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Co-Existence of Chaos-Based Communication Systems and Conventional Spread-Spectrum Communication Systems

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Abstract

This paper studies the performance of selected chaos-based systems which share their frequency bands with conventional spread-spectrum systems. Such a scenario may occur in normal practice when chaos-based systems are introduced while the conventional systems are still in operation. The particular chaos-based systems under study are the coherent chaos-shift-keying (CSK) system and the non-coherent differential CSK system, and the co-existing conventional system employs the standard direct-sequence spread-spectrum modulation scheme. Analytical expressions for the bit error rates are derived and computer simulations are performed to verify the analytical findings.

Index Terms — Chaos communication, chaos-shift-keying, differential chaos-shift-keying, direct-sequence spread-spectrum.

1. Introduction

Communication using chaos began to attract attention from researchers about a decade ago [1, 2]. Much of the research work has focused on the basic modulation processes and the noise performance assuming ideal channel conditions. Among the various digital chaos-based communication schemes, chaos-shift-keying (CSK) and differential CSK (DCSK) schemes have been most thoroughly analyzed [3, 4].

Being spread-spectrum, chaos-based communication systems are expected to perform reasonably well even in the presence of other wideband signals sharing the same bandwidth. Such a scenario may occur in normal practice, for example, when chaos-based systems are introduced while the conventional systems are still in operation.

The basic problem we wish to investigate in this paper is the performance of chaos-based digital communication systems when the channel is under the influence of (a) additive white Gaussian noise; and (b) a wideband signal generated from a co-existing conventional spread-spectrum communication system, which shares the same frequency band as the chaos-based system under study.

Fig. 1 shows a block diagram of the system under study. Our analysis will be carried out in a discrete-time fashion. At time k, the receiver receives signals consisting of three components, namely chaotic, conventional spread-spectrum and additive noise. This corrupted signal will be used by the receiver to recover the transmitted data stream. Coherent or non-coherent detection schemes may be applied in the receiver, depending upon the modulation method used.

In the following sections, we focus on the coherent CSK system [5] and the non-coherent DCSK system [4]. The co-existing conventional system is a direct-sequence spread-spectrum (DS-SS) system [6]. We will develop analytical expressions for the bit error rates of the recovered data streams for each of the chaos-based communication schemes under the aforedescribed channel conditions.

2. Coherent CSK System

We consider a discrete-time baseband binary CSK communication system in which one chaos generator is used to produce chaotic signal samples $\{x_k\}$. Suppose $\alpha_l \in \{-1,+1\}$ is the lth transmitted symbol and 2β is the spreading factor. Further, we assume that "-1" and "$+1$" occur with equal probabilities. During the lth bit duration, i.e., for $k = 2\beta(l-1)+1, 2\beta(l-1)+2, \ldots, 2\beta l$, the output of the CSK transmitter is $s_k = \alpha_l x_k$.

For the DS-SS system, we assume that the period of the pseudorandom (PN) sequence is very long. We denote the output power of the system by P_B. Essentially, we can model the output of the DS-SS system as a random binary symbol $b_k \in \{-1,+1\}$ multiplied by $\sqrt{P_B}$. Also, "-1" and "$+1$" occur with equal probabilities. Thus, for time k, the transmitted signal of the DS-SS system is represented by $u_k = \sqrt{P_B} b_k$.

The CSK and DS-SS signals are added, and corrupted by

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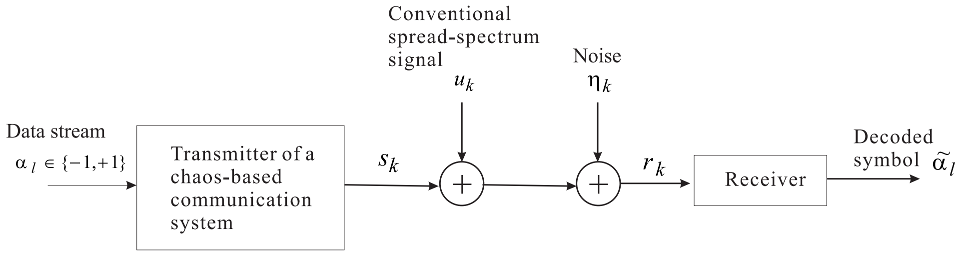


Fig. 1: Block diagram of a chaos-based digital communication system with conventional spread-spectrum signal and noise added.

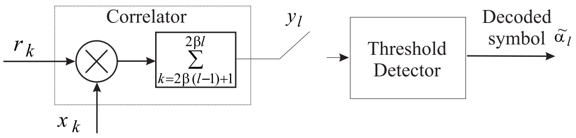


Fig. 2: Block diagram of a coherent CSK receiver.

an additive white Gaussian noise before arriving at the receiving end. The received signal r_k is thus given by $r_k = s_k + \sqrt{P_B} b_k + \eta_k$ where η_k is a Gaussian noise sample of zero mean and variance (power spectral density) $N_0/2$.

Assume that a correlator-type CSK receiver is employed. As shown in Fig. 2, the correlator output for the lth bit, y_l, is given by

$$\begin{aligned} y_l &= \underbrace{\alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2}_{\text{required signal}} + \underbrace{\sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} b_k x_k}_{\text{interfering DS-SS signal}} \\ &\quad + \underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k}_{\text{noise}} \\ &\equiv \alpha_l A + B + C \end{aligned} \tag{1}$$

where $\alpha_l A$, B and C are the required signal, interfering DS-SS signal and noise, respectively. Suppose a "+1" is transmitted in the CSK system during the lth symbol duration, i.e., $\alpha_l = +1$. The mean and variance of $y_l \mid (\alpha_l = +1)$ are readily shown equal to, respectively,

$$\mathrm{E}[y_l \mid (\alpha_l = +1)] = 2\beta P_s \tag{2}$$

and

$$\mathrm{var}[y_l \mid (\alpha_l = +1)] = 2\beta\Lambda + 2\beta P_B P_s + \beta N_0 P_s, \tag{3}$$

where $P_s = \mathrm{E}[x_k^2]$ denotes the average power of the chaotic signal, and $\Lambda = \mathrm{var}[x_k^2]$ is the variance of x_k^2. In the derivation of the variance, it has been assumed that the covariance of x_j^2 and x_k^2 vanishes for $j \neq k$, i.e.,

$$\mathrm{cov}[x_j^2, x_k^2] = \mathrm{E}[x_j x_k^2] - \mathrm{E}[x_j^2]\mathrm{E}[x_k^2] = 0 \quad \text{for} \quad j \neq k. \tag{4}$$

For the lth symbol, an error occurs when $y_l \leq 0 \mid (\alpha_l = +1)$. Since $y_l \mid (\alpha_l = +1)$ is the sum of a large number of random variables, we may assume that it follows a normal distribution when 2β is large. The error probability is thus given by

$$\begin{aligned} &\mathrm{Prob}(y_l \leq 0 \mid (\alpha_l = +1)) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{\mathrm{E}\left[y_l \mid (\alpha_l = +1)\right]}{\sqrt{2\mathrm{var}\left[y_l \mid (\alpha_l = +1)\right]}}\right) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{4\beta\Lambda + 4\beta P_B P_s + 2\beta N_0 P_s)}}\right) \end{aligned} \tag{5}$$

where erfc(.) is the complementary error function. Similarly, when $\alpha_l = -1$, the corresponding error probability can be shown equal to

$$\begin{aligned} &\mathrm{Prob}(y_l > 0 \mid (\alpha_l = -1)) \\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{4\beta\Lambda + 4\beta P_B P_s + 2\beta N_0 P_s)}}\right). \end{aligned} \tag{6}$$

Hence, the overall error probability of the lth transmitted symbol is

$$\begin{aligned} \mathrm{BER}_{\mathrm{CSK}}^{(l)} &= \mathrm{Prob}(\alpha_l = +1) \times \mathrm{Prob}(y_l \leq 0 \mid (\alpha_l = +1)) \\ &\quad + \mathrm{Prob}(\alpha_l = -1) \times \mathrm{Prob}(y_l \leq 0 \mid (\alpha_l = -1)) \\ &= \frac{1}{2}\mathrm{erfc}\left(\left[\beta\Omega + \left(\frac{E_b}{2P_B}\right)^{-1} + \left(\frac{E_b}{N_0}\right)^{-1}\right]^{-1/2}\right). \end{aligned} \tag{7}$$

where $E_b = 2\beta P_s$ denotes the average bit energy of the CSK system, and $\Omega = \mathrm{var}[x_k^2]/\mathrm{E}^2[x_k^2]$. It can be seen from (7) that $\mathrm{BER}_{\mathrm{CSK}}^{(l)}$ is independent of l. Thus, the error probability of the lth transmitted symbol is the same as the bit error rate (BER) of the system.

3. Non-coherent DCSK System

For a non-coherent DCSK system, the basic modulation process involves dividing the bit period into two equal slots. The first slot carries a reference chaotic signal, and the second slot bears the information. For a binary system, the second slot is the same copy or an inverted copy of the first slot

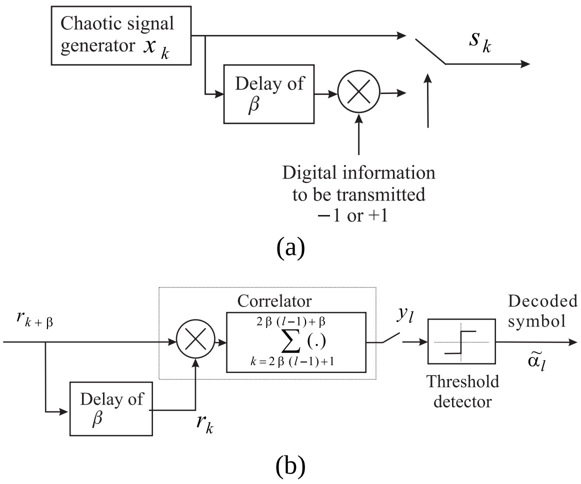


Fig. 3: Block diagram of a non-coherent DCSK system. (a) Transmitter; (b) receiver.

depending upon the symbol sent being "+1" or "−1". This structural arrangement allows the detection to be done in a non-coherent manner requiring no reproduction of the same chaotic carrying signals at the receiver. Essentially, the detection of a DCSK signal can be accomplished by correlating the first and the second slots of the same symbol and comparing the correlator output with a threshold. Fig. 3 shows the block diagrams of a DCSK transmitter and receiver pair.

Using similar procedures as in the previous section, it is readily shown that the means and variances of $y_l \mid (\alpha_l = +1)$ and $y_l \mid (\alpha_l = -1)$ are given by

$$\mathrm{E}[y_l \mid (\alpha_l = +1)] = \beta P_s \tag{8}$$

$$\mathrm{E}[y_l \mid (\alpha_l = -1)] = -\beta P_s \tag{9}$$

$$\begin{aligned} \mathrm{var}[y_l \mid (\alpha_l = +1)] &= \mathrm{var}[y_l \mid (\alpha_l = -1)] \\ &= \beta\Lambda + 2\beta P_B P_s + \beta P_B^2 + \beta P_s N_0 \\ &\quad + \beta P_B N_0 + \frac{\beta N_0^2}{4}. \end{aligned} \tag{10}$$

Assuming both $y_l \mid (\alpha_l = +1)$ and $y_l \mid (\alpha_l = -1)$ are normal, the bit error rate of the DCSK system under the interference of a DS-SS signal, denoted by $\mathrm{BER}_{\mathrm{DCSK}}$, equals the overall error probability of the lth transmitted symbol ($\mathrm{BER}_{\mathrm{DCSK}}^{(l)}$), i.e.,

$$\begin{aligned} & \mathrm{BER}_{\mathrm{DCSK}} = \mathrm{BER}_{\mathrm{DCSK}}^{(l)} \\ = \; & \mathrm{Prob}(\alpha_l = +1) \times \mathrm{Prob}(y_l \le 0 \mid (\alpha_l = +1)) \\ & + \mathrm{Prob}(\alpha_l = -1) \times \mathrm{Prob}(y_l > 0 \mid (\alpha_l = -1)) \\ = \; & \frac{1}{2}\mathrm{erfc}\left(\left[\frac{8\beta\Lambda + 8\beta P_B^2 + 8\beta P_B N_0 + 2\beta N_0^2}{E_b^2}\right.\right. \\ & \qquad \left.\left. + \frac{8P_B + 4N_0}{E_b}\right]^{-1/2}\right) \end{aligned} \tag{11}$$

where E_b is the bit energy defined in the previous section.

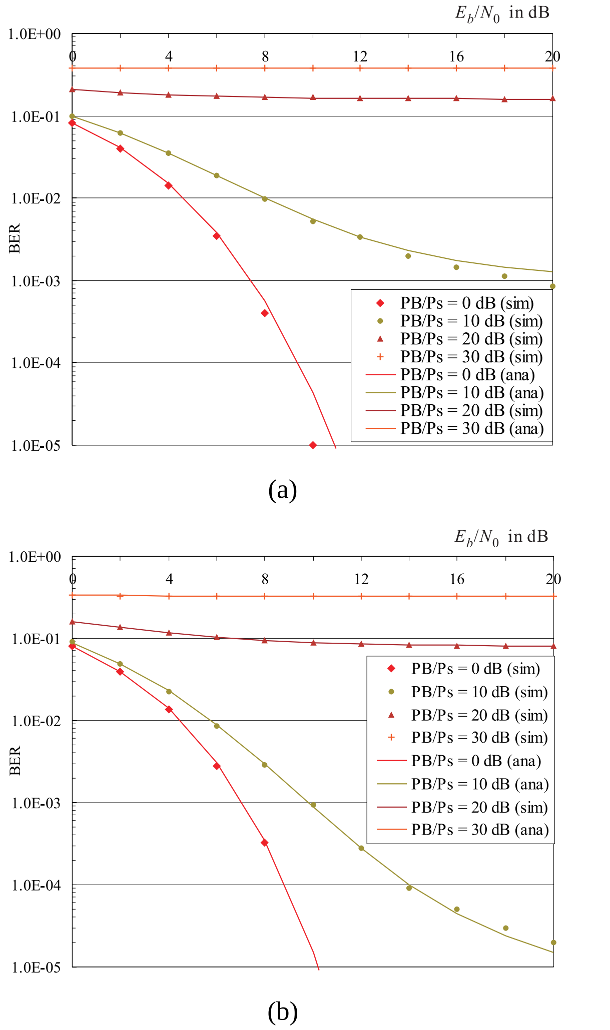


Fig. 4: BERs versus E_b/N_0 of the coherent CSK system under the interference of a direct-sequence-spread-spectrum signal. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 100; (b) spreading factor is 200.

4. Computer Simulations and Discussions

In this section the performance of the chaos-based digital communication systems under the influence of a DS-SS signal is studied by computer simulations. A logistic map is used for chaos generation. The form of the map is

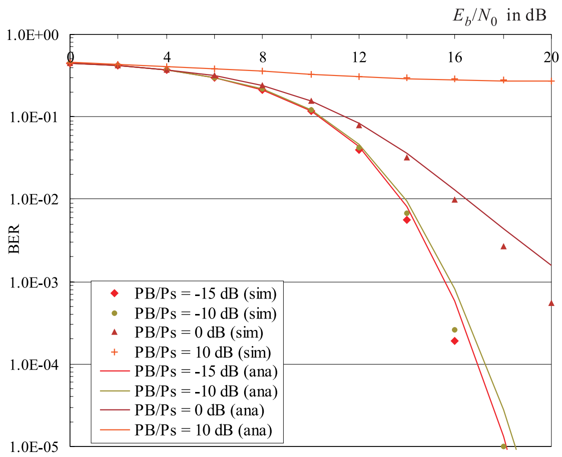
$$x_{k+1} = g(x_k) = 1 - 2x_k^2 \quad \text{where} \quad x_k \in (-1, +1). \tag{12}$$

Given that the invariant distribution function of x_k equals [7]

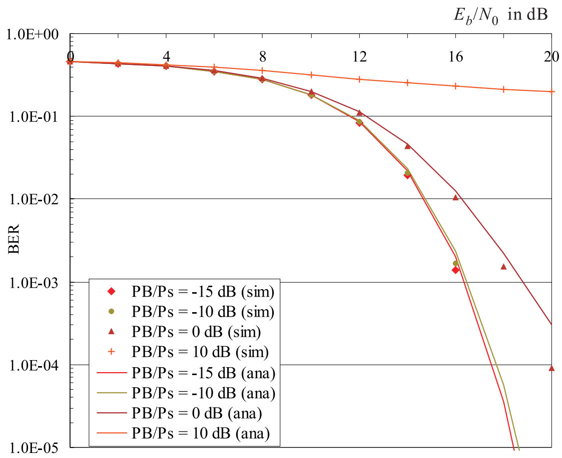
$$\rho(x_k) = \begin{cases} \dfrac{1}{\pi\sqrt{1 - x_k^2}} & \text{if } |x_k| < 1 \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

we obtain $P_s = 1/2$ and $\Lambda = 1/8$.

For comparison, we plot in each case also the analytical BERs obtained from the expressions derived in Sections 2



(a)



(b)

Fig. 5: BERs versus E_b/N_0 of the non-coherent DCSK system under the interference of a direct-sequence-spread-spectrum signal. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 100; (b) spreading factor is 200.

and 3. The relevant simulated BERs for the CSK system and DCSK systems are shown in Figs. 4 and 5. From these figures, we clearly see that the analytical and simulated BERs are in very good agreement for $2\beta = 100$ and 200, where the assumption of normal distribution of the conditional correlator output holds better. Also, as a general observation, the coherent CSK system consistently performs better than the non-coherent DCSK system under the influence of a co-existing conventional DS-SS signal. Further observations are summarized as follows.

1. The BERs increase (degrade) as P_B/P_s increases for a given E_b/N_0. This is apparently due to the increasing power of the DS-SS signal which causes more interference to the chaos-based systems, thus giving a higher BER.
2. For the CSK system, at $E_b/N_0 = 7$ dB and a spreading factor of 100, the BER degrades from about 10^{-3} to 10^{-2} when the conventional-to-chaotic-signal-power ratio (P_B/P_s) increases from 0 dB to 10 dB. For the DCSK system, with the same increase in P_B/P_s, the BER now degrades from about 10^{-3} to 0.3 at $E_b/N_0 = 20$ dB. Thus, the CSK system is more tolerant to wideband interfering signals compared with the DCSK system.

5. Conclusion

In this paper, the performance of chaos-based communication systems under the influence of a wideband signal generated from a co-existing conventional spread-spectrum system is investigated. The problem is important technically since chaos-based systems are spread-spectrum systems which are expected to resist interference and the kind of interference considered here, namely one being generated from another conventional spread-spectrum system such as a DS-SS system, represents a realistic (future) practical concern when chaos-based systems need to "co-operate" with existing systems. The present study provides useful performance data specifically for the coherent CSK and non-coherent DCSK systems when they are required to operate in channels already occupied by conventional wideband DS-SS systems.

References

[1] L. Kocarev, K.S. Halle, K. Eckert, L.O. Chua and U. Parlitz, "Experimental demonstration of secure communications via chaotic synchronization," *Int. J. Bifurc. Chaos*, vol. 2, pp. 709–713, 1992.

[2] U. Parlitz, L.O. Chua, L. Kocarev, K.S. Halle and A. Shang, "Transmission of digital signals by chaotic synchronization," *Int. J. Bifurc. Chaos*, vol. 2, pp. 973–977, 1992.

[3] H. Dedieu, M.P. Kennedy and M. Hasler, "Chaos shift keying: modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuit," *IEEE Trans. Circ. Syst. II*, vol. 40, no. 10, pp. 634–642, 1993.

[4] G. Kolumbán, B. Vizvari, W. Schwarz and A. Abel, "Differential chaos shift keying: A robust coding for chaos communications," *Proc., 4th International Workshop on Nonlinear Dynamics of Electronics Systems*, Seville, pp. 87–92, June 1996.

[5] G. Kolumbán, M.P. Kennedy and L.O. Chua, "The role of synchronization in digital communications using chaos - Part II: Chaotic modulation and chaotic synchronization," *IEEE Trans. on Circ. Syst. I*, vol. 45, no. 11, pp. 1129–1140, 1998.

[6] R.L. Peterson, R.E. Ziemer and D.E. Borth *Introduction to Spread Spectrum Communications*, Singapore: Prentice Hall, 1995.

[7] T. Kohda and A. Tsuneda, "Even- and odd-correlation functions of chaotic Chebychev bit sequences for CDMA," *Proc., IEEE Int. Symp. Spread Spectrum Tech. Appl.*, Oulu, Finland, pp. 391–395, 1994.