International Symposium on Nonlinear Theory and its Applications
Xi'an, PRC, October 7–11, 2002

Trajectory Stability-preserving and Dimension-reducing Method for Stability Analysis of Nonlinear Systems

Yusheng XUE† Haiqiang Zhou† and Guanrong CHEN‡

† *Corresponding Author*
Nanjing Automation Research Institute
210003 Nanjing, P. R. China
Phone: 86-25-3429900-2229, Fax: 86-25-3427142
Email: yxue@nari-china.com

‡ Department of Electronic Engineering
City University of Hong Kong
Kowloon, Hong Kong, P. R. China
Phone: 852-2788-7922, Fax: 852-2788-7791
Email: gchen@ee.cityu.edu.hk

Summary

A novel methodology, called the trajectory stability-preserving and dimension-reducing (TSPDR) method, is proposed for analyzing various stabilities of nonlinear systems, including bounded stability, unbounded-mode stability, and structural stability. The idea is to first get the time response curves of the system, and then divide the curves into complementary pairs and map them into subspaces of two-degree of freedom by using a suitable linear, nonsingular, and stability-preserving transformation, and finally determine the system stabilities from the projected trajectories. TSPDR combines the universal applicability of numerical integration with rigorous quantitative analysis.

Based on TSPDR, the complementary-cluster energy-barrier criterion (CCEBC) was employed to deal with the bounded stability of motion of nonlinear systems. The stability margin can be defined rigorously for the projected trajectories, and the stability margin of the original system is the minimum among all such margins. CCEBC has been implemented as a commercial soft-ware package, which is unique with quantitative assessment capability in the world today; it has been widely used in power systems in China, Canada, France, and USA.

In this study, the coordinate-plane projection (CPP) method is put forward for studying global bifurcation and higher-dimensional chaos. An n-dimensional space X is divided into $X_1 \in R^2$ and $X_2 \in R^{n-2}$ with $n-1$ independent patterns. For each pattern, the characteristic equation of X_1 is studied with X_2 as relative parameters, and the bifurcation sets of X_1 can be analytically expressed as functions of X_2 but independent of the changes of $X_2(t)$ in the process. After numerical integration, the actual trajectories $X_1(t)$ and $X_2(t)$ are shown respectively in the phase plane and the parameter space. Bifurcation occurs whenever $X_2(t)$ intersects a bifurcation set, thereby the points, at which the structures of $X_1(t)$ and hence $X(t)$ have changed, are identified. Since each bifurcation is rigorously located as well as characterized, the route to chaos can be uniquely described and symbolized for each trajectory of the system.

For illustration, micro-structures of Lorenz and Lorenz-like attractors are studied as examples.

Keywords Chaos, stability preserving, dimension reduction, numerical analysis, micro-structure of chaos

References

[1] Y. S. Xue: *Quantitative Stanility Analysis of Nonlinear Nonautonomous Multi-Rigid-Body Systems,* (in Chinese), Nanjing, 2000.

[2] G. Chen: *Stability of Nonlinear Systems*, Encyclopidea of Electrical and Electronic Engineering, Wiley, New York, Suplment Vol. 1, pp. 627-642, 2000.

This work was supported by Hong Kong CERG 9040579.