2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

A Nonlinear Dynamical Model of Two-Sites Electricity and Heat Supply System

Hikaru Hoshino†, Yoshihiko Susuki†‡, Takashi Hikihara†

†Department of Electrical Engineering, Kyoto University
Katsura, Nishikyo, Kyoto 615-8510 Japan
‡JST-CREST 4-1-8 Honcho, Kawaguchi, Saitama 332-0012 Japan
Email: hoshino@dove.kuee.kyoto-u.ac.jp, susuki.yoshihiko.5c@kyoto-u.ac.jp, hikihara.takashi.2n@kyoto-u.ac.jp

Abstract—This paper proposes a nonlinear dynamical model of electricity and heat supply in a simple two-sites system. Each site includes a CHP (Combined Heat and Power) plant with gas turbine-generator, and the two sites are coupled via a power transmission line and a steam pipe. The developed model represents short-term (zero to ten seconds) and mid-term (several tens of seconds) dynamics occurring in the system that are crucial for its operation with a specification of supply stability. A numerical simulation of the model is performed for investigating a feasibility of simultaneous supply of electricity and heat in the two-sites system.

1. Introduction

Energy systems integration has attracted a lot of interest in science and technology [1, 2]. In this integration, it is expected that multiple types of energy such as electricity, heat, and gas are managed simultaneously in order to increase the reliability and efficiency of energy supply at a variety of different scales. A key technology for this integration is the so-called Combined Heat and Power (CHP) [3] that has a gas-fired distributed generation system and utilizes the waste heat as a by-product of the conversion of fuel into electricity. It is shown in [3] that the installation of CHP plants can increase the reliability of electricity supply, and also in [4] that it can increase the energy efficiency of heat supply.

The purpose of this paper is to propose a nonlinear dynamical model for electricity and heat supply. The model can contribute to operational design of energy supply with multiple CHP plants. In particular, we intend the CHPs' operation for realizing fast change of energy supply with satisfying the stability specification: see [3]. In this case, the supply control possibly causes a rapid and large change in generated energy of multiple CHP plants. This brings about nonlinear problems on feasibility and stability of the energy supply against a large change of operating condition. The nonlinear model developed in this paper represents short-term (zero to ten seconds) and mid-term (several tens of seconds) dynamics, which correspond to electromechanical dynamics of synchronous generators [5] and internal dynamics of Heat Recovery Steam Generators (HRSGs) in heat supply systems [6]. To the best of our knowledge, mathematical modeling for the dynamics of multi-energy supply in this time domain has not been reported. In this paper, we derive a nonlinear dynamical model of a simple two-sites supply system, in which two CHP plants are operating and coupled via not only a power transmission line but also a steam pipe. A numerical simulation of the derived model is also performed for investigating a feasibility of simultaneous supply of electricity and heat in the two-sites system.

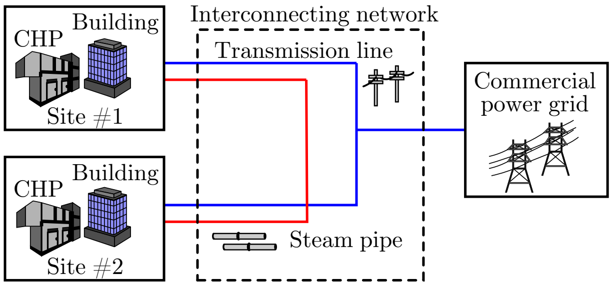


Figure 1: Two-sites electricity and heat supply system

2. Two-Sites Electricity and Heat Supply System

Figure 1 shows the two-sites system studied in this paper. The system is based on our previous study [7, 8]. The notion of *site* is a unit of energy system and consists of a CHP plant with gas turbine-generator, and electric and heat loads. One of practical examples of the site is a commercial or large residential building with own CHP plant. The two sites are interconnected via a transmission line and a steam pipe. The transmission line is connected to a commercial power grid. Understanding dynamics and stability of this simple system is of significance because it corresponds to a minimal structure where operational designs of the energy supply can be clearly apprehended.

3. Modeling

In this section, we derive a nonlinear dynamical model of the two-sites electricity and heat supply system. Figure 2 shows the block diagram of the system. Each arrow in the figure shows the positive direction of power flow. The time responses of the power flows are represented with the model derived below. For the operational design, it is desirable to derive a simple model capturing the essential physics behind the energy supply. With this aim, we make the following assumptions:

1) Dynamics of gas turbines, which are faster than electromechanical dynamics of generators and internal dynamics of HRSGs [9], are negligible.
2) Electromagnetic transients in generators and transmission lines, which are faster than the electromechanical dynamics of generators [5], are negligible.

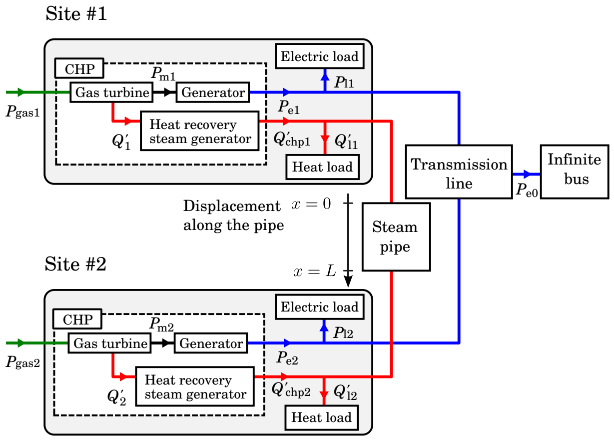


Figure 2: Block diagram of the two-sites electricity and heat supply system

3) The commercial power grid is regarded as a infinite bus, which is a voltage source with constant phase, magnitude, and frequency not affected by the amount of current [5].
4) The boiler-wall temperature in HRSGs is equivalent to steam/water mixture temperature, i.e. the saturation temperature [10].
5) Feedwater to the boiler is at the condition of saturated liquid [10].
6) The mass exchange between phases by condensation due to the heat losses in steam pipes is negligible [11].
7) The influence of compressibility of a working steam is negligible on its velocity profile because the steam velocity used in standard heat management is sufficiently small than the sound speed [12].

3.1. Gas turbine

The gas turbine at site #i converts the fuel flow rate $P_{\mathrm{gas}i}$ to both the mechanical power $P_{\mathrm{m}i}$ and the heat flow rate Q'_i. The conversion of energy at each gas turbine is represented by

$$\begin{bmatrix} P_{\mathrm{m}i} \\ Q'_i \end{bmatrix} = \begin{bmatrix} \eta_{\mathrm{e}i} \\ \eta_{\mathrm{h}i} \end{bmatrix} P_{\mathrm{gas}i}, \tag{1}$$

where $\eta_{\mathrm{e}i}$ stands for the thermal efficiency of the CHP plant at site #i, and $\eta_{\mathrm{h}i}$ the ratio of recovered heat flow rate to gas input rate. The coefficients $\eta_{\mathrm{e}i}$ and $\eta_{\mathrm{h}i}$ are constant and satisfy $\eta_{\mathrm{e}i} + \eta_{\mathrm{h}i} < 1$. All the losses due to conversion of energy in the CHP plants are included in these coefficients.

3.2. Electric Sub-system

The electric sub-system in Figure 2 consists of the two generators, electric loads, transmission lines, and infinite bus. The generator at each site #i converts the mechanical input power $P_{\mathrm{m}i}$ to the electric output power $P_{\mathrm{e}i}$. Here, we denote by δ_i the angular position of rotor with respect to the infinite bus. The deviation of rotor speed relative to the system angular frequency is denoted by ω_i. The electromechanical dynamics of the generator #i are represented by the so-called swing equations [5]:

$$\frac{\mathrm{d}\delta_i}{\mathrm{d}t} = \omega_i, \quad \frac{\mathrm{d}\omega_i}{\mathrm{d}t} = \frac{1}{2H_i}(P_{\mathrm{m}i} - D_i\omega_i - P_{\mathrm{e}i}), \tag{2}$$

where H_i is the per-unit time constant of the generator, and D_i the damping coefficient of the generator. The electric output power $P_{\mathrm{e}i}$ is given by

$$\begin{aligned} P_{\mathrm{e}i} = G_{ii}E_i^2 + \sum_{j\in\mathcal{I}\setminus\{i\}} E_iE_j\{G_{ij}\cos(\delta_i-\delta_j) \\ + B_{ij}\sin(\delta_i-\delta_j)\}, \end{aligned} \tag{3}$$

where $\mathcal{I} = \{0,1,2\}$, and $\delta_0\ (=0)$ represents the infinite bus. The constant E_i is the terminal voltage of the generator, and $G_{ij} + \mathrm{j}B_{ij}$ are the transfer admittances between generator #i and #j. The influence of electric load consumptions and transmission losses on $P_{\mathrm{e}i}$ are represented by G_{ij}.

3.3. Heat Sub-system

The heat sub-system in Figure 2 consists of the two HRSGs, heat loads and a steam pipe. A model of the heat sub-system is derived from the standard boiler model [6, 10] and one-dimensional gasflow equations [13]. The state variables of the model are the steam pressure p_i in the HRSGs and the steam velocity u in the pipe.

First of all, we briefly review the nonlinear dynamical model of the HRSG developed in [6, 10]. Let V represent volume, ρ density, h specific enthalpy, T temperature, and m' mass flow rate. Furthermore, the four subscripts s, w, f and m represent steam, water, feedwater, and metal, respectively. According to [6], the pressure dynamics are formulated as

$$e_i\frac{\mathrm{d}p_i}{\mathrm{d}t} = Q'_i - m'_{\mathrm{f}i}(h_{\mathrm{w}i} - h_{\mathrm{f}i}) - m'_{\mathrm{s}i}(h_{\mathrm{s}i} - h_{\mathrm{w}i}), \tag{4}$$

with

$$\begin{aligned} e_i = (h_{\mathrm{s}i} - h_{\mathrm{w}i})V_{\mathrm{s}i}\frac{\partial\rho_{\mathrm{s}i}}{\partial p_i} + \rho_{\mathrm{s}i}V_{\mathrm{s}i}\frac{\partial h_{\mathrm{s}i}}{\partial p_i} + \rho_{\mathrm{w}i}V_{\mathrm{w}i}\frac{\partial h_{\mathrm{w}i}}{\partial p_i} \\ + m_{\mathrm{t}i}C_{\mathrm{p}}\frac{\partial T_{\mathrm{m}i}}{\partial p_i} - V_{\mathrm{s}i} - V_{\mathrm{w}i}, \end{aligned} \tag{5}$$

where m_t stands for the total mass of the metal of the HRSG, and C_{p} the specific heat of the metal. The thermodynamic properties h_{s}, h_{w}, ρ_s, ρ_w, and T_{s} are evaluated from the steam table. In this paper, according to [6], the steam tables are approximated by quadratic functions, and these properties are represented as functions of p_i, for example, $h_{\mathrm{s}i} = h_{\mathrm{s}}(p_i)$. Furthermore, from the fifth assumption implying $h_{\mathrm{f}} = h_{\mathrm{w}}$, Equation (4) becomes

$$e_i(p_i)\frac{\mathrm{d}p_i}{\mathrm{d}t} = Q'_i - m'_{\mathrm{s}i}h_{\mathrm{c}}(p_i), \tag{6}$$

where $h_{\mathrm{c}} = h_{\mathrm{s}} - h_{\mathrm{w}}$ corresponds to the condensation enthalpy.

The transient steam flow in a pipe is described by one-dimensional mass and momentum continuity equations. From the sixth assumption, the mass continuity equation is represented as follows:

$$\frac{\partial\rho_{\mathrm{s}}}{\partial t} + \frac{\partial}{\partial x}(\rho_{\mathrm{s}}u) = 0, \tag{7}$$

where u is the velocity of the steam in the pipe and x the displacement variable along the pipe. Also, the momentum equation is represented by

$$\frac{\partial}{\partial t}(\rho_s u)+\frac{\partial}{\partial x}(\rho_s u^2)+\frac{\partial p}{\partial x}+\Delta p=0, \tag{8}$$

where Δp is the shear force acting on a gas element due to friction and condensation. Based on the sixth assumption, Δp is approximated in [11, 13] as follows:

$$\Delta p=\frac{\lambda\rho_s u|u|}{2d}, \tag{9}$$

where λ is the friction coefficient, and d the diameter of the pipe. From the seventh assumption, Equation (7) and (8) become

$$\frac{\partial u}{\partial x}=0,\quad \rho_{s0}\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}+\frac{\lambda\rho_{s0}u|u|}{2d}=0, \tag{10}$$

where ρ_{s0} stands for the density of the steam. Therefore, by integrating the second equation of Equation (10) from $x = 0$ (site #1) to $x = L$ (site #2), the following ordinary differential equation is derived for representing the dynamics of heat transfer:

$$\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{1}{\rho_{s0}L}(p_1-p_2)-\frac{\lambda u|u|}{2d}, \tag{11}$$

where L stands for the distance between the sites and becomes an important parameter that characterizes the influence of spatial scales on dynamics and stability of the system. Thus, the model of heat sub-system is derived from Equation (6) and (11) with the mass continuity equation at each site #i represented by

$$\left.\begin{aligned} m'_{s1}-\frac{Q'_{l1}}{h_c(p_1)}-\frac{\pi d^2}{4}\rho_s(p_1)u=0,\\ m'_{s2}-\frac{Q'_{l2}}{h_c(p_2)}+\frac{\pi d^2}{4}\rho_s(p_2)u=0,\end{aligned}\right\} \tag{12}$$

where the second term on the left-hand sides represents the mass flow rate of the steam consumed in the load, and the third term represents the steam entering into the pipe. The heat flow rate $Q'_{\mathrm{chp}i}$ is derived as a product of the condensation enthalpy h_c and the mass flow rate as follows:

$$\left.\begin{aligned} Q'_{\mathrm{chp1}}&=Q'_{l1}+\frac{\pi d^2}{4}h_c(p_1)\rho_s(p_1)u,\\ Q'_{\mathrm{chp2}}&=Q'_{l2}-\frac{\pi d^2}{4}h_c(p_2)\rho_s(p_2)u.\end{aligned}\right\} \tag{13}$$

3.4. Derived Model

Consequently, the following nonlinear dynamical model is derived for representing the short-term (zero to ten seconds) and mid-term (several tens of seconds) dynamics of the electric and heat sub-system shown in Figure 2: for the electric sub-system,

$$\left.\begin{aligned} &\frac{\mathrm{d}\delta_1}{\mathrm{d}t}=\omega_1,\quad \frac{\mathrm{d}\delta_2}{\mathrm{d}t}=\omega_2,\\ &\frac{\mathrm{d}\omega_1}{\mathrm{d}t}=\frac{1}{2H_1}\Big\{\eta_{e1}P_{\mathrm{gas1}}-D_1\omega_1-P_{e1}(\delta_1,\delta_2)\Big\},\\ &\frac{\mathrm{d}\omega_2}{\mathrm{d}t}=\frac{1}{2H_2}\Big\{\eta_{e2}P_{\mathrm{gas2}}-D_2\omega_2-P_{e2}(\delta_1,\delta_2)\Big\},\end{aligned}\right\} \tag{14}$$

and for the heat sub-system,

$$\left.\begin{aligned} e_1(p_1)\frac{\mathrm{d}p_1}{\mathrm{d}t}&=\eta_{h1}P_{\mathrm{gas1}}-Q'_{\mathrm{chp1}}(p_1,u),\\ e_2(p_2)\frac{\mathrm{d}p_2}{\mathrm{d}t}&=\eta_{h2}P_{\mathrm{gas2}}-Q'_{\mathrm{chp2}}(p_2,u),\\ \frac{\mathrm{d}u}{\mathrm{d}t}&=\frac{1}{\rho_{s0}L}(p_1-p_2)-\frac{\lambda}{2d}u|u|.\end{aligned}\right\} \tag{15}$$

Based on the state dynamics, this model describes the time responses of the electric output power P_{ei} and the heat flow rate $Q'_{\mathrm{chp}i}$ via Equation (3) and (13).

4. Feasibility Study

This section performs a numerical simulation of the derived model for investigating simultaneous supply of electricity and heat. A possible strategy for energy supply in the two-sites system is firstly described. The feasibility of the strategy is then studied numerically.

4.1. Possible Supply Strategy

In the two-sites system we have two supply objectives: electricity supply and heat supply. These objectives are formulated as follows. In a steady operating condition, the total outputs of electricity and heat in the two-sites system are represented by

$$\begin{bmatrix} P_{e1}+P_{e2}\\ Q'_{\mathrm{chp1}}+Q'_{\mathrm{chp2}}\end{bmatrix}=\begin{bmatrix}\eta_{e1} & \eta_{e2}\\ \eta_{h1} & \eta_{h2}\end{bmatrix}\begin{bmatrix}P_{\mathrm{gas1}}\\ P_{\mathrm{gas2}}\end{bmatrix}. \tag{16}$$

Without any loss in the heat supply, the condition to balance the heat supply from the CHP plants and the load consumption Q'_{li} is described by $Q'_{\mathrm{chp1}}+Q'_{\mathrm{chp2}}=Q'_{l1}+Q'_{l2}$. By introducing a new parameter Q'_n to represent the heat transfer reference from one site to the other, the above condition is rewritten as follows:

$$\eta_{h1}P_{\mathrm{gas1}}=Q'_{l1}+Q'_n,\quad \eta_{h2}P_{\mathrm{gas2}}=Q'_{l2}-Q'_n. \tag{17}$$

Therefore, the total output of electricity $P_{e1}+P_{e2}$ is parametrized by Q'_n:

$$P_{e1}+P_{e2}=\frac{\eta_{e1}}{\eta_{h1}}Q'_{l1}+\frac{\eta_{e2}}{\eta_{h2}}Q'_{l2}+\left(\frac{\eta_{e1}}{\eta_{h1}}-\frac{\eta_{e2}}{\eta_{h2}}\right)Q'_n. \tag{18}$$

Equation (18) implies that by operation of the CHP plants with different values of η_{ei}/η_{hi}, it is possible to vary the total electricity output while maintaining the total heat output. In the rest of this paper, we refer to η_{ei}/η_{hi} as the electricity-heat ratio of the CHP plant at site #i.

4.2. Numerical Simulation

For exploring a feasibility of the supply strategy presented above, we investigate the dynamics of the system under a change of the heat transfer reference Q'_n. Here, we suppose that the CHP plant at site #1 has a higher electricity-heat ratio than that of the CHP plant at site #2. The setting of parameters for simulations is given in Table 1. For the electric sub-system, all the parameters except G_{ij} are based on the distribution networks in conventional electric power systems [5]. The setting $G_{ij}=0$ implies

Table 1: Parameters for simulation

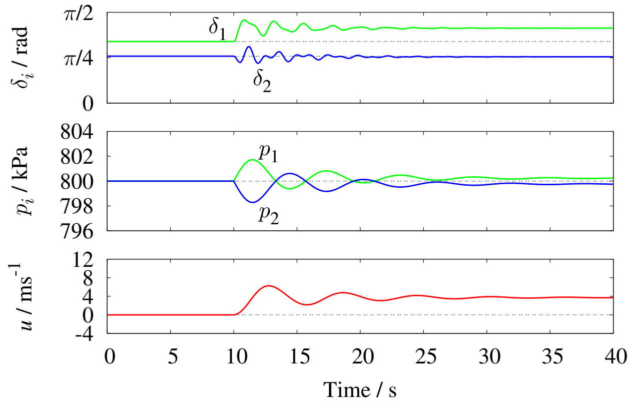
P_{base}	5.0 MW	1.0	Q'_{base}	6.25 MJ/s	1.0
η_{e1}		0.5	η_{h1}		0.4
η_{e2}		0.3	η_{h2}		0.4
D_i		8.7	H_i	10 s	3770
E_i		1.0	G_{ij}		0
B_{12}		0.5	B_{i0}		1.0
p_{base}	800 kPa		ρ_{s0}	4.16 kg/m^3	
d	0.2 m		L	200 m	
λ		0.02	$Q'_{\mathrm{l}i}$	5.0 MJ/s	0.8

no loss in the transmission lines and no consumed electric power in the loads. The parameters for the heat sub-system are based on the steam network for district heating systems presented in [11].

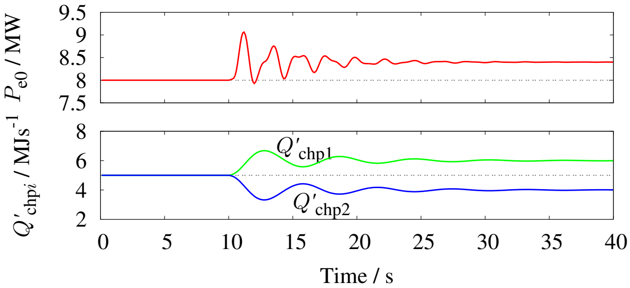
Figure 3a shows the step responses of the state variables δ_1, δ_2, p_1, p_2, and u. The heat transfer reference Q'_{n} suddenly changes from 0 to 0.16 (1.0 MJ/s) at $t = 10$ s. Initially, the system has no heat transfer and is at a steady operating condition. After Q'_{n} changes, in the electric sub-system, the phase angles δ_1 and δ_2 exhibit oscillatory responses and move to a new steady operating condition, where the value of δ_1 (or δ_2) becomes larger (or smaller) than the previous value. In the heat sub-system, after the oscillations of pressure p_1, p_2 and velocity u, the values of $p_1 - p_2$ and u become positive. This implies that the steam is transfered from site #1 to site #2 accompanied with the pressure drop $p_1 - p_2$. Figure 3b shows the power flows P_{e0} to the infinite bus and Q'_{chp1}, Q'_{chp2} from the CHP plants.The value of P_{e0} becomes larger by 0.08 (400 kW). In the heat sub-system, Q'_{chp1} and Q'_{chp2} change symmetrically, which implies that the heat supply to the heat loads is compensated by the heat transfer from site #1 to site #2. This result clearly shows that the supply strategy presented above is feasible.

5. Conclusions

This paper reported a mathematical model of the dynamics occurring in the two-sites system for investigating the simultaneous supply of electricity and heat. The nonlinear dynamical model was derived by combining the standard dynamic models of a synchronous generator and a heat recovery steam generator. A numerical simulation of the derived model was performed to show the feasibility of regulating the total electricity output while maintaining the total heat output by heat transfer between the two sites. This result helps us to design a CHPs' controller for simultaneous supply of electricity and heat with satisfying the stability specification. Future directions of the work are generalization of the derived model for a system with multiple sites and detailed analysis of nonlinear dynamics and stability of the system.



(a) state variables



(b) power flows

Figure 3: Step responses of (a) the state variables and (b) the power flows

References

[1] M. Geidl, G. Koeppel, P. Fravre-Perrod, B. Klöckl, G. Andersson, and K. Fröhlich, "Energy hubs for the future", *IEEE Power Energy M.*, vol. 5, no. 1, pp. 24–30, 2007.

[2] M. O'Malley and B. Kroposki, "Energy comes together: The integration of all systems", *IEEE Power Energy M.*, vol. 11, no. 5, pp. 18–23, 2013.

[3] IEA, *Co-generation and renewables: Solutions for a low-carbon energy future*, 2011.

[4] T. Nagota, Y. Shimoda, M. Mizuno, "Comparison study on energy conservation effect between district heating and cooling system and individual heat source system—Simulation evaluation of co-generation system incorporated absorption chiller & boiler system" , *Trans. of SHASE*, no. 118, pp. 37–47, 2007 (in Japanese).

[5] J. Machowski, J. W. Bialek, and J. R. Bumby, *Power system dynamics: Stability and control*, 2nd ed., John Wiley & Sons, 2008.

[6] K. J. Åström and R. D. Bell, "Drum-boiler dynamics", *Automatica*, vol. 36, no. 3, pp. 363–373, 2000.

[7] H. Hoshino, Y. Susuki, T. Hikihara, "A study on dynamical model of power and heat supply networks" *Proc. 13th SICE Control Division Conference*, PS-025, 2013 (in Japanese).

[8] ——, "Numerical studies on dynamical model of regional power and heat supply system" *IEICE Technical Report*, NLP, vol. 113, no. 15, pp. 19–24, 2013 (in Japanese).

[9] F. P. de Mello and D. J. Ahner, "Dynamic models for combined cycle plants in power system studies", *IEEE T. Power Syst.*, vol. 9, no. 3, pp. 1698–1708, 1994.

[10] T. S. Kim, D. K. Lee, and S. T. Ro, "Dynamic behavior analysis of a heat recovery steam generator during start up", Int. J. Energy Res., vol. 24, no. 2, pp. 137–149, 2000.

[11] JSME, *Hydraulic losses in pipes and ducts*, Maruzen, 1979 (in Japanese).

[12] L. D. Landau and E. M. Lifshitz, *Fluid mechanics*, Pergamon, 1959.

[13] A. J. Osiadacz, *Simulation and analysis of gas networks*, E. & F. N. Spon, 1987.