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Analysis of Resonator Modes in a Penrose Unilluminable Room

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Abstract—Resonator modes in a Penrose unilluminable room are calculated using the finite-difference time-domain method. Spatially chaotic wavefunctions are obtained in the regions where the ray trajectories are confined. A chaotic wavefunction for which light spreads across the whole cavity was also derived. The existence of a widely spreading chaotic wavefunction shows that the properties of the unilluminable room derived from ray dynamical simulations do not always represent all of the resonator modes.

1. Introduction

The ray-wave correspondence in two-dimensional dielectric optical cavities is an interesting research topic from a viewpoint of fundamental physics [1–3]. In 1958, Penrose considered a two-dimensional cavity that always has dark regions which light rays cannot reach when a point light source is placed in the cavity [4,5]. This configuration is called a Penrose unilluminable room. Recently the ray dynamics in such a room have been investigated and have been found to exhibit three kinds of chaotic ray trajectories confined in different regions [6]. The light wave propagation has been modeled for a point light source using the finite-difference time-domain (FDTD) method [7]. A small amount of light leaks into the dark regions due to diffraction at the edges of the cavity. It would be interesting to investigate the modal properties of this optical cavity. In the present work, the resonator modes in a Penrose unilluminable room are calculated using the FDTD method.

2. Model

Figure 1 is a diagram of the Penrose unilluminable room studied here [7]. The left and right curved mirrors are half-ellipses whose semimajor and semiminor axes are 1.60 and 1.00 μm, respectively. Points F_1 and F_2 are the foci of the left half-ellipse, while points F_3 and F_4 are those of the right half-ellipse. The cavity has four arm regions labeled A, A', B, and B'. The top and bottom curved mirrors connecting points F_1 and F_3 and points F_2 and F_4 are half-ellipses whose semimajor and semiminor axes are 1.00 and 0.649 μm, respectively. The refractive index of the room is set to 3.3 to match that of GaAs laser diode cavity at a target wavelength of 860 nm. To ensure total internal reflection at the cavity edges, the region outside the cavity is taken to be a perfect electrical conductor.

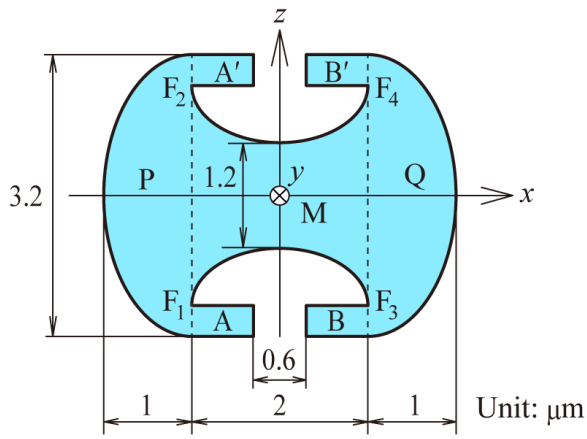

Figure 1: Schematic of the Penrose unilluminable room.

The cavity has three kinds of chaotic ray trajectories [6]. One set of rays starts in region A or A', and is confined to regions A, P, and A' as illustrated in Fig. 2(a). Another set starts in region B or B', and is confined to regions B, Q, and B' as shown in Fig. 2(b). The third kind consists of trajectories starting in region M, and is confined to regions P, M, and Q as sketched in Fig. 2(c). The cavity also has three stable periodic orbits, namely the axial orbit in Fig.

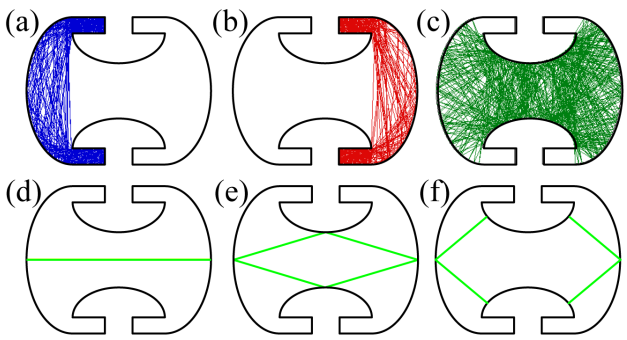

Figure 2: Ray trajectories confined in the cavity: (a) chaotic ray trajectories in regions A, P, and A'; (b) chaotic ray trajectories in regions B, Q, and B'; (c) chaotic ray trajectories in regions P, M, and Q; (d) a stable axial periodic orbit; (e) a stable diamond-shaped orbit; and (f) two stable V-shaped orbits.

2(d), the diamond-shaped orbit in Fig. 2(e), and two V-shaped orbits in Fig. 2(f).

The commercial software package FullWAVE [8] is used for the FDTD calculations. To obtain resonator modes associated with the chaotic ray trajectories, an impulsive light source having a Gaussian beam profile and a point monitor of the temporal waveform are positioned inside the cavity. The waveform of the magnetic field is measured and its spectrum is calculated from a Fourier transform. The mode patterns of the magnetic field distribution inside the cavity at the resonance wavelengths are calculated. Two combinations are used for the positions of the light source and waveform monitor. Combination 1 consists of the light source in region A (at $x = -0.65$ µm and $z = -1.4245$ µm) with the waveform monitor in region P (at $x = -1.6$ µm and $z = 0$) to excite resonator modes associated with the chaotic ray trajectories of Fig. 2(a). Combination 2 has the light source in region M (at $x = 0.1$ µm and $z = 0.2$ µm) with the waveform monitor in region P (at $x = -1.6$ µm and $z = 0$) to excite resonator modes associated with the chaotic ray trajectories of Fig. 2(c). The impulsive light is emitted in the negative x direction in both cases. The light propagating in the cavity is p-polarized with electromagnetic components E_x, H_y, and E_z.

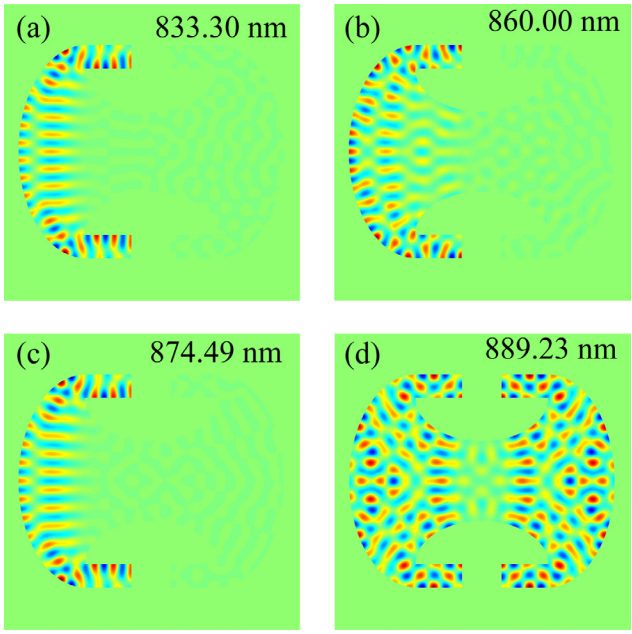

Figure 4: Magnetic field patterns for resonator modes at the indicated wavelengths.

3. Results

Figure 3 plots the H_y^2 spectrum for combination 1 in the wavelength region from 800 to 900 nm. Several resonance peaks can be seen. Figure 4 shows the magnetic field distribution for four resonator modes corresponding to the peaks found in Fig. 3. Spatially chaotic wavefunctions confined to regions A, P, and A' are shown in Fig. 4(a), (b), and (c). A small amount of light leaks into region M in Fig. 4(b). A spatially chaotic wavefunction spreading across the whole cavity is shown in Fig. 4(d). In the ray dynamical simulations, optical rays starting in region A are completely confined to regions A, P, and A' and can never reach regions M, Q, B, and B', as seen in Fig. 2(a). In the modal analysis, on the other hand, the light is evidently not strictly confined to regions A, P, and A'.

Figure 5 graphs the H_y^2 spectrum for combination 2 in the wavelength region from 800 to 900 nm. The four peaks indicated by the blue arrows correspond to resonator modes quantized along the stable axial periodic orbit, as shown in Fig. 6. We also found higher order axial resonator modes, as shown in Fig. 7(a), and complex spatially chaotic wavefunctions that spread into regions P, M, and Q, as shown in Fig. 7(b). The chaotic wavefunctions are associated with the chaotic ray trajectories of Fig. 2(c). However, a small amount of light leaks into regions A, A', B, and B'.

The Penrose unilluminable room has three kinds of chaotic ray trajectories confined to different regions. These trajectories are independent of each other. In the modal analysis based on the FDTD method, some spatially chaotic wavefunctions arise that are associated with the chaotic ray trajectories. Moreover, a chaotic wavefunction is found that spreads across the whole cavity. This widely spreading

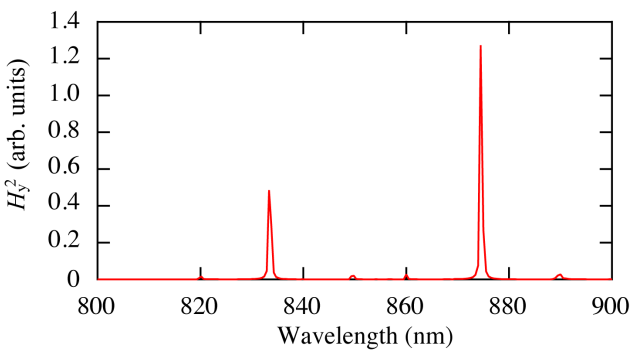

Figure 3: Spectrum of H_y^2 for combination 1.

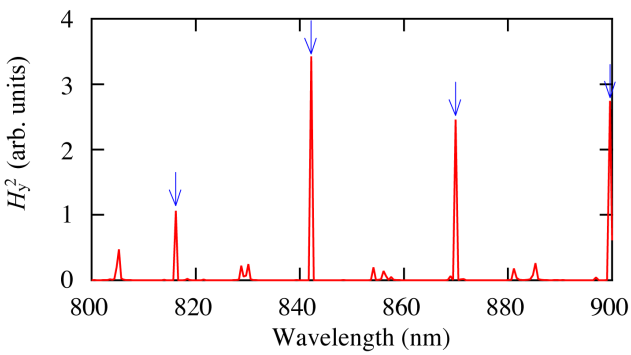

Figure 5: Spectrum of H_y^2 for combination 2.

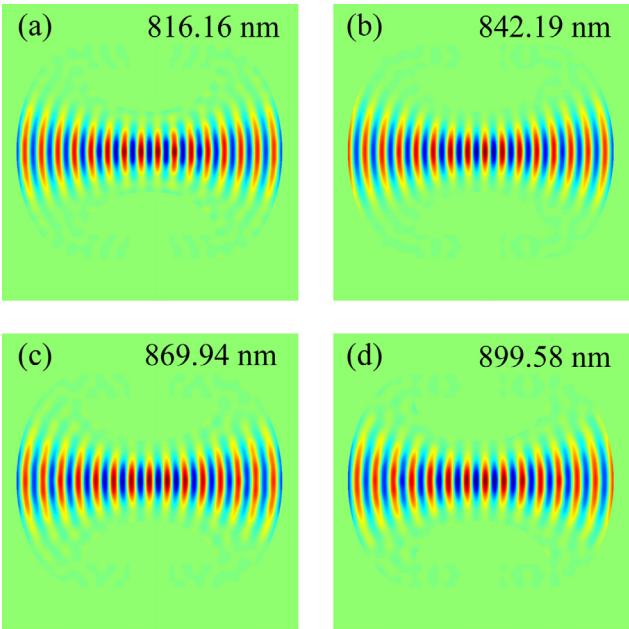

Figure 6: Magnetic field patterns for resonator modes quantized along the stable axial periodic orbit at the indicated wavelengths.

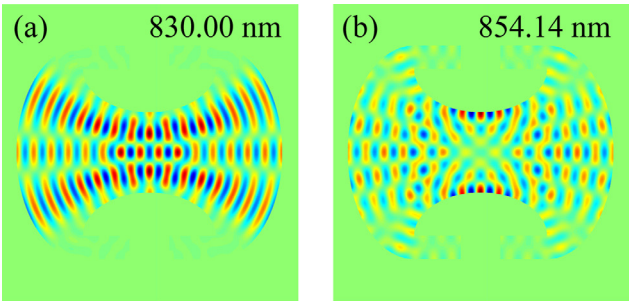

Figure 7: Magnetic field patterns for resonator modes at the indicated wavelengths.

wave implies that coupling exists among the three kinds of chaotic ray trajectories. In previous work, it was shown that a small amount of light leaks into the dark regions due to diffraction at the cavity edges [7]. The coupling among the three chaotic ray trajectories may also be caused by diffraction.

4. Conclusions

Resonator modes have been calculated for a Penrose unilluminable room using the FDTD method. Spatially chaotic wavefunctions associated with three kinds of chaotic ray trajectories have been found. Moreover, a chaotic wavefunction has been discovered that spreads across the whole cavity, implying coupling among the three chaotic ray trajectories. This coupling may arise from diffraction at the cavity edges. The Penrose unilluminable room is expected to have a wide variety of resonator modes in addition to those obtained here. Thus the correspondence between rays and waves in this type of optical cavity may reveal new aspects of fundamental physics.

Acknowledgment

T. F. first learned about Penrose unilluminable room from Prof. A. E. Siegman in 2007. The authors thank Mr. Yoshiki Mihara for assistance with the numerical calculations.

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