Numerical Experimentation of Image Recognition Using Quantum Computation

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1. Introduction

In 1985, for the first time, Deutsch presented the concept of Quantum Turing Machine as a mathematical computer model based on quantum mechanics [1]. Since then, in 1994, Shor proposed a quantum computing algorithm of factoring large integers, which is more efficient than classical computing algorithm [2]. And, in 1996, Grover developed a method of high-speed information retrieval using quantum computer [3], [4]. They built quantum algorithms that bring about the solutions of integer factorization problem and database search one with arbitrarily high probability in polynomial time order.

However, when we have to deal with the data having high correlation each other, such as face image data, we cannot cope with it in the same way as Grover's. In the literature [5], an idea of image recognition using quantum logic was proposed. The idea is based on the fact that similarity between two images can be defined to be the inner product between the two normalized vectors (or quantum state) corresponding to those two images.

Still, the proposal is somewhat conceptual because it does not concretely describe the method for image representation and matching between two images, using quantum logic. Also, like in the aforementioned database search method, it assumes that natural image data are orthogonal (or non-correlative) to each other. We have to need more sophisticated method for the recognition of practical data.

In this paper, we describe a concrete dealing way of image recognition problem in quantum computer [6]. We show how to build the quantum state representing image by unitary transformation and show some algorithms of image recognition, that is, how to construct the unitary transformations for matching between input image and reference image data. Moreover, we present some different algorithms for the matching. The difference of the algorithms depends on the way to correspond each of registered images to each state of orthonormal base. The orthonormal base can be obtained using Gram-Schmidt's orthogonalization of registered images, and so on. One of the algorithms includes the Square-Root Measurement method for the correspondence, which is well known in the field of quantum communication theory [7], [8]. Also in this paper, we show experimental results by applying those image recognition algorithms to face image data, for the comparison of those algorithms.

2. Image Recognition

2.1. Preliminaries

In quantum mechanics, we suppose that a target physical system and the physical state correspond to a complex linear vector space and the element vector, respectively. And we represent the state vector as $|\alpha\rangle$. Usually in the finite dimensional linear space, the $|\alpha\rangle$ represents a row vector and $\langle\alpha|$ represents the dual (column) vector.

In an electronic spin (1/2) system, two physical states are observed. And, they are represented as $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. This shows quantum bit (qubit). Moreover, we can suppose a superposition state between $|0\rangle$ and $|1\rangle$, and the state is represented as, $|\ \rangle = \alpha|0\rangle + \beta|1\rangle$ $\left(|\alpha|^2 + |\beta|^2 = 1\right)$, where α, β is called amplitude. If the state is observed, either state $|0\rangle$ or $|1\rangle$ is observed with the probability $|\alpha|^2$ and $|\beta|^2$, respectively.

2.2. Similarity and Measurement of Quantum State

In pattern recognition, similarity between two patterns is usually defined as the inner product of the two vectors representing the features of each pattern. Let two feature vectors (patterns) be f and g, the similarity *simil* between f and g is defined by the following equation.

$$simil\ (f, g) = \frac{(f, g)}{\|f\| \cdot \|g\|},$$

where (f,g) means the inner product, and $\|f\| = \sqrt{(f,f)}$, $\|g\| = \sqrt{(g,g)}$.

On the other hand, in quantum mechanics, inner product has the following meaning. Suppose that orthonormal bases in N dimensional linear vector space are $|0\rangle, |1\rangle, \cdots, |N\rangle$, and that arbitrary quantum superposition state $|a\rangle$ is represented as a linear combination of them, as follows.

$$|a\rangle = a_0|0\rangle + a_1|1\rangle + \cdots + a_N|N\rangle .$$

If the basic vectors $|0\rangle, |1\rangle, \cdots, |N\rangle$ correspond to basic states in the physical system of a detector unit, the inner product between $|i\rangle$ and $|a\rangle$, i.e. $\langle i|a\rangle$, is the coefficient of $|i\rangle$. And it also means that state $|a\rangle$ will be measured as eigenvalue i of $|i\rangle$ with the probability $|\langle i|a\rangle|^2$.

Therefore, the more similarity exists between two vectors (patterns), the more detection probability of the basic state is observed.

2.3. Quantum State Representation of Image

Let the pixel value of image at x, y coordinates be real-valued $f(x, y)$, the representation is considered to be $f(x, y)|x\rangle|y\rangle$. Then, the image can be represented as a superposition state as follows.

$$\frac{1}{\sqrt{\sum_{x=0}^{L-1}\sum_{y=0}^{L-1}(f(x,y))^2}}\sum_{x=0}^{L-1}\sum_{y=0}^{L-1} f(x,y)|x\rangle|y\rangle,$$

where the size of the input image is L x L pixels.

2.4. Unitary Transformation for Normalized Image Representation

We need a unitary transformation that will generate an arbitrary superposition state $|f\rangle$ from a basic state, in order to represent a normalized image. We show such a transformation can be synthesized. Suppose that the initial state to be transformed is $|0\rangle = (1,0, \cdots, 0)^T$. The goal is to obtain the vector $[a_1, a_2, ..., a_N]^T$, starting the state $|0\rangle$, (or $|0\rangle|0\rangle$ in the case of the aforementioned image).

Taking suitable θ_1 in order that $\cos\theta_1 = a_1$, first transformation will be

$$U_1 = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & \cdots & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & & \vdots \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \end{pmatrix}.$$

Similarly, taking $\theta_2 = \cos^{-1}(a_2 / x)$, where $x = \sin\theta_1$, second transformation will be

$$U_2 = \begin{pmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 & & \vdots \\ \vdots & \sin\theta_2 & \cos\theta_2 & 0 & & \vdots \\ \vdots & 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1 \end{pmatrix}$$

In the same way as above, iterate this operation N-1 times, and the quantum superposition state for arbitrary image representation will be obtained. The total transformation will be the following:

$$U_{N-1}\cdots U_2 \cdot U_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \cos\theta_2 \\ \sin\theta_1 \sin\theta_2 \cos\theta_3 \\ \vdots \\ \sin\theta_1 \sin\theta_2 \cdots \sin\theta_{N-2} \cos\theta_{N-1} \\ \sin\theta_1 \sin\theta_2 \cdots \sin\theta_{N-2} \sin\theta_{N-1} \end{pmatrix}$$

These transformations are unitary, and it is trivial that the generating algorithm takes execution steps of order (N)= order (L x L).

3. Image Recognition Schema

We propose a concrete image recognition schema on quantum spin register typed computer or some other quantum system.

3.1. Analysis Stage

Step 1: Choose the appropriate a set of orthonomal bases, which corresponds to the set of feature vectors of registered image pattern.

Step 2: Obtain the unitary transformation ***U*match** that makes correspondence from the above orthonormal bases to the quantum basic states in physical system of detector unit. If the detector is spin detector, the physical system becomes a quantum register.

3.2. Recognition Stage

Step 3: Make the unitary transformation ***U*input** for image representation.

Step 4: Apply the ***U*input** to the initial state $|0\rangle|0\rangle$, and generate a superposition state (input state).

$$|input\rangle = U_{\mathbf{input}} \cdot |0\rangle|0\rangle$$
$$= \frac{1}{\sqrt{\sum_{x=0}^{L-1}\sum_{y=0}^{L-1}(f(x,y))^2}} \sum_{x=0}^{L-1}\sum_{y=0}^{L-1} f(x,y)|x\rangle|y\rangle$$

Step 5: Apply the ***U*match** to the input state vector, and construct a state vector what we call "matched vector".

$$|matched\rangle = U_{\mathbf{match}} \cdot |input\rangle$$

Step 6: Measure the matched vector.

If the detector is a quantum register, we only have to read the qubits (integer number shown by the observed bit pattern on the register). In this way, with high probability, we can observe the number (basic state of quantum register) corresponding to the registered image pattern that has high similarity with the input image pattern.

4. Some Image Recognition Methods and Experimentation

As mentioned in the previous section, the correspondence between a set of registered image patterns and a set of orthonormal bases is very important for realizing the image recognition with high probability. In this section, we consider some methods, which are variations in the sense of the above correspondence.

If the registered image patterns are linearly independent, then complete orthogonalization for them is possible, by the well-known Gram-Schmidt's Orthogonalization Method (GSOM). However, the resultant patterns (basic vectors) depend on the applying order of GSOM. Suppose that there are m images and each image is numbered from 1 to m. If the GSOM is applied from image 1 to image m, and if the resultant orthogonal bases are named from V1 to Vm, corresponding to the image1 through m. In this case, the last image m has correlation with the all vectors from V1 to Vm, in general. Then, recognition probability at once observation becomes lower.

4.1. Dual Orthogonalization Method (DOM)

In order to avoid the difficulty, we present a dual orthogonalization method. That is, we obtain the two sets of bases by applying the GSOM to registered image patterns in two ways: from 1 to m and from m to 1. Let the former bases be $t_0, t_1, \cdots, t_m$ and let the latter bases be $b_0, b_1, \cdots, b_m$. As at the Step 2 in the Section 3, we have to obtain the two unitary transformations (U_t, U_b) which makes correspondence from the image bases to a quantum bases.

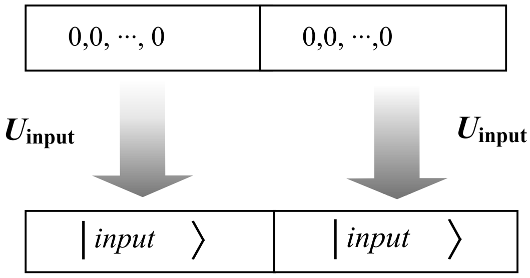

Fig.1. Doubled quantum states for input image.

The quantum state representation for input image on the two sections of a quantum register is shown in Fig.1.

Let the quantum state of an input image pattern be $|f_n\rangle$. And we assume that the dimension of the quantum space is m. We make the unitary transformations U_t and U_b operate the $|f_n\rangle$ on each section, then we have the following superposition state.

$$(|1\rangle\langle t_1|f_n\rangle + \cdots + |n\rangle\langle t_n|f_n\rangle + \cdots + |m\rangle\langle t_m|f_n\rangle)$$
$$\otimes (|1\rangle\langle b_1|f_n\rangle + \cdots + |n\rangle\langle b_n|f_n\rangle + \cdots + |m\rangle\langle b_m|f_n\rangle)$$

If $|f_n\rangle$ is similar to the registered image pattern *n*, only $|n\rangle|n\rangle$ has the amplitude in $|0\rangle|0\rangle, \cdots, |m\rangle|m\rangle$. The amplitude is relatively high. But, even when the amplitude is small, we only have to repeat until we observe $|n\rangle|n\rangle$, i.e. the same number on the two sections.

4.2. Gram-Schmidt Orthonormal Bases Embedding Method (GSEM)

Let *M* be the number of registered images. A set of feature vectors is obtained by iteration of the following steps.

Step 1: Initialize k=*M*-1.

Step 2: Orthogonalize the images by GSOM in the order of $|f_0\rangle, |f_1\rangle, \cdots, |f_{M-1}\rangle$.

Step 3: If k =0 then Stop, else Replace the order of the images, and Decrement the number of j, and Go to the Step 1.

For example, in the Step 3, the set of the registered image vectors $|f_0\rangle, |f_1\rangle, \cdots, |f_{M-1}\rangle$ are replaced in the order of the following: $|f_1\rangle, |f_2\rangle, \cdots, |f_{M-1}\rangle, |f_0\rangle$.

Then we have *M*×*M* bases. And $|f_k^i\rangle$ means the base that is obtained by orthogonalizing $|f_k\rangle$ in the *i*th order in the process of GSOM. Then, we can embed those bases into M x M dimensional direct product space utilizing other orthonormal bases $\{ |e_i\rangle \}$. So, if the input image is the same as registered image $|f_k\rangle$, the input quantum state vector in the direct product space associating the input image $|f_k\rangle$ will be given as follows.

$$|input\rangle = |f_k\rangle \otimes \left(\frac{1}{\sqrt{M}} |e_0\rangle + \cdots + \frac{1}{\sqrt{M}} |e_{M-1}\rangle \right)$$

And, the unitary transformation ***U*match** will be given as follows.

$$U_{match} = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} |i\rangle \otimes |j\rangle \cdot \langle e_i | \otimes \langle f_j^k |, \ (k = (j-1) \bmod M)$$

Then, the correct recognition probability is

$$P_k = \frac{1}{M} \sum_{i=0}^{M-1} \left| \left\langle f_k^i \middle| f_k \right\rangle \right|^2 .$$

4.3. Square Root Measurement (SRM)

The Square Root Measurement is well known as a quasi-optimal signal detection method in the field of quantum communication field [7], [8].

Let $|f_0\rangle, \cdots, |f_{M-1}\rangle$ be a set of linear independent image vectors (quantum image representation state). We can construct the correspondent orthonormal system for the given set.

We define a linear operator (or density matrix or autocorrelation matrix) as follows.

$$\hat{\mathbf{G}} = \sum_{i=0}^{M-1} |f_i\rangle\langle f_i|$$

Let the eigenvalues and its associated eigenvectors of the matrix $\hat{\mathbf{G}}$ be λ_i and $\mathbf{u}_i$ ($i = 0, 1, \cdots, M-1$), respectively. We can define the following operator $\hat{\mathbf{G}}^{-1/2}$.

$$\hat{\mathbf{G}}^{-1/2} = \sum_{i=0}^{M-1} \frac{1}{\sqrt{\lambda_i}} |\mathbf{u}_i\rangle\langle \mathbf{u}_i|$$

Then, we can construct the correspondent orthonormal bases $\{|\mu_i\rangle\}$ from the given set of $\{|f_0\rangle, \cdots, |f_{M-1}\rangle\}$ as in the following.

$$|\mu_i\rangle = \hat{\mathbf{G}}^{-1/2} |f_i\rangle \quad (i = 0, 1, \cdots, M-1)$$

In the case of this correspondence, the measurement probability $P(i \mid j)$ for detecting the state $|\mu_i\rangle$ for the input state $|f_j\rangle$ will be given as follows.

$$P(i \mid j) = \left| \left\langle \mu_i \middle| f_j \right\rangle \right|^2$$

So, if we apply this correspondence to our image recognition system, the absolute value of the proposed similarity measure between the state $|\mu_i\rangle$ and input image $|f_i\rangle$ will be given as

$$|Simil(i \mid j)| = \sqrt{P(i \mid j)}$$

4.4. Experimentation

We have experimented those methods for one hundred face images whose pixels are 256x256. As a result of our experimentation, the GSEM is better than the DOM in the sense of the recognition rate. However, the SRM has shown the best correct recognition rate in those three methods. Although the SRM is the best of the above methods, the recognition rate decreases as the number of face images increases. So, if we have too large number of registered images, the recognition rate will considerably decrease. Thus we may need to introduce some efficient mechanics into quantum computing system, such as positive feed back one, for the realization of more sophisticated image recognition system in quantum computing.

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