OPTIMISATION OF A SLIDING MODE-BASED GNC USING ROBUST EIGENSTRUCTURE ASSIGNMENT

O. Anaya-Lara and E. W. McGookin

Centre for Systems and Control &
Department of Electronics and Electrical Engineering
The University of Glasgow
Glasgow, G12 8LT, U. K.
Phone: +44 141 330 6137, Fax: +44 141 330 6004
Email: oanaya@elec.gla.ac.uk, e.mcgookin@elec.gla.ac.uk

Abstract: This paper presents the design and implementation of a robust guidance and navigation control (GNC) system for a 190 000 dwt oil tanker. The control methodology is based on the Sliding Mode methodology, which relies on the *pole placement theory*. The associated eigenstructure assignment algorithm is properly amended, optimising the controller's response for the particular tanker application. Relevant results are presented to assess the increased functionality and reliability of the automatic controller presented in this paper.

1. Introduction

Guidance and navigation control of super tankers have been an issue of major concern for the marine industry during the last few decades [1][2]. The manoeuvring of these huge vessels involves particular navigational safety issues, as even slight errors in the navigation control systems may result in disastrous environmental and financial losses. In order to prevent these incidents from happening, it is necessary to significantly improve the performance and reliability of the navigation controllers. As opposed to conventional PID-based techniques used by the marine industry to control the manoeuvres of actual super tankers [2][3], Sliding Mode (SM) control [4] has been selected in this investigation. It provides improved system response and robustness, therefore being an ideal method for handling nonlinear operating variables such as speed and water depth, which affect ships control systems [2][3].

The Sliding Mode control action developed in this work relies substantially on the *pole placement theory* [5]. Therefore, the eigenstructure assignment algorithm was adjusted in order to obtain the best possible solution for the closed-loop matrix. To this end, new code lines were added to the Matlab function *place* [6], to include a broader set of convergence conditions to the iterative process that develops the pole placement assignment. Desired requirements regarding controller robustness, stability, and overall system response performance, have been of major concern.

The paper also describes the automatic control navigation system, which provides appropriate rudder commands for the vessel in order to manoeuvre in the desired manner. The resulting optimised control algorithm is tested under realistic sailing conditions, considering external disturbances such as wind-generated ocean waves [2].

2. Tanker Simulation Model

The vessel investigated in this study is a 190 000 dwt oil tanker for which mathematical representations of the heading and propulsion dynamics are available [1]-[3]. Its nonlinear mathematical model is represented in standard state space form as:

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x},\mathbf{u}\right) \tag{1}$$

where $\mathbf{x}$ is the state vector and $\mathbf{u}$ represents the inputs to the tanker. The kinetic and kinematic states for this model are defined in Table 1 along with their associated units.

Table 1. State and input definitions

Body fixed states (kinetic)	Units
u (surge velocity)	m/s
v (sway velocity)	m/s
r (yaw/heading rate)	rad/s
dr (actual rudder deflection)	rad
n (actual propeller speed)	rps
Earth fixed states (kinematic)	
x_{pos} (x position co-ordinate)	m
y_{pos} (y position co-ordinate)	m
y (yaw/heading angle)	rad
Commanded inputs	
dr_c (commanded rudder deflection)	rad
n_c (commanded propeller speed)	rps
h (water depth)	m

This table also defines the inputs for this vessel. It should be observed that as well as having controllable inputs (i.e., commanded rudder deflection, dr_c, and propeller angular velocity, n_c) this model is dependent on the depth of water (h) [1]. Its external environment therefore directly influences the vehicle's dynamics and motion. Moreover, there is a distinction between commanded inputs and the actual state versions of the same variables (i.e., rudder deflection and propeller speed). This is because the actuator dynamics of these inputs are also represented in this model by rate equations, which determine the dynamic response of the input signal. In addition to the rate equations, physical saturation limits of maximum amplitude and rate are implemented, which define the operational envelope for the actuator systems and restrict the input signals [2][3].

3. Navigation System

The navigation system relies on course-changing commands from an autopilot, which are applied to a control system that manoeuvres the tanker in the required manner, by providing necessary rudder command signals. The considered autopilot is called a line of sight (LOS) autopilot [2][3], and it is integrated into the navigation system in the way shown in Figure 1. The LOS autopilot directs the tanker along a course made up of destination coordinates. These co-ordinates are called *waypoints* and are used as reference points for the autopilot to calculate the desired heading angle, y_{d}, for the tanker to follow.

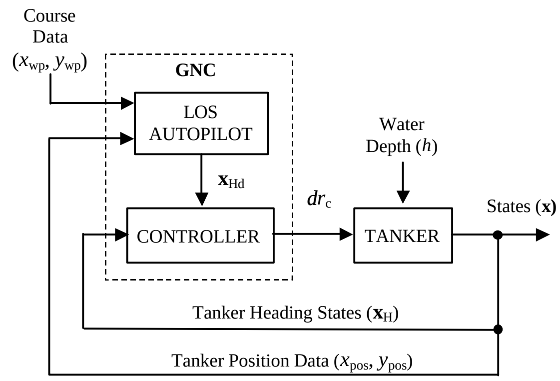

Figure 1. Tanker and navigation system.

The desired heading angle y_{d} is obtained from the following equation:

$$y_{\mathrm{d}} = \tan^{-1}\left(\frac{y_{\mathrm{wp}} - y_{\mathrm{pos}}}{x_{\mathrm{wp}} - x_{\mathrm{pos}}}\right) \tag{2}$$

where (x_{pos}, y_{pos}) are the current-position co-ordinates of the tanker obtained from a global positioning system (GPS), and (x_{wp}, y_{wp}) are the waypoint co-ordinates [2][3].

4. Sliding Mode Control

The controller designed in this investigation is derived from Sliding Mode control theory [4]. Having a characteristic discontinuous switching action, it inherently provides robustness to matched uncertainties caused by unmodelled dynamics or external disturbances. This robust behaviour enables the controller to operate with acceptable accuracy under a variety of operating conditions [2][3]. The only consideration in meeting this control objective is that the rudder usage must be kept to a minimum. Hence, a trade-off between accurate heading tracking and minimised rudder usage must be considered.

The SM controller is derived from a single-input single-output (SISO) linear representation of the heading dynamics of the vessel [2][3]. This is obtained by decoupling the yaw rate (r) and the heading angle (y) dynamics from the entire tanker model by creating the following single input state space equation:

$$\begin{bmatrix} \dot{r} \\ \dot{y} \\ \dot{dr} \end{bmatrix} = \mathbf{A}_{\mathrm{H}}\left(r, y, dr\right)\begin{bmatrix} r \\ y \\ dr \end{bmatrix} + \mathbf{b}_{\mathrm{H}} \cdot dr_{\mathrm{c}} \tag{3}$$

where $\mathbf{A}_{\mathrm{H}}$ is the corresponding system matrix, $\mathbf{b}_{\mathrm{H}}$ the input vector and dr_{c} the commanded rudder input. The control action provided by the controller is applied to the tanker via the rudder input command dr_{c}, which has two distinct components: the equivalent term (dr_{eq}) and the switching term (dr_{sw}), i.e.,

$$dr_{\mathrm{c}} = dr_{\mathrm{eq}} + dr_{\mathrm{sw}} \tag{4}$$

The equivalent component is chosen as a state feedback gain controller of the following form [2][3]:

$$dr_{\mathrm{eq}} = -\mathbf{k}_{\mathrm{H}}^{\mathrm{T}}\mathbf{x}_{\mathrm{H}} \tag{5}$$

where $\mathbf{k}_{\mathrm{H}}$ is a feedback gain obtained from *pole placement theory* [5]. The switching control signal in (4) that accounts for variations in the vessel's operating conditions, is derived from a state hyperplane called *sliding surface* (s_{H}) [4], and is represented by the following equation:

$$s_{\mathrm{H}}\left(\Delta\mathbf{x}_{\mathrm{H}}\right) = \mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\Delta\mathbf{x}_{\mathrm{H}} = \mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\left(\mathbf{x}_{\mathrm{H}} - \mathbf{x}_{\mathrm{Hd}}\right) \tag{6}$$

This is a function of the state error $\Delta\mathbf{x}_{\mathrm{H}}$ and the gain vector $\mathbf{h}_{\mathrm{H}}$. After some derivations the switching component is given as [3]:

$$dr_{\mathrm{sw}} = \left(\mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\mathbf{b}_{\mathrm{H}}\right)^{-1}\left[\mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\dot{\mathbf{x}}_{\mathrm{Hd}} - h_{\mathrm{H}}\operatorname{sgn}\left(s_{\mathrm{H}}\left(\Delta\mathbf{x}_{\mathrm{H}}\right)\right)\right] \tag{7}$$

where $\mathbf{h}_{\mathrm{H}}$ is the right eigenvector of the closed-loop system matrix created by the feedback gain. In (7) the switching action is provided by the discontinuous function *signum* with h_{H} as the switching gain, which determines the amount of switching action provided by the controller. Finally, by combining equations (5) and (7) the controller structure for the commanded rudder deflection has the following form:

$$dr_{\mathrm{c}} = -\mathbf{k}_{\mathrm{H}}^{\mathrm{T}}\mathbf{x}_{\mathrm{H}} + \left(\mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\mathbf{b}_{\mathrm{H}}\right)^{-1}\left[\mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\dot{\mathbf{x}}_{\mathrm{Hd}} - h_{\mathrm{H}}\operatorname{sgn}\left(s_{\mathrm{H}}\left(\Delta\mathbf{x}_{\mathrm{H}}\right)\right)\right] \tag{8}$$

A full development of this expression can be found in [3].

5. Oil Tanker Eigenstructure Assignment

The feedback gain $\mathbf{k}_{\mathrm{H}}$ and right eigenvector $\mathbf{h}_{\mathrm{H}}$ in (8) are calculated using state-feedback eigenstructure assignment. This methodology aims to achieve certain system properties such as robustness and stability by using a state-feedback control of the form

$$\mathbf{u} = \mathbf{K}\mathbf{x} + \mathbf{v} \tag{9}$$

where $\mathbf{K}$, the *feedback* or *gain matrix*, is chosen such that the modified dynamic system

$$\dot{\mathbf{x}} = \left(\mathbf{A} + \mathbf{B}\mathbf{K}\right)\mathbf{x} + \mathbf{B}\mathbf{v} \tag{10}$$

now with the reference input $\mathbf{v}$, has the desired closed-loop poles. Hence, the eigenstructure assignment problem is to find a real matrix $\mathbf{K}$ and a non-singular matrix $\mathbf{X}$ such that the eigenvalues of $\mathbf{A}+\mathbf{BK}$ are $l_j,\ j=1,2,\ldots,n$, satisfying

$$\left(\mathbf{A}+\mathbf{BK}\right)\mathbf{X}=\mathbf{X}\Lambda \tag{11}$$

where $\Lambda = diag\{l_1, l_2, \ldots, l_n\}$, such that some measure ν of the conditioning, or robustness, of the eigenproblem is optimised. The conditioning measures are [5],

$$n_1 = \left\|\mathbf{c}\right\|_\infty \tag{12}$$

$$n_2 = k_2\left(\mathbf{X}\right) \tag{13}$$

$$n_3 = \frac{\left\|\mathbf{X}^{-1}\right\|_{\mathrm{F}}}{n^{1/2}} = \frac{\left\|\mathbf{c}\right\|_2}{n^{1/2}} \tag{14}$$

where $\mathbf{c}=\left[c_1,c_2,\ldots,c_n\right]$ is the vector of condition numbers and $\mathbf{X}=\left[\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n\right]$ is the matrix of eigenvectors. These measures all attain their minimal values simultaneously when the eigenproblem (11) is perfectly conditioned and the assigned eigenvalues are as insensitive as possible. An iterative process using the Matlab function *place* calculates the feedback matrix $\mathbf{K}$ as,

$$\mathbf{k}_{\mathrm{H}} = place\left(\mathbf{A}_{\mathrm{H}}, \mathbf{b}_{\mathrm{H}}, p\right) \tag{15}$$

where $\mathbf{A}_{\mathrm{H}}$ and $\mathbf{b}_{\mathrm{H}}$ are defined in (3) and p represents the desired closed-loop system eigenvalues. The right eigenvector is chosen as to satisfy the condition

$$\mathbf{h}_{\mathrm{H}}^{\mathrm{T}}\mathbf{A}_{\mathrm{c}} = \left(\mathbf{A}_{\mathrm{c}}^{\mathrm{T}}\mathbf{h}_{\mathrm{H}}\right)^{\mathrm{T}} = 0 \tag{16}$$

where $\mathbf{A}_{\mathrm{c}}$ is the desired closed-loop matrix [3]:

$$\mathbf{A}_{\mathrm{c}} = \mathbf{A}_{\mathrm{H}} - \mathbf{b}_{\mathrm{H}}\mathbf{k}_{\mathrm{H}} \tag{17}$$

The *place* function aims to minimise only the conditioning measure n_2, however for this investigation, the function is adjusted with new code lines, so that the calculation process continues for an appropriate number of iterations to satisfy not only n_2, but also n_1 and n_3. By doing so, it is possible to achieve a better solution and well-conditioned closed-loop matrix $\mathbf{A}_{\mathrm{c}}$ [5].

6. Simulation Results

Several digital simulations were performed using Matlab [6]. Table 2 shows the controller parameters used throughout the simulations. It shows the desired closed-loop poles, which must be allocated by eigenstructure assignment, and the desired heading angle provided by the LOS autopilot.

Table 2: Controller parameters.

1st Heading closed loop pole	-0.1
2nd Heading closed loop pole	-0.2
Desired heading angle y_{d}	20°

The SM control switching action was provided by the discontinuous *signum* function. The optimised controller was also tested considering the effect of external disturbances such as wind-generated ocean waves. The simulations were performed for a period of 1000 sec. In the first case of simulations the Matlab *place* function is used with no modification and generates the following results for the controller parameters shown in Table 2:

Feedback gain vector:

$$\mathbf{k}_{\mathrm{H}} = \left[-11.9047 \quad 0.0 \quad -0.75\right]$$

Matrix of Right Eigenvectors:

$$\mathbf{X} = \begin{bmatrix} 0.0 & 0.0125 & 0.0042 \\ 1.0 & -0.125 & -0.021 \\ 0.0 & 0.9921 & 0.9998 \end{bmatrix}$$

Conditioning measures ($\mathbf{A}_{\mathrm{c}}$):

$$n_1 = 119.99,\ n_2 = 240.044,\ n_3 = 97.874$$

It is significant to observe that these results have high values for the conditioning measures, since the aim of this design approach is to have them as small as possible to reduce the sensitivity of the system eigenvalues to disturbances. The corresponding responses for the heading angle and rudder response are shown in Figure 2. It can be seen that the actual heading angle response accurately follows the desired heading reference. Unfortunately, there is considerable rudder usage with high chattering [2][3] due to the switching action.

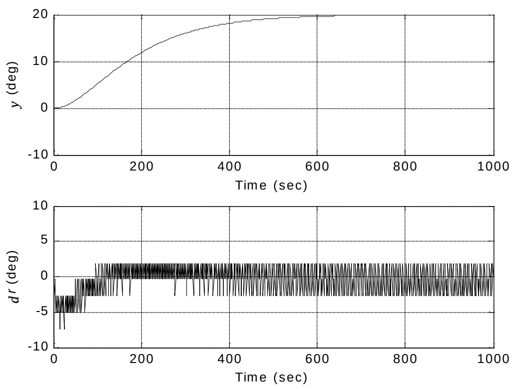

Figure 2: Heading results for the values shown in **Table 2**.

A new set of simulations was carried out using the modified Matlab *place* function to minimise simultaneously the three conditioning measures. The eigenstructure assignment results are as follow:

Feedback gain vector:

$$\mathbf{k}_{\mathrm{H}} = \left[0.00 \quad 0.0 \quad -0.9\right]$$

Matrix of Right Eigenvectors:

$$\mathbf{X} = \begin{bmatrix} 0.0 & 0.0499 & 0.0125 \\ 1.0 & -0.9987 & -0.125 \\ 0.0 & 0.0 & 0.9921 \end{bmatrix}$$

Conditioning measures ($\mathbf{A}_{\mathrm{c}}$):

$$n_1 = 20.026,\ n_2 = 40.331,\ n_3 = 16.361$$

These results show that the conditioning measures have been reduced noticeably, by almost a factor of six. The closed-loop eigenvalues become less sensitive to disturbances thus improving the robustness and stability of the overall system. Figure 3 shows the responses for the heading and rudder angle. Once more, the controller accurately tracks the reference but in this case, the requirements for the switching gain are reduced due to a better-conditioned system and less sensitive poles. This can be observed in the better-behaved rudder response with moderate chattering content.

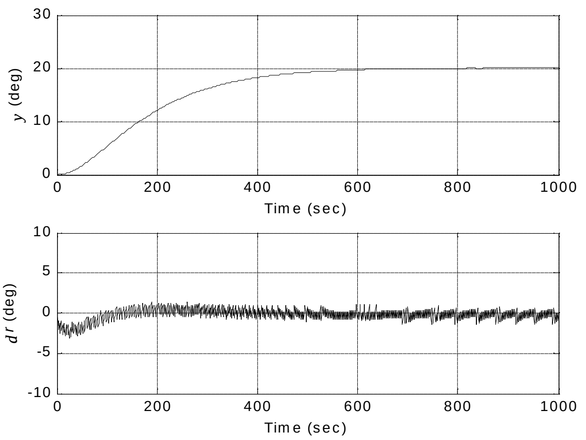

Figure 3: Heading results with optimised conditioning measures.

6.1. Performance Under Disturbances

In order to evaluate the controller's robustness, simulations of manoeuvres in presence of environmental disturbances were carried out. The test scenarios considered the same controller parameters as those shown in Table 2. The model that has been used to simulate the waves action on the oil tanker derives the forces and moments induced by a regular sea on a block-shaped ship, and it is based on the modified version of the Pierson-Moskowitz spectrum [2].

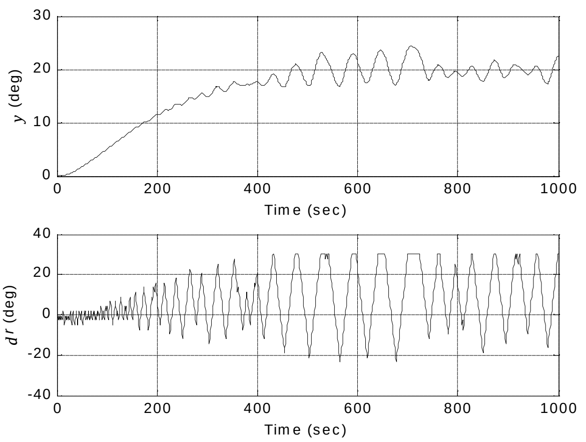

Figure 4: Non-optimised heading controller performance with disturbances.

The action of the forces generated by the waves is directly added to the input vector using the principle of superposition. Waves with a height of 4.0-5.0 m have been applied, which corresponds to a very rough sea description [2]. Figure 4 shows the results when the unmodified *place* function is used. It can be observed that this unoptimised SM controller design generates poor and inaccurate responses under the presence of external disturbances. The responses are considerably improved when the optimised heading controller is used to manoeuvre the tanker in the presence of disturbances (see Figure 5). Here the controller manages to drive the heading angle to the desired reference with an acceptable accuracy. The rudder action is highly demanded due to the presence of the waves forces. Nevertheless, the amplitude of the rudder signal is within the operational envelope of the actuator while maintaining acceptable manoeuvring performance.

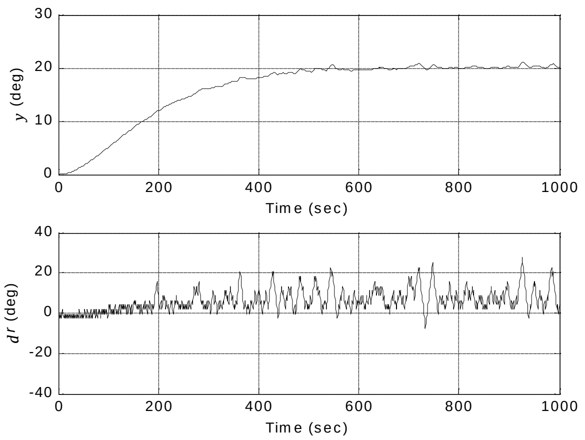

Figure 5: Optimised heading controller performance with disturbances.

7. Conclusions

This paper has presented the application of state-feedback eigenstructure assignment to enhance the response of an oil tanker SM-based navigation controller. Suitable amendments were made on the Matlab *place* function to include new conditioning measures. This proved to provide better and well-conditioned solutions for the tanker application. External disturbances in the form of wind-generated waves were used to test the performance robustness of the optimised controller attaining sensibly satisfactory results.

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