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Tradeoff Between Bit Error Rate and Acquisition Rate in an Asynchronous DS/CDMA System

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1. Introduction

One of the recent interesting facts about asynchronous direct sequence/code division multiple access (DS/CDMA) systems [1][2] is this: spreading sequences (SS) codes generated by Markov chains are superior to ones generated by sequences of i.i.d. binary random variables as well as LFSR sequences, such as Kasami codes, Gold codes in terms of multiple access interference (MAI) [3]. In the paper [3], chaos-based sequences generated by piecewise linear (PL) Markov maps were proven to minimize the average nterference parameter (AIP). This was supported by several following papers [4, 5, 6]. Corresponding with such results including almost all of the previous ones [4]–[7], is the fact that the receiver is assumed to be synchronized. It is natural to ask: to what extent do the designers of sequences sacrifice acquisition and tracking performance of the receiver for smaller bit error rate (BER) in using Markov sequences?

We gave an affirmative answer to this question using simple expressions for estimating BER of incompletely synchronized correlator: Markovian SS codes are superior to LFSR and/or i.i.d. codes regardless of the synchronization achievement [8]. Such an estimated BER, however, requires information about the time delay.

Another natural question concerns BER, in the following sense: does there exist a tradeoff between BER and acquisition rate in terms of Markovity if we do not assume any synchronization-achievement? The literal answer for this is negative as will be shown in this paper. We, however, examine the tradeoff carefully by discussing an accumulated version of the serial search acquisition [9] as one of simple acquisition methods (See Fig. 1) and give the BER of the correlation receiver based on this method. Simulation results show that the distribution of deviation of estimated time delay from the synchronization is almost irrelevant to kinds of SS codes: Markov, i.i.d., and Gold sequences. Such results lead to the conclusion that SS codes generated by a Markov chain look promising.

2. Asynchronous DS/SS System

We consider baseband direct sequence/spread spectrum (DS/SS) communications of J users. We define the data signal of the j-th user ($j = 1, 2, \cdots, J$) and its assigned SS code signal, respectively, by $d^{(j)}(t) = \sum_{p=-\infty}^{\infty} d_p^{(j)} u(t/T - p)$, and $X^{(j)}(t) = \sum_{q=-\infty}^{\infty} X_q^{(j)} u(t/T_c - q)$, where $u(t) = 1$ for $0 \le t < 1$ and $u(t) = 0$ otherwise. The j-th user's SS code sequence $\boldsymbol{X}^{(j)} = \{X_q^{(j)}\}_{q=-\infty}^{\infty}$ has period $N = T/T_c$. Without loss of generality, we assume $T_c = 1$ through this paper. We assume both data symbols $d_p^{(j)}$ and code symbols $X_q^{(j)}$ take values in $\{+1, -1\}$. The transmitted signal for the j-th user is $s^{(j)}(t) = d^{(j)}(t)X^{(j)}(t)$. For asynchronous systems the received signal $r(t)$ is given by $r(t) = \sum_{j=1}^{J} s^{(j)}(t - t^{(j)}) + n(t)$, where $n(t)$ is the channel noise assumed to be a white Gaussian process with two-sided spectral density $N_0/2$. We assume the time delay of j-th user's signal, denoted by $t^{(j)}$, is expressed as $\ell_j + \frac{k_j}{M}$ with $\ell_j \in \{0, 1, \cdots, N-1\}$, $k_j \in \{0, 1, \cdots, M-1\}$ (M is some positive integer), where $\frac{1}{M}$ is referred to as the *microchip*.

Let $I_p^{(i,j)} = \int_{pT+\tau^{(i)}}^{(p+1)T+\tau^{(i)}} s^{(j)}(t - t^{(j)})\ X^{(i)}(t - \tau^{(i)})\mathrm{d}t$, then we get the output of the i-th correlation receiver with its time delay $\tau^{(i)}$ during the p-th time interval as

$$Z_p^{(i)} = S_p^{(i)} + I_{J,p}^{(i)} + \eta^{(i)}, \qquad (1)$$

where $S_p^{(i)} = I_p^{(i,i)}$ is the self-interference component, $I_{J,p}^{(i)} = \sum_{\substack{j=1 \\ j \ne i}}^{J} I_p^{(i,j)}$ is the multiple-access interference (MAI) from the other $J-1$ channels, and $\eta^{(i)}$ is the noise component. Note that if $\tau^{(i)}$ is equal to $t^{(i)}$, then $S_p^{(i)}$ is equal to $d_p^{(i)}N$. Using the *up-sampled sequence by a factor of* M for $\boldsymbol{X}$, defined by $\widehat{\boldsymbol{X}} = \{\underbrace{X_0, \cdots, X_0}_{M}, \underbrace{X_1, \cdots, X_1}_{M}, \cdots, \underbrace{X_{N-1}, \cdots, X_{N-1}}_{M}\}$,

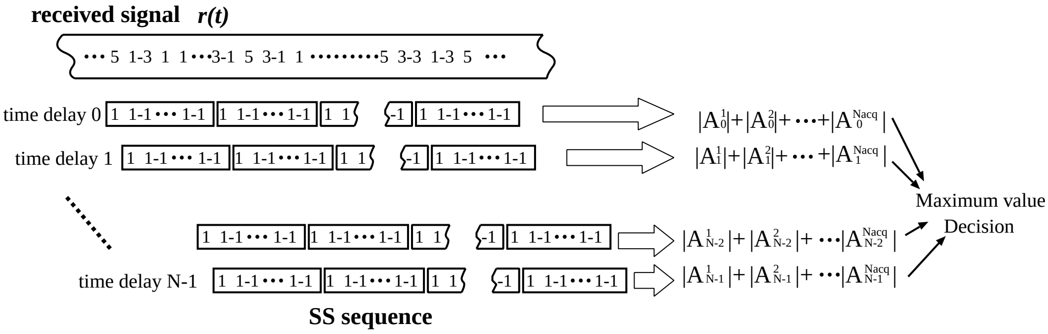

Fig. 1: Accumulated serial search acquisition method

we have

$$
\begin{aligned}
I_p^{(i,j)} = \frac{1}{M}\big\{ & d_p^{(j)} R_{NM}^{A}(\ell_{ij}M + k_{ij}; \widehat{\boldsymbol{X}}^{(i)}, \widehat{\boldsymbol{X}}^{(j)}) \\
& + d_{p+1}^{(j)} R_{NM}^{A}(NM - \ell_{ij}M - k_{ij}; \widehat{\boldsymbol{X}}^{(j)}, \widehat{\boldsymbol{X}}^{(i)})\big\},
\end{aligned} \tag{2}
$$

where ℓ_{ij} and k_{ij} are the relative time delay parameters with $\tau^{(i)} - t^{(j)} = \ell_{ij} + \frac{k_{ij}}{M}$. Especially, ℓ_{ii} and k_{ii} are denoted by ℓ_S and k_S, respectively.

3. SS Codes Generated by a Markov Chain

Let $\boldsymbol{X} = \{X_n\}_{n=0}^{\infty}$ and $\boldsymbol{Y} = \{Y_n\}_{n=0}^{\infty}$ be sequences of $\{-1, 1\}$-valued stationary random variables. Suppose that $\boldsymbol{X}$ and $\boldsymbol{Y}$ are stationary 2-state Markov chains with 2-dimensional transition matrix P, and mutually independent. Let their probabilities be $\mathrm{Prob}\{X_n = -1\} = \mathrm{Prob}\{Y_n = -1\} = \mathrm{Prob}\{X_n = 1\} = \mathrm{Prob}\{Y_n = 1\} = \frac{1}{2}$. For simplicity, suppose irreducible, aperiodic Markov chains, then for $\ell, m, k \geq 0$ we have

$$\mathbf{E}_X[X_n] = \mathbf{E}_Y[Y_n] = 0,\ \mathbf{E}_{XY}[X_n Y_{n+\ell}] = 0, \tag{3}$$

$$\mathbf{E}_X[X_n X_{n+\ell}] = \lambda^{\ell}, \mathbf{E}_Y[Y_n Y_{n+\ell}] = \lambda^{\ell}, \tag{4}$$

$$\mathbf{E}_X[X_n X_{n+\ell} X_{n+\ell+k}] = 0, \tag{5}$$

$$\mathbf{E}_X[X_n X_{n+\ell} X_{n+\ell+k} X_{n+\ell+k+m}] = \lambda^{l+m} \tag{6}$$

where $\mathbf{E}_Z[\cdot]$ denotes the expected value with respect to the distribution of a random variable Z, and λ is the eigenvalue of P other than 1.

4. Bit Error Rate in an Incompletely Synchronized Correlator

Let σ^2 and σ_S^2, respectively, denote the variances of $I_p^{(i,j)}/\sqrt{N}$ and $S_p^{(i)}/\sqrt{N}$ with respect to SS codes $\boldsymbol{X}^{(j)}$ $(j = 1, 2, \cdots, J)$. Applying the central limit theorem, Kohda and Fujisaki [5] showed that $I_p^{(i,j)}/\sqrt{N}$ tends to Gaussian distribution with mean 0 and variance $\sigma^2(\lambda) \stackrel{\text{def}}{=} \mathbf{E}_K[\mathbf{Var}_X[I_p^{(i,j)}/\sqrt{N}]] = \frac{2}{3}\frac{1+\lambda+\lambda^2}{1-\lambda^2}$ (for $M \gg 1$) and $\sigma^2(\lambda) = \frac{1+\lambda^2}{1-\lambda^2}$ (for $M = 1$) as N approaches to infinity, where K is a uniform random variable for k_{ij} taking values in a set $\{0, 1, \cdots, M-1\}$. Further, we have recently shown [8] that the expectation and the variance of $S_p^{(i)}/\sqrt{N}$, respectively, given by $E_S(k_S, \ell_S, \lambda) \stackrel{\text{def}}{=} \mathbf{E}_D[\mathbf{E}_X[S_p^{(i)}/\sqrt{N}]] = \frac{1}{\sqrt{N}}\left\{\left(1 - \frac{k_S}{M}\right)(N - \ell_S)\lambda^{\ell_S} + \frac{k_S}{M}(N - \ell_S - 1)\lambda^{\ell_S+1}\right\}$, and $\sigma_S^2(k_S, \ell_S, \lambda) \stackrel{\text{def}}{=} \mathbf{E}_D[\mathbf{Var}_X[S_p^{(i)}/\sqrt{N}]] = \left(1 - \frac{k_S}{M}\right)^2 \times \mathcal{G}_+(\ell_S) + \left(\frac{k_S}{M}\right)^2 \mathcal{G}_+(\ell_S+1) + 2\frac{k_S}{M}\left(1 - \frac{k_S}{M}\right)\mathcal{H}_+(\ell_S)$, where we assume the data symbol $d_p^{(i)}$ is a sequence of balanced i.i.d. binary random variables. The expression of $\mathcal{G}_+(\ell_S)$ and $\mathcal{H}_+(\ell_S)$ was given by [8].

By the additive property of the Gaussian distribution, the output of the correlator $Z_p^{(i)}(\ell_S, k_S)/\sqrt{N}$ tends to Gaussian distribution with mean $E_S(k_S, \ell_S, \lambda)$ and total variance $\sigma_T^2(\ell_S, k_S, \lambda) = \mathbf{E}_K[\mathbf{Var}_X[Z_p^{(i)}/\sqrt{N}]] = \sigma_S^2(\ell_S, k_S, \lambda) + (J-1)\sigma^2(\lambda) + \frac{N_0}{2}$. If ℓ_S and k_S are previously known, then the BER of the correlator with its time delay $\ell_S + \frac{k_S}{M}$ is given by

$$P_e(\ell_S, k_S, \lambda) = Q\left(\frac{E_S(\ell_S, k_S, \lambda)}{\sigma_T(\ell_S, k_S, \lambda)}\right), \tag{7}$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\omega^2}{2}\right] \mathrm{d}\omega$. This leads the average bit error rate $\overline{P_e} = 2\int_0^{\frac{1}{2}} P_e(0, k_S, \lambda)\mathrm{d}\varepsilon$, where $\varepsilon = \frac{k_S}{M}$ provided that correlator is assumed to be nearly synchronized within a fraction of a chip, *i.e.* $\ell_S = 0$ or $N-1$. On the contrary, not using the above synchronization assumption we have to discuss *a posterior* BER defined by

$$\widehat{P}_e(\lambda) = \sum_{\ell_S=0}^{N-1} \sum_{k_S=0}^{M-1} f_{acq}(\ell_S, k_S, \lambda) P_e(\ell_S, k_S, \lambda), \tag{8}$$

where $f_{acq}(\ell_S, k_S, \lambda)$ is the distribution function of deviations from synchronization which depends on acquisition methods to be used.

5. Distribution of Deviations from Synchronization and Acquisition Rate

Fig. 2 and 3 show the empirical BERs of the correlator which is assumed to be synchronized completely, *i.e.*, $f_{acq}(\ell_S, k_S, \lambda) = \delta_{\ell_S,0}\delta_{k_S,0}$, where $\delta_{i,j}$ is the Kronecker delta function for integers i and j. In actual cases, because the synchronization timing is not previously known to the receiver, $f_{acq}(\ell_S, k_S, \lambda)$ differs from the above one. Then we will give BERs based on the estimated time delay of synchronization in the following simulations.

We performed the acquisition simulation by using an accumulated version of the serial search acquisition method. In the usual serial search acquisition, the receiver de-spreads at every time-delay from $\ell_S = 0, k_S = 0$ to $\ell_S = N-1, k_S = M-1$ and the one which has the maximum absolute value is selected as the estimated time-delay of synchronization. The accumulated serial search acquisition is defined as follows. For a given data period of the acquisition, the receiver calculates the sum of the absolute values for each time delay over the data periods (See Fig. 1). Let N_{acq} denote the data periods, then the sum for time delay $\ell_S + \frac{k_S}{M}$ is $\zeta(\ell_S, k_S, \lambda) = \sum_{p=1}^{N_{acq}} |Z_p^{(i)}(\ell_S, k_S, \lambda)|$. The distribution of deviations from synchronization for accumulated serial search acquisition is, then given by $f_{acq}(\ell_S, k_S, \lambda) = \mathrm{Prob}\big(\zeta(\ell_S, k_S, \lambda) = \max_{\ell,k} \zeta(\ell, k, \lambda)\big)$.

We give $f_{acq}(\ell_S, k_S, \lambda)$ for a special case where $\boldsymbol{X}^{(i)}$'s are sequences of i.i.d. binary random variables and the system is chip-synchronous state, *i.e.* $\lambda = 0$ and $M = 1$. For simplicity, $f_{acq}(\ell_S, 0, 0)$ and $\zeta(\ell_S, 0, 0)$ are denoted as $f_{acq}(\ell_S)$ and $\zeta(\ell_S)$, respectively. The analysis of $f_{acq}(\ell_S, k_S, \lambda)$ for the general case is more complicated and remains to be solved. Letting $f(x)$ and $g(x)$ denote the probability distribution of $\zeta(\ell_S)$ for $\ell_S \neq 0$ and $\ell_S = 0$, respectively, then we get

$$f_{acq}(0) = \int_{-\infty}^{\infty} g(x) \cdot F(x)^{N-1} \mathrm{d}x, \quad (9)$$

$$f_{acq}(\ell_S) = \int_{-\infty}^{\infty} f(x) \cdot F(x)^{N-2} G(x) \mathrm{d}x, \ (\ell_S \neq 0) \quad (10)$$

where $F(x) = \int_{-\infty}^{x} f(x)dx$ and $G(x) = \int_{-\infty}^{x} g(x)dx$.

Fig. 4–7 are the distributions of deviations from synchronization of the correlator, $f_{acq}(\ell_S, k_S, \lambda)$ caused by this method, where $N = 63$, $J = 12$, $\lambda = -2 + \sqrt{3}$. Note that Fig. 6 and 7 show that Gold sequence and Markov sequence have the almost same acquisition performance in asynchronous systems. Fig. 8 and 9, respectively, show the success rates of the accumulated serial search acquisition and the BERs based on the synchronization estimated by this method using Gold sequences, i.i.d. sequences, and Markov sequences, where $N = 63$

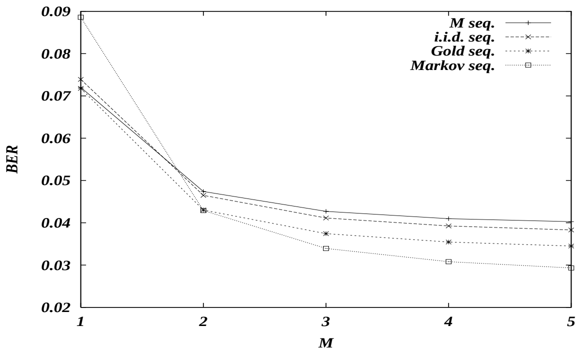

Fig. 2: Bit error rate ($N = 63, J = 30$). $P_e(0, 0, \lambda)$ *vs* M

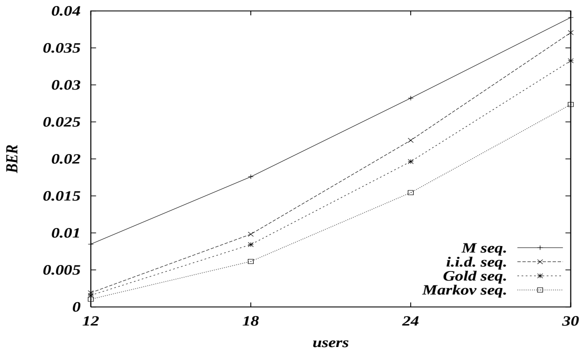

Fig. 3: Bit error rate ($N = 63, M = 10$). $P_e(0, 0, \lambda)$ *vs* J

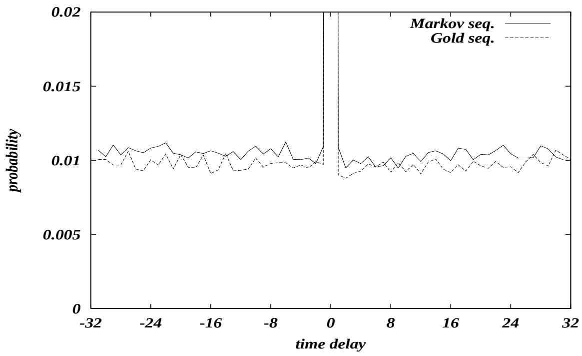

Fig. 4: Distribution of $f_{acq}(\ell_S, k_S, \lambda)$ by using serial search acquisition ($M = 1$, $N_{acq} = 1$).

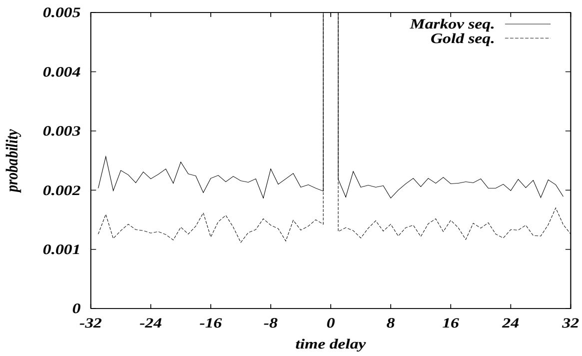

Fig. 5: Distribution of $f_{acq}(\ell_S, k_S, \lambda)$ by using serial search acquisition ($M = 1$, $N_{acq} = 5$).

and $J = 30$. Note that in an asynchronous system, the number of time-delays to be searched for acquisition is M times as much as that of a chip-synchronous system ($M = 1$), nevertheless the acquisition rate of the former system is drastically improved than the latter.

6. Concluding Remarks

In this paper, we have given *a posteriori* BER together with the distribution of deviations from synchronization using the accumulated serial search acquisition method. We have shown that in a chip-synchronous state ($M = 1$) Markov sequences are inferior to Gold sequences in terms of both the acquisition rates and BER; on the contrary, in an asynchronous state ($M \gg 1$) Markov sequences is superior to Gold sequences and i.i.d. sequences in terms of BERs primarily because Markov sequences and Gold sequences have the almost same distribution of deviations from synchronization.

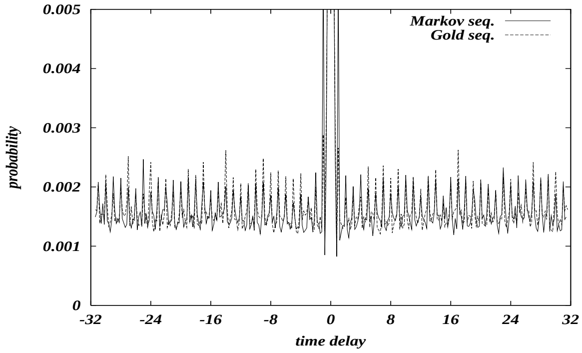

Fig. 6: Distribution of $f_{acq}(\ell_S, k_S, \lambda)$ by using serial search acquisition ($M = 5$, $N_{acq} = 1$).

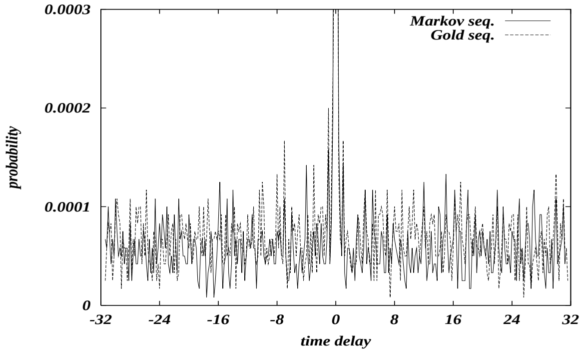

Fig. 7: Distribution of $f_{acq}(\ell_S, k_S, \lambda)$ by using serial search acquisition ($M = 5$, $N_{acq} = 5$).

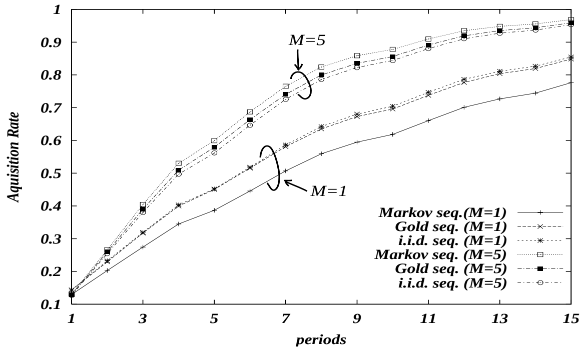

Fig. 8: Acquisition rate. $f_{acq}(0, 0, \lambda)$ *vs* N_{acq}

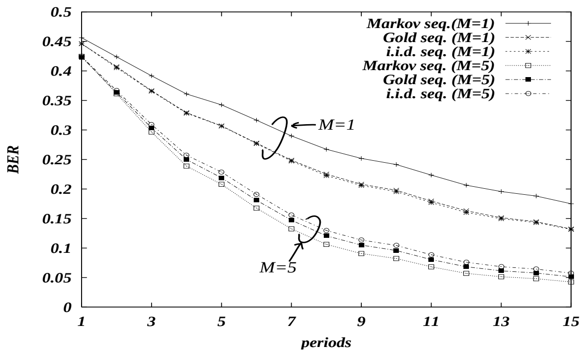

Fig. 9: A posteriori bit error rate of the correlator, $\widehat{P}_e(\lambda)$, using the accumulated serial search acquisition.

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