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Application of Nonlinear Modeling Methods for Event Sizes and Event Timings

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1. INTRODUCTION

Although many methods for analyzing possible chaotic time series have already been proposed, these methods are mainly applied to amplitude information of observed time series, or to event timing. However, in the real world, we often have to treat different type of information, which incorporates both amplitude information and event timings. For example, they are seismic events, financial indices, neural firings and so on. Then, it is very natural to consider that if we can have multivariable information, i.e. both of event sizes and event timings, it might be better to handle them simultaneously for realizing higher prediction accuracy.

From the view point described above, we have already proposed a nonlinear modeling method which uses not only event sizes but also event timings [1, 2]. The method considers complex phenomena as a realization of event occurrences [1, 2]. In order to confirm the effectiveness of our scheme, we produced artificial event time series from mathematical models, then applied a nonlinear prediction algorithm to them. As a result, even if we use a simple nonlinear prediction algorithm, our method has potential ability for analysing complex phenomena by comparing prediction performance of the conventional and the proposed schemes. Namely, our proposed scheme exhibits higher predictability for complex phenomena of event time series.

In this paper, we analyze availability of our modeling scheme for continuous time series in the real world. We consider the situation that we predict occurrence timings of maxima and their sizes from the continuous time series. In case of predicting a next maximum from observed maximum, we can calculate prediction accuracy using next two methods. As the first method, we predict the maximum using all the information of the continuous time series and using $m(>1)$-step ahead prediction. As the second method, we extract maxima from the continuous time series and predict maxima using only the maxima information. In both cases we can observe two information, maximum values and maximum timings, and we apply these information to the proposed modeling scheme. We have already shown that the proposed modeling method is available for predicting maxima of continuous time series produced from mathematical models [3], then we evaluate whether our modeling scheme is applicable for real time series in this paper.

As results, if essential information of the dynamical system is expressed in maxima, the prediction accuracy becomes better when we apply our modeling scheme to the continuous time series. In this paper, we show availability of our proposed modeling scheme for predicting maxima of continuous time series in the real world.

2. PROPOSED METHOD

2.1. How to describe the observed phenomena

First, we consider a dynamical system and an observation function described by the following equations,

$$\begin{cases} \boldsymbol{x}(n+1) = \boldsymbol{f}_0(\boldsymbol{x}(n), \boldsymbol{\xi}(n)), & (1) \\ v(n) = \boldsymbol{h}(\boldsymbol{x}(n)) + \boldsymbol{\eta}(n), & (2) \end{cases}$$

where n is discrete time, $\boldsymbol{x}$ is a state vector, $\boldsymbol{f}_0$ is a dynamical system, $\boldsymbol{h}$ describes an observation function, and $v(n)$ is an observed time series. $\boldsymbol{\xi}$ is a dynamical noise and $\boldsymbol{\eta}$ is a observational noise. The embedding theorems proved that we can recover the dynamics of an unknown system $\boldsymbol{f}_0$ from the information of observed time series v [4, 5]. If we can measure at least a single variable time series, we can reconstruct the dynamics of the original dynamical system $\boldsymbol{f}_0$. Although the method is now widely acknowledged and used for analyzing possible nonlinear time series, there exists a different kind of system from which we can observe not only amplitude

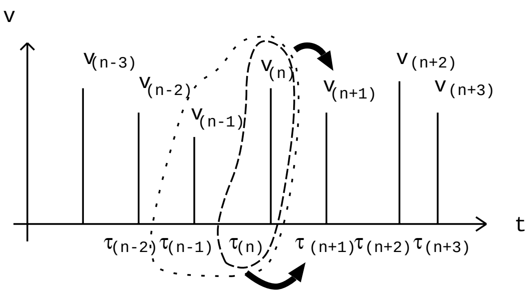


Figure 1: How to deal with event series.

values but also event timings. Let us describe such a dynamical system for event series as follows,

$$\begin{cases} \boldsymbol{x}(n+1) = \boldsymbol{f}(\boldsymbol{x}(n), \boldsymbol{t}(n), \boldsymbol{\xi}_1(n)), & (3) \\ \boldsymbol{t}(n+1) = \boldsymbol{g}(\boldsymbol{x}(n), \boldsymbol{t}(n), \boldsymbol{\xi}_2(n)), & (4) \\ v(n) = \boldsymbol{h}_1(\boldsymbol{x}(n)) + \boldsymbol{\eta}_1(n), & (5) \\ \tau(n) = \boldsymbol{h}_2(\boldsymbol{t}(n)) + \boldsymbol{\eta}_2(n), & (6) \end{cases}$$

where $\boldsymbol{x}$ is a state vector of event sizes, $\boldsymbol{f}$ is a dynamical system for event sizes, $\boldsymbol{t}$ is a state vector of event timings, $\boldsymbol{g}$ is a dynamical system for event timings, and $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are observation functions, respectively. $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are dynamical noises and $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are observational noises. In Fig.1, we show how to deal with event series for prediction. Here, we assume that $\boldsymbol{f}$, $\boldsymbol{g}$, $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are unknown, and we can only access to $v(n)$ and $\tau(n)$, but we can recover the combined dynamics $\boldsymbol{f} \times \boldsymbol{g}$, if we use a mixed delay coordinate,

$$\begin{aligned} V(n) = \{&v(n), v(n-1), \ldots, v(n-m_v), \\ &\tau(n), \tau(n-1), \ldots, \tau(n-m_\tau)\}. \end{aligned} \qquad (7)$$

When we predict $v(n+1)$, we can use not only $v(n)$ but also $\tau(n)$. Namely, we use both $v(n)$ and $\tau(n)$ for elements of a reconstructed state space [1].

2.2. Prediction method

As for the prediction algorithm, we utilize a simple technique of local linear projection called the modified Lorenz's method of analogues [6, 7] in this paper. In this prediction method, first, we consider an orbit of an attractor in a reconstruction state space. Let us consider a point on the orbit as $\boldsymbol{w}(T)$, which will be predicted. Next, we search for M near neighbors of $\boldsymbol{w}(T)$ from the reconstructed attractor. These points are denoted as $\boldsymbol{w}(k_i), (i = 1, 2, \ldots, M)$. We sort $\boldsymbol{w}(k_i)$ in the ascending order, then the point $\boldsymbol{w}(k_1)$ is the nearest point of $\boldsymbol{w}(T)$. Last, we calculate the predicted point $\hat{\boldsymbol{w}}(T+p)$ by the following equation,

$$\hat{\boldsymbol{w}}(T+p) = \frac{\sum_{i=1}^{M} d_i^{-1} \boldsymbol{w}(k_i + p)}{\sum_{i=1}^{M} d_i^{-1}}, \qquad (8)$$

where $d_i = |\boldsymbol{w}(k_i) - \boldsymbol{w}(T)|$ is a distance between $\boldsymbol{w}(T)$ and $\boldsymbol{w}(k_i)$, which is introduced for weighting the information of the near points. If d_i equals to zero, we will not use the corresponding point $\boldsymbol{w}(k_i)$. It is very important to decide an optimal dimension value of the reconstructed state space and appropriate number of near neighbors in order to obtain better prediction results [6].

2.3. Evaluation

If we predict event time series, we have to treat two components for real series and predicted series, not only for event sizes but also for event timings. Then we use the following equation to quantitatively evaluate the error:

$$E = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{(v(i)-\hat{v}(i))^2}{V_v^2} + \frac{(\tau(i)-\hat{\tau}(i))^2}{V_\tau^2}\right\}}}{\sqrt{\frac{1}{V_v} + \frac{1}{V_\tau}}}, \qquad (9)$$

where v is a real event size, $\hat{v}$ is a predicted event size, τ is a real event timing, $\hat{\tau}$ is a predicted event timing, V_v is a variance of v, V_τ is a variance of τ and N is the number of events. Namely, E is a normalized distance between real events and predicted events. If we have well-behaved time series and apply a suitable prediction scheme to the time series, the value of E takes almost 0.

3. ANALYSIS METHOD

In case that we predict a next maximum value from observed maximum values, we compare the prediction accuracy of the following two methods to confirm the availability of the proposed modeling scheme.

3.1. The first method

If we apply prediction schemes, first we can obtain predicted continuous time series $\{\hat{s}\}$ from observed continuous time series $\{s\}$ using $m(>1)$-step ahead direct prediction. Next we can extract maxima directly from $\{s\}$. Let us denote $s_{max}(n)$ as the n-th maximum from $\{s\}$, and also describe $\hat{s}_{max}(n)$ the predicted n-th maximum in a window, the center of which is at $T_{s_{max}}$ (Fig.

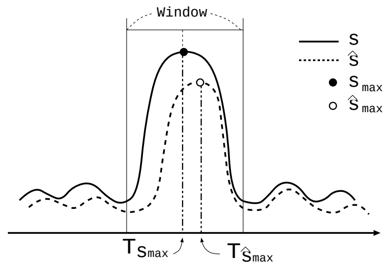


Figure 2: How to make a predicted maximum.

2). Then we define $s_{max}(n)$ as $v(n)$, $\hat{s}_{max}(n)$ as $\hat{v}(n)$, $T_{s_{max}(n)} - T_{s_{max}(n-1)}$ as $\tau(n)$ and $T_{\hat{s}_{max}(n)} - T_{s_{max}(n-1)}$ as $\hat{\tau}(n)$, respectively, and we use these variables to Eq.(9).

3.2. The second method

First, we extract maxima series $\{s_{max}\}$ from the observed continuous time series $\{s\}$. Then we define the maximum values as the event sizes v and the maximum timings as event timings τ. Next we use event sizes and event timings as inputs to our proposed modeling scheme. Finally we can get predicted event sizes $\hat{v}$ and event timings $\hat{\tau}$. Namely, we use these four variables to Eq.(9).

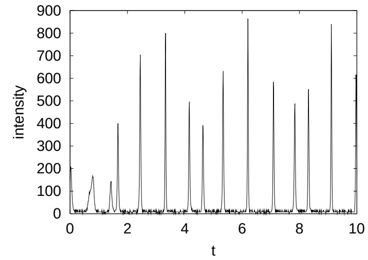
If we can get better prediction accuracy with the second method, we can conjecture that essential aspects of the complex behavior of the observed phenomena are well expressed in the maxima as event timings. Then we can consider that in these maxima included are important information from underlying dynamics of a system which produces observed time series.

4. LASER CHAOS

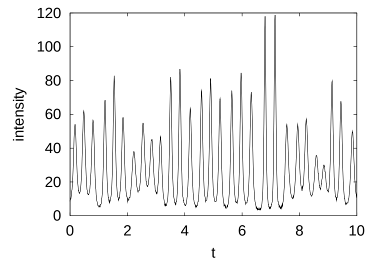
4.1. Data

We analyze laser chaos as real world complex phenomena to confirm the availability of our proposed method for predicting maxima. We use a laser diode-pumped Nd:YVO_4 microchip laser. The output of the microchip laser oscillates chaotically when the injection current of the pumping laser diode is modulated around the relaxation oscillation frequency. We observe a single longitudinal mode (single-mode) oscillation at weak pumping power, whereas two longitudinal mode (two-mode) is observed at high pumping power [8]. A wave form of single-mode data looks like spike oscillations (Fig. 3(a)). Two-mode data show a smooth wave form (Fig. 3(b)).

In the followings, we analyze the prediction performance of both the single-mode time series and the two-mode time series by the first method and the second method.



(a) Single-mode



(b) Two-mode

Figure 3: Wave forms of laser chaos.

4.2. Single-mode

In Table 1, we show the results of prediction performance calculated at various prediction steps ($1 \sim 40$) and several sampling rates ($0.01 \sim 0.04$). At the bottom of Table 1, we also show the results calculated by the proposed modeling scheme. Numbers in parentheses are mean values of maxima timings. Bold numbers denote prediction accuracy by the first method evaluated at the mean value of maxima timing. We compare the prediction performance of the first method in the bold number with the prediction performance of the second method shown in the bottom of Table 1. It is clearly seen that the proposed scheme exhibits better than the prediction method in case of using all the information of continuous time series.

4.3. Two-mode

In Table 2, we show the results of prediction performance calculated at various prediction steps and with several sampling rates. From these results, we can see that the first method results are better than or nearly equal to our proposed method. It maybe because the information of the dynamical system is well expressed not only in maxima but also whole of the time series.

Table 1: Prediction accuracy of the single-mode.

step	E			
	0.01 (73)	0.02 (37)	0.03 (25)	0.04 (19)
1	0.141084	0.370329	0.607613	0.975396
10	1.638282	1.246934	1.177046	1.401648
20	1.767223	1.509762	1.313009	**1.435322**
30	1.560680	1.406435	**1.594936**	1.492033
40	1.562477	**1.484924**	1.512803	1.496401
the proposed method errors				
1	0.586990	0.666685	0.756828	0.795934

Table 2: Prediction accuracy of the two-mode.

step	E			
	0.01 (35)	0.02 (18)	0.03 (12)	0.04 (9)
1	0.055431	0.094244	0.137768	0.220336
10	0.532304	0.443730	0.503365	**0.579416**
20	0.749267	**0.625520**	**0.842022**	0.985072
30	0.657236	0.786681	1.059395	0.991168
40	**0.857367**	1.026111	0.963151	1.019595
the proposed medhod errors				
1	0.672681	0.658680	0.700780	0.718779

5. CONCLUSIONS

In this paper, we investigate predictability of maxima from continuous time series in order to confirm the availability of our proposed modeling method for continuous time series in the real world. We compare the prediction accuracy in the following two cases: using all of the continuous time series with the conventional method and using only maxima (sizes and timings) with our modeling method. From the results of single-mode laser, it is confirmed that our method is more appropriate to predict maxima of continuous time series if the maxima are realizations of essential information of a dynamical system. These results have a good correspondence to the fact that flat parts in time series of single-mode laser indicate noise, so the prediction accuracy is not good calculated by the first method which uses whole of the time series. On the other hand, the results for multi-mode laser, we can see that it is better to use whole information of the continuous time series if we predict its maxima. In conclusion, if essential information of the dynamical system is described in maxima, we can use our modeling method to predict them. Then we can make better modeling and we can realize better prediction.

Although we evaluate our proposed modeling method by estimating prediction accuracy in this paper, we can use another nonlinear indices, for example, Lyapunov exponents, for more precise analysis of our proposed method. It is an important future work to evaluate our modeling method with such measures.

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