding of case 1. Figures 9, 10, and 11 show the proceeding of case 2. Figures 12, 13, and 14 show the proceeding of case 3.

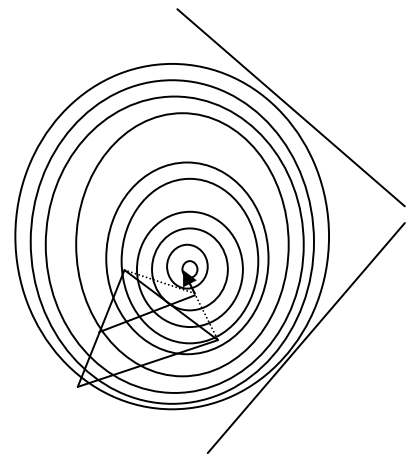


Figure 6. Processing of case 1 (1/3)

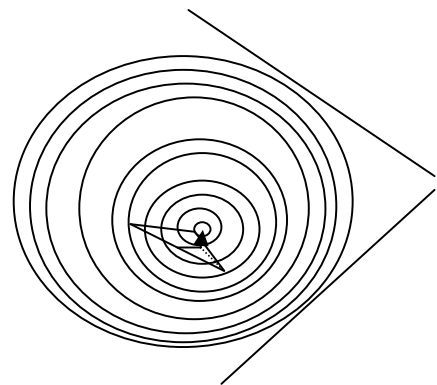


Figure 7. Processing of case 1 (2/3)

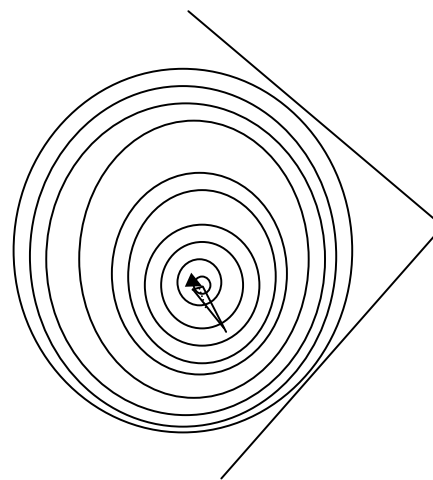


Figure 8. Processing of case 1 (3/3)

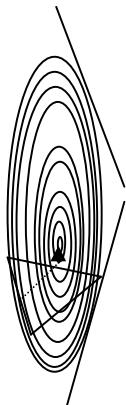


Figure 9. Processing of case 2 (1/3)

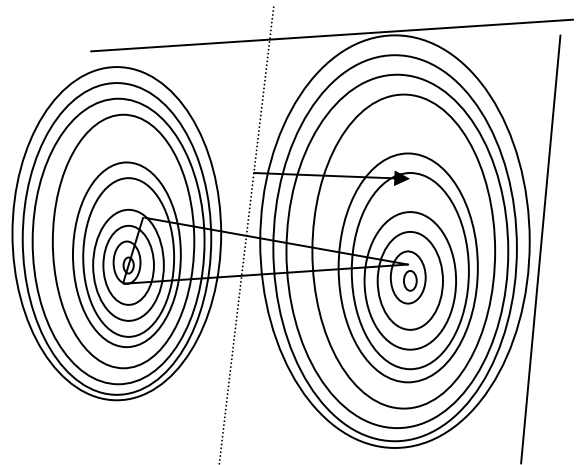


Figure 12. Processing of case 3 (1/3)

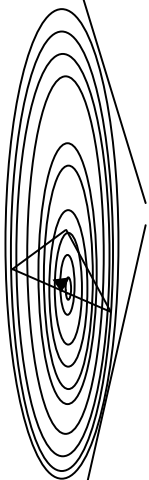


Figure 10. Processing of case 2 (2/3)

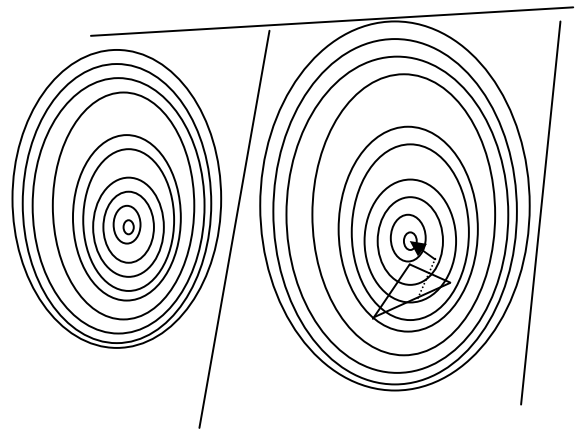


Figure 13. Processing of case 3 (2/3)

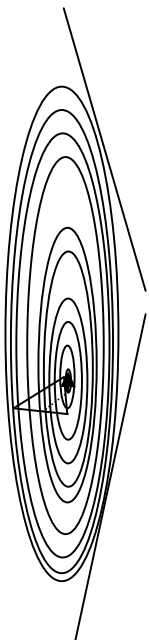


Figure 11. Processing of case 2 (3/3)

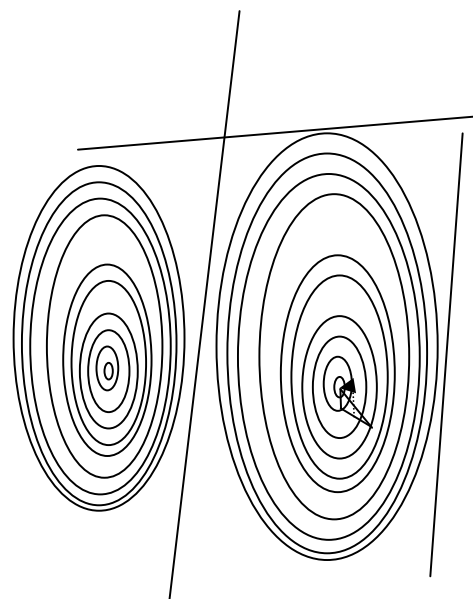


Figure 14. Processing of case 3 (3/3)

## IV. CONCLUSION

In this study, we proposed the networking factor of e-science.

The three main contributions of this study are as follows: First, we confirmed the networking factor of e-science. Second, we described the advanced constrained simplex method. Finally, the advanced constrained simplex method was employed to resolve the problem of MIMO. Our proposed method is relatively well adapted to any optimization problem. The results obtained by using this new technique could be very useful.

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