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Applications of Nonlinear Signal Processing by DSP

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Abstract

We give two applications utilizing a digital signal processor (DSP) for nonlinear signal processing which are not mentioned before as real implementations. Firstly we develop a chaos control system based on the delayed feedback control method. We develop its realization and investigate incidental problems. Secondly we try to construct a simple digital filter by a chaotic difference system. Some results of its dynamical properties are given. We show concrete examples for both applications with laboratory experiments.

1. Introduction

Digital signal processor (DSP) becomes a one of remarkable processor recently since DSP can handle real time signal processing. Therefore, it is applied to many high-tech equipment, instuments, machinery as an embedded system, e.g., digital oscillographs, cellular phones, modems, and PWM controllers. We can point out that there are two useful advantages of DSP for nonlinear signal processing; availability of fast and sufficient memory, and high performance of mutiply and accumulate (MAC) operation.

We firstly apply the former feature to controlling chaos. The external feedback control and the delayed feedback control, stated next section, needs much memory to store the orbit signal to perform chaos cotrol. In this paper, we develop a DFC simulator with DSP systems as an preliminary of DFC for continuous-time system.

Secondly, we propose a very simple digital filter to utilize fast MAC operations. The chaotic difference equation can be regarded as an infinite impulse response (IIR) filter, so that DSP can perform a nonlinear oscillator or fileter. We investigate its dynamical properties and show experimental results.

In the following, we use consistently TMS320-C6711DSK as a DSP board. This provide floating point arithmetic and analog I/O peripherals.

2. Implementation of Controlling Chaos on DSP

Controlling chaos is the typical nonlinear problem and have studied with much attention. The problem can be stated as follows; stabilizing an unstable periodic orbit embedded in the given chaotic attractor.

There are following chaos control methods: the OGY method proposed by Ott, Grebogi and Yorke[1], external force control and delayed feedback control (DFC) developed by Pyragas[2]. The former is based on geometrical property of a return map, thus it must requires the location of target fixed point. Whereas, the latter is based only on periodicity of the target, no information about the target orbit anymore. The weakest point of DFC is requirement of much memory for stroring a long time series of the orbit. In this paper, we realize this feedback control as a circuit implementation with DSP.

In this paper, we use delayed feedback control(DFC) as method of chaos control on DSP. DFC is used as method of continuous-time system actually, but, we can use easily constitutive idea of DFC as method of discrete-time system. DFC is method applying difference between output signal of current time and output signal of τ times ago to chaos system, and the method can control chaos signal into τ periodic orbit.

2.1. Implementation of DFC on DSP

Pyragas[2] proposed the following feedback system to stabilize an unstable periodic orbit (UPO) embedded in a chaotic attractor.

$$\frac{dx}{dt} = f(x(t)) + u(t), \quad u(t) = K(x(t) - x(t-\tau)), \quad (1)$$

where, $x \in \boldsymbol{R}^n$, $\tau > 0$ is a delay time, and K is a control matrix. In this system, any UPO can be stabilized by choosing appropreate K and τ. We have to implement a controller that produce $u(t)$ from the observed $x(t)$.

In this paper, we provide a chaotic generator which is also designed by DSP. Because it is easy to realize chaotic motion from mathematical model and it is very robust with no component drift, gains fixed performance.

Needless to say, the control value $u(t)$ is calculated another DSP. Figure 1 shows configuration of our control system.

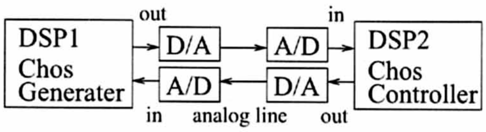


Figure 1: A block diagram of the DSP systems.

Chaos generator (DSP1):

$$\begin{cases} x_k = y_{k-1} + 1 - a{x_k}^2 + u_k \\ y_k = bx_{k-1} \end{cases} \quad (2)$$

Controller (DSP2):

$$u_k = K(x_k - x_{k-\tau}) \quad (3)$$

We choose Hénon map as a chaos generator DSP1, Only the information of $x(k)$ in DSP1 is transmitted to DSP2 via an analog signal line. Immediately the control input is calculated in DSP2, the output of DSP2 is transmitted to DSP1 via an analog signal line.

Then we consider that DSP2 stabilize a fixed point in the chaotic attractor via analog (noisy) signal line. Since the chaotic system in DSP1 is written in a difference equation, we consider stability of fixed point of Eqs (2).

We consider the case $\tau = 1$. Substitute Eq.(3) into Eq.(2) then, the fixed point are given by

$$x_{1,2} = \frac{(b-1) \pm \sqrt{(1-b)^2 + 4a}}{2a},$$

and the corresponding eigenvalues are as follows:

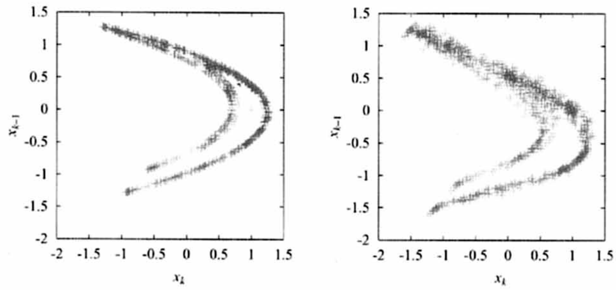
$$\mu_{1,2} = \frac{(K - 2ax) \pm \sqrt{(2ax - K)^2 + 4(b + K)}}{2}$$

. At the parameter choice of $a = 1.4$, $b = 0.3$, both are unstable and embeded in the chaos attractor. By using a stability condition $|\mu_{1,2}| < 1$, we can solve K as:

$$0.53388 < K < 1.3$$

. Within this range, we choose $k = 0.64$ to minimize the transient response by the trial-and-error.

The DSP motherboard provides a CODEC (TLC320AD535), however, the sampling rate is fixed as 8 [KHz]. From the Nyquest theory, the output signal can be contain frequency components under 4 [KHz]. We try to transmit the signal generated in DSP1 with 8 [KHz], but the received signal in DSP2 is not preserve the orinal signal waveform. This is caused by dropping frequency components over 4 [KHz] in the signal in DSP1. To overcome this distortion, we simply do down sampling. The sampling rate of the CODEC in the DSP mother board is hardwired, and cannot be changed. We therefore simulate 1 [KHz] sampling rate by the software programming; we put same sampled signal for 8 times to the D/A with 8 [KHz] sampling rate. (The chaotic system Eq.(2) runs 1 [KHz] equivalently.) Figure 2 (a) shows the chaotic attractor in Eq. (2) without control, and Fig. 2 (b) shows the received signal. They both are reconstructed by the embedding method from the signal x_k. By this down-sampling, the wave form of the original chaos signal may be preserved.



(a) (b)

Figure 2: Signals in DSPs. (a): original data in DSP1, (b): processed signal in DSP2.

Next, we investigate validity of the control input $u(k)$. In DSP1, we can estimate an expected $u(k)$ to stabilize the fixed point since Eq. (3) can be also calculated in DSP1. Figure 3 depicts the desired control input signal $u(k)$ which can stabilize the fixed point. While, Fig. 4 show the received control input from DSP2.

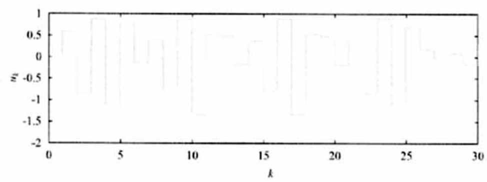


Figure 3: The expected feedback singnal calculated in DSP1.

It is needless to say that we expect no difference between Figs. 3 and 4. However the signal transmitted from DSP2 is delayed about 10 unit times although DSP2 performs with the maxmum speed. Hence we cannot use this u_k as a control input.

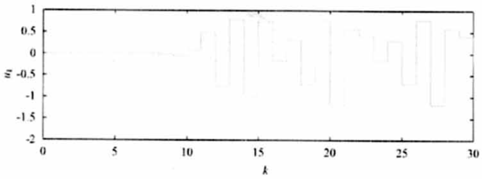


Figure 4: The received feedback signal u_k from DSP2.

This delay is also caused by the characteristics of the CODEC. By consulting from the data sheet of TLC320AD535, an A/D conversion spends $17/f_s$ [sec], and a D/A conversion spends $18/f_s$ [sec], where $f_s = 8$ [KHz]. Thus totally twice A/D, D/A conversions are needed, about 8.75 [msec] must be delayed. This limitation is inevitable and cannot be removable for this system.

Here we simply insert a delay to the updating timing of DSP1, i.e., we just let the dynamical system Eq. (2) be delayed. Figure 5 show the expected signal and control input signal u_k from DSP2 with a dalay. It seems that both signals are coincident anytime. Figure 6 illustrates transient response of u_k with arbitrarily chosen initial point, and it ensures the control system Eq. (2) and Eq. (3) is useful as DFC system if some hardware limitation can be removed.

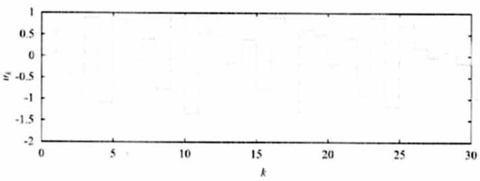


Figure 5: The expected signal and control input signal from DSP2 with a dalay.

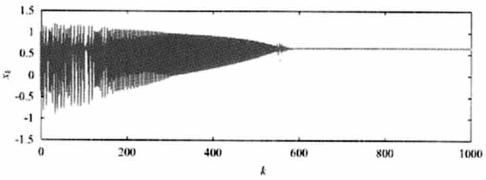


Figure 6: the transient response of u_k.

3. Application of Nonlinear Digital Filter

Linear digital filters have been developed from 60s, and they are implemented in many embedded systems nowaday. However, some nonlinear digital filters also have been developed becouse of it can clear nonstationary noise what linear digital filters can't clear. Since linear IIR (Infinite impulse response) filters are written in a difference equation, so that nonlinear IIR fileter can be realized by modifing some parameters and states of the linear IIR filter. For instance, if we change a product of a constant and a state variable into a product of any two state variables (or delayed states), we already have a chaotic filter.

We study time responses and frequency characteristics of a nonlinear digital filter based on bifurcation theory. Some engineering applications of this filter are shown.

3.1. Difference Equation and Filter

We consider a simple nonlinear difference equation (4) whose origin is a fixed point:

$$x_{k+1} = ax_k^2 + y_k, \quad y_{k+1} = bx_k \tag{4}$$

The fixed points are given by:

$$(x_1, y_1) = (0, 0), \quad (x_2, y_2) = (-(b-1)/a, -b(b-1)/a), \tag{5}$$

and the corresponding eigenvalues are:

$$\mu_1, \mu_2 = ax_i \pm \sqrt{a^2x_i^2 + b} \quad (i = 1, 2) \tag{6}$$

The origin is stable under $|b| < 1$ and the stability is not switched by any value of a. But the basin of attraction of the origin is depend on the values of a and b. So we assume that the filter is used with a sufficiently small input which is not beyond this basin of attraction.

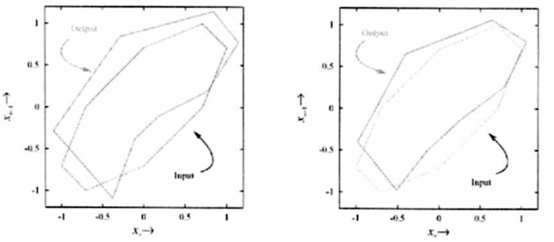
3.2. Nonlinear Digital Filter

Usually the filter assume x_k in Eq. (4) as an input pulse train. From similarity of the non-autonoumous system of ODE, and motivated from the precede work Ref.[3], we attach an input as follows:

$$x_{k+1} = ax_k^2 + y_k + c\sin\omega kT \quad y_{k+1} = bx_k \tag{7}$$

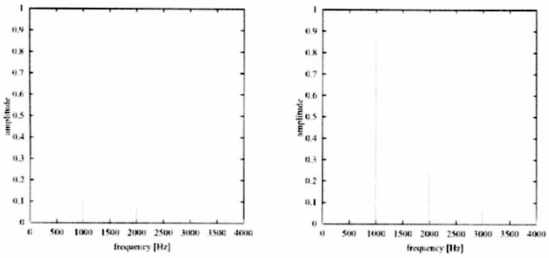
where, T is the sampling period.

Equation (7) is a nonlinear difference equation with a sampled sine wave. We fix filter parameter $(a, b) = (0.3, 0.5)$ and $(a, b) = (0.3, -0.5)$, and investigate the frequency response with 1 [KHz] sine wave input with 8 [KHz] sampling frequency. Figure 7 (a) and (b) show the results, respectively. These orbit is reconstracted by the embedding method. It is clear that the output orbit is little bit distorted by nonlinearity of the filter.



(a): $(a, b) = (0.3, 0.5)$ (b): $(a, b) = (0.3, -0.5)$

Figure 7: The input and output signal with 1 [kHz] input.



(a):$(a, b) = (0.3, 0.5)$ (b):$(a, b) = (0.3, -0.5)$

Figure 8: DFTs according to Fig. 7 .

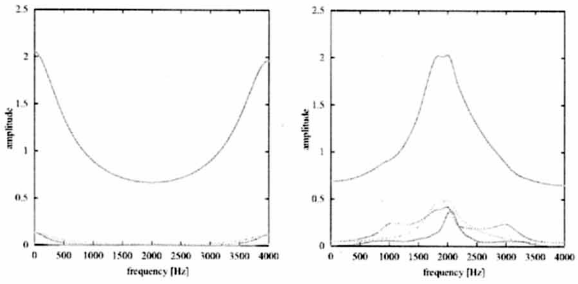
We show DFT of each output in order to check the frequency components in an output, see Fig. 8. It is obvious that the output contains not only an input frequency component, but also direct current, and high frequency components. Figure 9 shows frequency characteristics. Figure 9 (a) behaves as a band eliminator, and (b) is as a resonator.

3.3. Implementation Filter on DSP

Actually we implement difference equation with Parameter of $(a, b) = (0.3, -0.5)$ on DSP. Fig. 10 shows input and output signals of DSP. An upper wave form is input signal and a lower wave form is output signal on each figure. Fig. 10 shows frequency characteristics shown by Fig. 9. 1 [KHz] sine wave input gives about same amplitude of input signal. And 2 [KHz] sine wave input gives about about double smplitude of input signal.

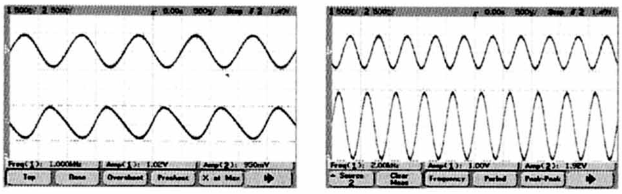
4. conclusions

In this paper, we show two applications using DSP as nonlinear signal processing. As the future problems,



(a):$(a, b) = (0.3, 0.5)$ (b):$(a, b) = (0.3, -0.5)$

Figure 9: Frequency characteristics in the output of Eq.(7).



(a):1000Hz input (b):2000Hz input

Figure 10: Input and output signals on DSP

implementation of DFC for continuos-time chaotic signals and application of nonlinear IIR filters should be investigated.

About, we think that DSP can be used widly by other applications.

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