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An Application of Automaton with Variable Membership Functions

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Abstract

The paper introduces a fuzzy automaton where internal state is n-tuple of fuzzy sets. Each fuzzy set records the difference between expected and actual input. This enables adaptation to variations between processed unknown pattern and template. Some experiments and results of utilizing the proposed automaton in handwritten character recognition problem are presented.

1. Introduction

In the present paper we introduced a fuzzy automaton whose internal state is defined as the n-tuple of fuzzy sets. With such an extension of internal state, more information can be kept in it. Each element of the n-tuple is the fuzzy set which records the difference between the expected and actual fuzzy input string. This means that automaton in fuzzy sets keeps additional information about the history of fuzzy symbols. The information is used during the processing phase for computing parameters of next expected input symbols. The automaton is called - automaton with variable membership functions. It has applicability to problems of recognition of inexactly defined sequences, where only approximate interdependence of input symbols is known. An example of inexactly defined sequence is the progress of illness, where only an approximate sequence of events is known which can not be precisely described.

An element of the n-tuple that represents the state of the automaton is the fuzzy set. The set may be extended so that automaton can accept multidimensional input symbols.

As an example of application where defined fuzzy automata may be utilized, we have chosen the recognition of hand-written alphanumeric characters. The processing capability of defined fuzzy automaton was tested by recognition of the letter S and the digit 5. The pair was chosen because of great similarity. In the case of high recognition rate, the correct processing of automaton is confirmed and thus its utilization in the field is appropriate.

2. The Definition of Fuzzy Automaton With Variable Membership Functions

What we have in mind when designing the automata is automata capable of solving certain class of pattern recognition problems. The designer of the automata determines by definition of the shape and position of the membership functions of input sets what difference between given and expected input could be tolerated. Defined membership functions of input sets are not constant but subject of changing. Modifications of input sets in form of translations along x-axis and reductions of height are dependent on difference between given and expected input in the previous step. This enables automata to adapt to variations between processed unknown pattern and templates.

Fuzzy automaton is defined as 5-tuple $\mathcal{A} = \langle Q, E, \delta, \widehat{s}^0, Q_F \rangle$, where:

$Q = \{q_1, q_2, ..., q_n\}$ finite set of internal states.

$E = \{E_1, E_2, ..., E_m\}$ finite set of input symbols, defined as fuzzy sets on interval $X = [x_1, x_2] \subset \Re$.

$E_i(x)$ is membership function of fuzzy set E_i and e_i is the point of its maximum value, $E_i(e_i) = 1$.

δ is the relation, which defines transitions between states, $\delta : Q \times E \times Q \to \{0, 1\}$.

The state of automata $\mathcal{A}$ at step k, denoted $\widehat{s}^k$, is n-tuple $\widehat{s}^k = \langle S_1^k, S_2^k, ..., S_n^k \rangle$, where S_i^k is the fuzzy set on interval $S = [s_1, s_2] \subset \Re$, at step k, that is assigned to state q_i.

$\widehat{s}^0$ is fuzzy initial state.

$Q_F \subseteq Q$ is set of final states.

Input to the automaton in the step k, denoted X^k, is the fuzzy set on interval $X = [x_1, x_2] \subset \Re$.

The figure 1 shows the basic scheme of fuzzy automaton and the following algorithm, explained in detail [11], presents one processing cycle of the automaton.

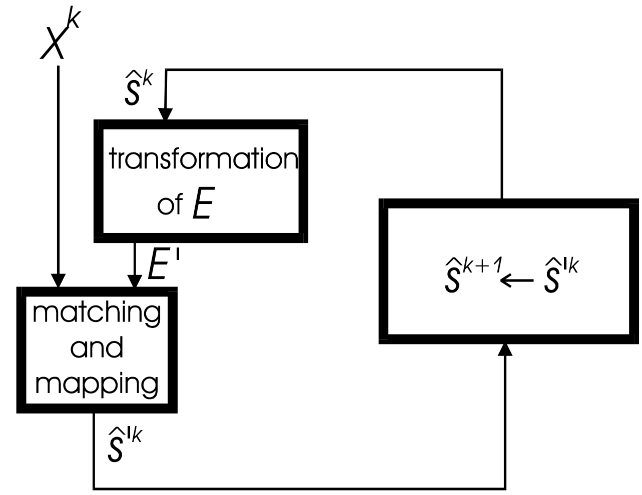

Figure 1: The basic scheme of the fuzzy automaton.

$$
\begin{array}{l}
\text{FOR EACH } state\ q_i \text{ WHERE } height(S_i^k) > 0 \\
\quad \text{FOR EACH } input\ \ symbol\ \ E_l \\
\quad\quad \text{FOR EACH } state\ \ q_j \\
\quad\quad\quad \text{IF } (q_i, E_l, q_j) \in \delta \\
\quad\quad\quad\quad E_{li}^{'} \leftarrow Transform\ \ [E_l(S_i^k)] \\
\quad\quad\quad\quad E_{li}^{''} \leftarrow Match\left[E_{li}^{'}, X^k\right] \\
\quad\quad\quad\quad S_{ji}^{\prime k}(s) \leftarrow Move\left[E_{li}^{''}\right] \\
\text{FOR EACH } state\ \ q_j \\
\quad S_j^{\prime k} \longleftarrow \ \bigcup_i S_{ji}^{\prime k} \\
\quad S_j^{k+1} \longleftarrow \ \ S_j^{\prime}
\end{array}
$$

3. The fuzzy automaton for accepting 2-dimensional inputs

Considering the application, the proposed model of the fuzzy automaton should be generalized so that it could accept 2-dimensional inputs. The former automaton definition needs these alterations:

$E = \{E_1, E_2, ..., E_m\}$ finite set of input symbols, defined as fuzzy sets on Cartesian product $X \times Y = [x_1, x_2] \times \left[y_1, y_2\right] \subset \Re^2$.

$E_i(x, y)$ is membership function of fuzzy set E_i and $e_i = (ex_i, ey_i)$.

The state of automata $\mathcal{A}$ at step k, denoted $\widehat{s}^k$, is n-tuple $\widehat{s}^k = \langle S_1^k, S_2^k, ..., S_n^k \rangle$, where S_i^k is the fuzzy set on Cartesian product $S_x \times S_y = [s_{x1}, s_{x2}] \times \left[s_{y1}, s_{y2}\right] \subset \Re^2$, at step k, that is assigned to state q_i.

Input to the automaton in the step k, denoted X^k, is the fuzzy set on Cartesian product $X \times Y \subset \Re^2$.

Let us illustrate the processing of fuzzy automaton which accepts 2-dimensional inputs. The example presents the transition from the state q_1 to q_2 by expected input symbol E_1, $(q_1, E_1, q_2) \in \delta$. Fig. 2 shows the initial state $\widehat{s}^0$, $\widehat{s}^0 = \langle S_1^0, S_2^0 \rangle$, where S_1^0, the fuzzy set assigned to q_1, is the fuzzy singleton and S_2^0, the fuzzy set assigned to q_2 is an empty fuzzy set. Fig. 3 presents the membership function of expected input symbol E_1 and actual input X^0. Considering the matching between E_1 and X^0 the next state $\widehat{s}^1$ is then computed and presented in fig. 4.

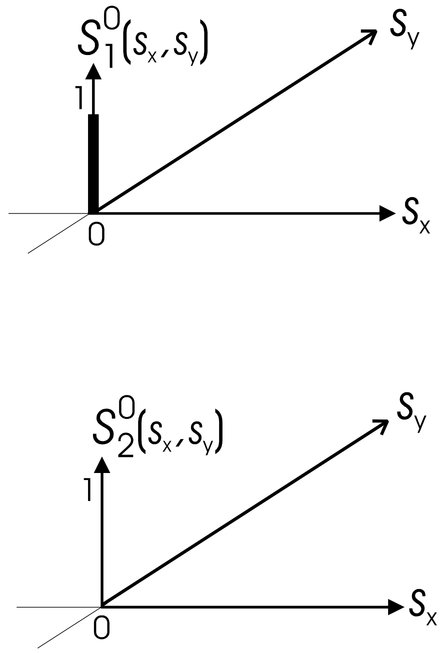

Figure 2: The initial state $\widehat{s}^0$.

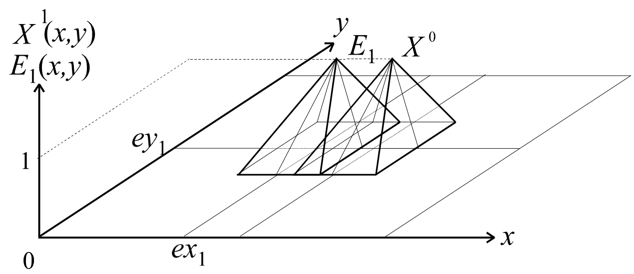

Figure 3: The expected E_1 and actual input X^0.

4. Experiments

The idea of applying the theory of fuzzy logic and fuzzy reasoning to pattern recognition is not new. The authors in [1, 2, 3] introduce different methods and approaches to fuzzy

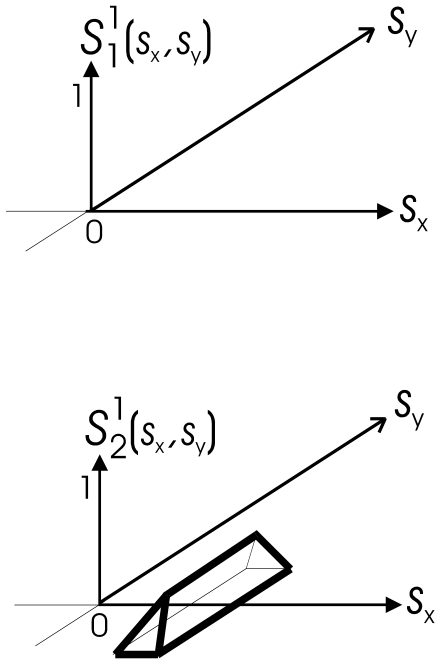

Figure 4: The new state $\widehat{s}^1$.

pattern recognition, whereas the authors in [4, 5, 6, 7] use these methods for character recognition.

The automata, described in the previous section , can also be used in character recognition problems. To test their capability in this field, we made experiments of classifying handwritten characters *5* and *S*. These characters were chosen because of their great similarity. They can easily be misclassified even by humans.

The database for our experiments consists of 100 characters written by 5 individuals. Half of the characters from database belong to the class *5*, half to the class *S*. The character images were scanned as 200 × 200 pixels 256-gray level image. We did not employ the preprocessing, even the noise filtering, nor feature extraction methods, known from literature [8, 9]. The images were only normalized, so that all characters were of the same height and of the same width.

From the database we chose 4 prototypes for class *S* and 5 prototypes for class *5* for which automata were built. Automaton for certain prototype was defined so that it accepted the prototype with maximum membership, namely 1. It is worth mentioning that none of the characters of 2 out of 5 individuals, who wrote characters in database, have been chosen as prototype.

The support of fuzzy sets of input symbols is a square of 20 × 20 pixels and the core is its central point. An example of membership functions of input symbols built from one of the prototype for class *S* and the character under recognition are given in Fig. 5. Fig. 6 and 7 present the third step of

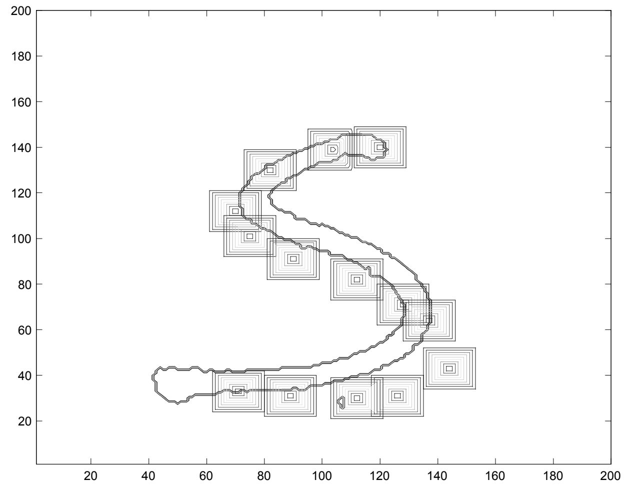

Figure 5: Character under recognition and membership function of expected input symbols.

recognition process.

5. Results

For each class of 50 characters, written by five subjects, several characters were chosen (prototypes) and for each of them an automaton was built. The recognition rate achieved by this algorithm is 95%. Unrecognizied characters were mainly written by subjects whose characters were not chosen as prototypes for which automata were built. As the number of prototypes is not limited, it would be possible for each type (differently written same character) to set up a separate automaton. In this way the recognition rate could reach 100%. On the other hand, this would lead to many prototypes for which automata should be set up, which is practically impossible. A special attention should be paid to time, required for computation of the automaton's next state. The complexity of the automaton demands much processing time. The software tool (MATLAB) and the personal computer of moderate capacity used in simulations might also be the reason for the amount of the processing time. By optimizing the program and using a more powerful computer, the processing time would definitely shorten. Considering the strucure of the algorithm, the algorithm can easily be paralellized and therefore run on several processors simultaneously.

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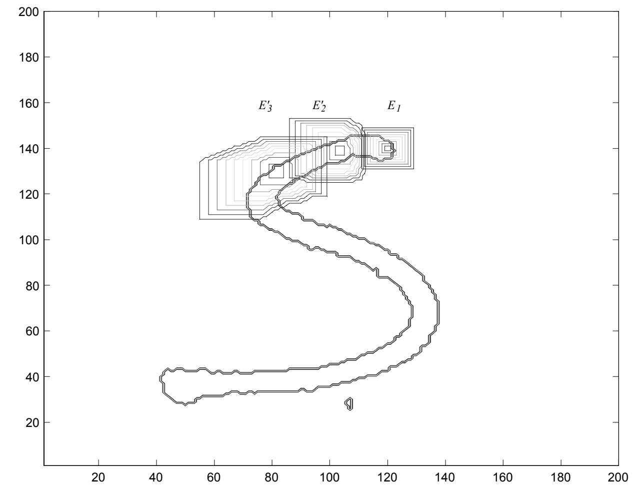

Figure 6: Membership function of the first expected (E_1) and transformed membership function of the second ($E_2^{'}$) and the third ($E_3^{'}$) expected input symbol.

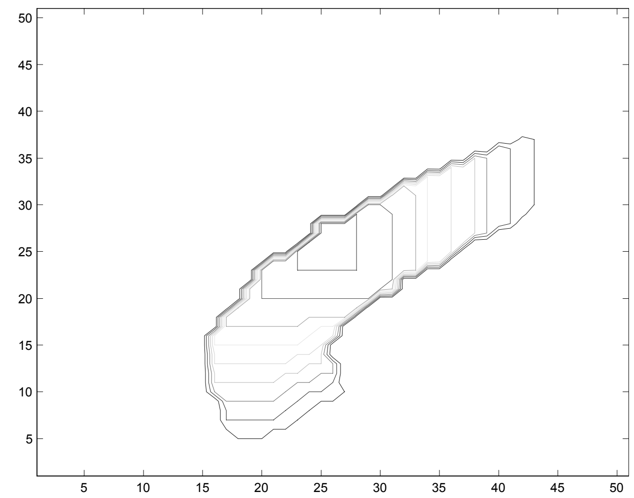

Figure 7: The state of automaton at 3rd step.

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