THE ESTIMATION OF HARMONIC CONTENTS OF A SINGLE PHASE TRANSFORMER BY HARMONIC DESCRIBING FUNCTIONS

Sy Ruen Huang, Chien-Cheng Yu*, Chin Hsien Lee , Yung-Cheng Change, Tsung-Lung Chen
Dept. of Electrical Engineering Feng-Chia University,
* Dept. of Electrical Engineering Hsiu-Ping Institute of Technology,
E-MAIL:srhuang@fcu.edu.tw

Abstract

In an electric power system, harmonics are often caused by several reasons; for example, the magnetic nonlinearities of transformer core, non-symmetric winding distribution, rotor structure of the generators, and large nonlinear loads. Specifically, the large nonlinear loads often result in the distortion of voltage and current. On the other hand, the magnetic nonlinearities of the transformer core also significantly results in lower-order harmonics especially for the third, fifth and seventh harmonics

To solve the harmonics resulted by the magnetic nonlinearities for the transformer core, several studies have been proposed by the numerical iteration methods. These methods allow the accurate estimation of the harmonic contents; however, they require time-consuming and complicated calculations. In this present research, the describing function method is used to express the harmonic components of the magnetization current. It allows the harmonic components of the magnetization current to be expressed approximately and explicitly. But, the most important is that it is easily manipulated. In addition, when considering the limitation of the harmonic components of the magnetization current, this paper introduce a formula to select the appropriate voltage rating for a single phase transformer. Experimental results verified of the effectiveness of the proposed method.

Indexing terms: Describing function, Transformers, Harmonic current

1. Introduction

Harmonic voltages and currents in an electric power system will result in the over-voltage of the capacitor bank, the harmonic frequency resonance, the improper operations of the protective relay, system equipment overheat [1]. In an electric power system, the presence of harmonics is mainly due to non-symmetric winding, non-symmetric rotor structure of the generators, the magnetic nonlinearities of the transformer core and the use of nonlinear loads [1]. Especially, when a transformer operating in the limit of linear region, the over-exciting becomes an important affect on low order harmonics in the power systems. Therefore, in this article, we focuses on the analysis of the magnetic nonlinearities for the transformer core. In the previous literatures, the harmonics resulting from the magnetic nonlinearities of the transformer core have been analyzed by the several methods, such as: solving the differential equations of the transformer core [2], the use of Norton equivalent method [3], and a quasi-stationary approaches [4], etc. However, to obtain the harmonic contents of the magnetization current, most of these methods require time consuming and complicated calculations.

The second section introduces the notion of describing function and express the harmonics of the magnetic nonlinearities. In the third section, we try to measures the flux linkage, harmonics and the magnitude of the magnetization current with a no-load magnetization experiment. The fourth section analyzes the harmonics in a no-load magnetization experiment. Here, we uses the mathematical expression derived from the second section to calculate the various order harmonics of the magnetization current, and to compare them with the experimental results in the third section.

2. The Harmonic Describing Function and Magnetic Non-linearities

2.1 The Describing Functions for the Single-Phase Transformer Magnetization Current

In a single-phase transformer, the relationships between the flux linkage and the magnetization current, is called the magnetization characteristic [5] or the $\lambda - i_e$ curve, where $\lambda = N\phi$ is the flux linkage, ϕ is the flux and i_e is the magnetization current. When the magnetization current is large, the $\lambda - i_e$ curve, in general, is nonlinear. Consider a transformer with the nonlinear magnetic curve $\lambda - i_e$ shown in Fig.1 and there is a sinusoidal input of the flux linkage, i.e. $\lambda(t)$ ($\lambda(t) = E\sin(\omega t)$). Then, a non-sinusoidal but periodic magnetization current, $i_e(t)$ can be obtained as shown in Fig.1. We will show that the magnetization current $i_e(t)$ can be expressed by the describing functions very concisely. The calculation steps can be written as follows.

Step 1: First, perform piecewise linearity to the $\lambda - i_e$ curve and define the slopes, k_{sat} and k , as the saturation region and the non-saturated region, respectively. Next, find the knee point (λ_k, i_k) in the $\lambda - i_e$ curve as shown in Fig. 1. Specifically, λ_k, i_k are the saturated flux linkage and current respectively.

Step 2: According to Fig. 1, the output $i_e(t)$ can be expressed by the following equations:

$$c(t)=i_e(t)=\begin{cases} \frac{1}{k} E\sin(\omega t) & , 0<\omega t<\beta \\ \frac{1}{k_1}(E\sin(\omega t)-\lambda_k)+\frac{\lambda_k}{k} & , \beta<\omega t<\pi-\beta \\ \frac{1}{k} E\sin(\omega t) & , \pi-\beta<\omega t<\pi \end{cases} \quad (1)$$

where $\beta=\sin^{-1}(\lambda_k/E)$, λ_k is the flux linkage at the knee point, the saturated flux linkage and E is the peak value of the flux linkage Note that: β, λ_k, and E can be measured from the voltage directly.

Step 3: In this study, we only focus on harmonic analysis for the magnetization current $i_e(t)$. Then we have,

$$b_n=\begin{cases} 0; & n \text{ is even} \\ \frac{4E}{\pi}(\frac{1}{k}-\frac{1}{k_{sat}})\left\{\left[\frac{1}{n-1}\sin(n-1)\beta-\frac{1}{n+1}\sin(n+1)\beta\right]+\frac{\lambda_k}{n}\cos n\beta\right\} & , else, n\neq 1 \end{cases} \quad (2)$$

where $\beta=\sin^{-1}(\lambda_k/E)$, k, k_{sat} are the slope in the $\lambda - i_e$ curve, before and after the saturation respectively. From Eq.(2), we can observe that there are no even harmonics and in general, the3[rd], 5[th], and 7[th] harmonic contents are our major concerns. Then, we have the describing functions as follows:

(i) Fundamental Describing function:

$$N_1(E)=\frac{1}{k}\left\{\frac{1}{\alpha}+\frac{1}{\pi}\times[2\beta+\sin(2\beta)]\times(1-\frac{1}{\alpha})\right\} \quad (3)$$

where $\alpha \equiv k_{sat}/k$ is the ratio between the saturated and the non-saturated slope; i.e. α is the ratio of the saturated and non-saturated inductance for the transformer core and $\beta=\sin^{-1}(\lambda_k/E)$.

(ii) Third-harmonic Describing function:

$$N_3(E)=\frac{E}{k}\left\{\frac{2\sin(2\beta)+\sin(4\beta)}{6\pi}\times(1-\frac{1}{\alpha})\right\} \quad (4)$$

(iii) Fifth-harmonic Describing function:

$$N_5(E)=\frac{E}{k}\left\{\frac{3\sin(4\beta)+2\sin(6\beta)}{30\pi}\times(1-\frac{1}{\alpha})\right\} \quad (5)$$

(iv) Seventh-harmonic Describing function:

$$N_7(E)=\frac{E}{k}\left\{\frac{4\sin(6\beta)+3\sin(8\beta)}{84\pi}\times(1-\frac{1}{\alpha})\right\} \quad (6)$$

From Eqs.(12)-(15), the fundamental and harmonic mode of the magnetization current can be expressed explicitly; thus, we can express the ratio of the various order harmonic content of the magnetization current by $N_3(E)/N_1(E)E$, $N_5(E)/N_1(E)E$, $N_7(E)/N_1(E)E$, ..., respectively. As shown in Fig.2 where $\alpha=0.01$, the percentage of the various order harmonic contents versus the ratio of the flux linkage E/λ_k, where λ_k is the saturated flux linkage, (i.e. the voltage ratio if the frequency ω is fixed) are represented by red, dark blue, pink and blue lines respectively. As can be seen in Fig. 2, the total harmonic distortion (THD) is represented by the green line. When considering the frequency is constant, let's denote $\lambda_k=\omega V_k, \lambda=\omega V$, where V_k is defined as the saturated voltage and V is the applied voltage for the transformer, the ratio of the flux linkage E/λ_k becomes the ratio of the voltage V/V_k.

From Fig.2, it can be observed that the third-harmonic component of the magnetization current can reach up to 60 % and THD can reach up to 80% even if the ratio of the flux linkage (the voltage ratio if ω is fixed) is only 1.06. This means that if the applied voltage is larger only 6% than the saturated voltage, there exist 60% third order harmonic contents and 80% THD for magnetization current of the magnetic core.

Similar to Fig.2, various values of α, $\alpha=0.03, \alpha=0.05, \alpha=0.097$, the percentage of the various order harmonic contents and THD versus the ratio of the flux linkage are shown in Figs. 2-4 respectively. As $\alpha=0.097$, from Fig. 5, it can be observed that if the applied voltage is larger only 6% than the saturated voltage, the third-harmonic component and THD are only 13% and 15% respectively.

2.2 Voltage Rating for the Single-Phase Transformer Based on The Harmonic Components of the Magnetization Current

For a transformer, the voltage rating V_{max} is an important issue since the values of inductance and resistance in a transformer model rely on the Voltage Rating. Conventionally, there are two major concerns for the selection of the voltage rating of the transformer. The first is to protect the winding insulation from breakdown due to excessive voltage applied to it. The second is related to the magnetization curve and magnetization current of the transformer. If a steady-state voltage

$$v(t)=V\cos(\omega t) \quad (V) \quad (7)$$

is applied to a transformer's primary winding, the flux linkage of the primary side of the transformer is given by

$$\lambda(t)=\int V\cos(\omega t)dt=\frac{V}{\omega}\sin(\omega t) \quad (8)$$

Then, from Eq.(3), considering the maximum fundamental component of the magnetization current [5], we have

$$\left|N_1(\frac{V_{max}}{\omega})\frac{V_{max}}{\omega}\right|\leq I_{max} \quad (9)$$

where I_{max} is the maximum fundamental component of the magnetization current. Based on Eq.(7), one can use to select the appropriate voltage rating V_{max} [5].

However, in this paper, we further point out that when considering the limitation of the harmonic contents of the magnetization currents, we have

$$THD\left(\frac{V_{max}}{V_k}\right) \le permissible\ THD\ \% \qquad (10)$$

where $V_k = \lambda_k / \omega$, λ_k is the flux linkage at the knee point or the saturated linkage in the magnetization curve. According to Eq.(10), one can select the appropriate voltage rating V_{max}. Assume that THD (Total Harmonic Distortion) is dominated by the third harmonics, Eq.(10) can be simplified as

$$\left|\frac{N_3(E)}{N_1(E)E}\right| = \left|\frac{[2\sin(2\beta)+\sin(4\beta)](\alpha-1)}{6[\pi+(2\beta+\sin(2\beta))(\alpha-1)]}\right| \le \text{permissible percentage } \% \qquad (11)$$

where $\beta = \sin^{-1}(\lambda_k / E) = \sin^{-1}(V_k / V_{max})$. As seen in Figs.2-5, it can be observed that if consider 5% over-voltage for the power systems and as α is very small, the voltage rating should be to select to satisfy $V_{max} \le V_k$; otherwise, it will result in large amounts of harmonic components of magnetization currents.

3 Saturation Curve and Harmonic Content of Experimental Transformer

3.1 Saturation Curve

The saturation curve of the transformer can be obtained by a no-load experiment. Here, an experimental transformer with a rated voltage of 110 V and a rated capacity of 36VA is used. In this experiment, we records each values of λ and i_e and the results can be shown in Fig.6.

3.2 Harmonic Content of The Experimental Transformer

In the experiment, a 1 Ω sampling resistor is used to measure the harmonic components of the transformer magnetization current. In the harmonic analysis, it should be noted that the current signal is converted to the voltage signal by this sampling resistor. The harmonic components are shown in Table 1. As shown in Table 1, the third, fifth and seventh harmonics are 43.5656%, 15.7633% and 2.3% respectively.

Table 1. Experimental results of the harmonic components of the magnetization current

Harmonic order	Harmonic content (peak values)	
Fundamental	45.358mA(0.098pu)	100%
3^{rd} harmonic	19.76mA(0.04269pu)	43.5656%
5^{th} harmonic	7.15mA(0.01545pu)	15.7633%
7^{th} harmonic	1.0434mA(0.00225pu)	2.3%

4 The Comparison of Proposed Method and The Results of Experiment

The values of parameters k_{sat}, k and β used in Eqs. (1)-(6) can be obtained from the magnetic saturation curve shown in Fig.6. Among these parameters, k is the slope close to the linear region of the saturation curve; k_{sat} is the slope of a straight line which is similar to the non-linear region of the saturation curve; β defined in Eq.(1). In practice, the transformer is designed to operate at the limit of linear region and is designed with a minimal over-exciting, and then it has an important affect on the harmonic contents. Hence, as shown in Fig. 6, k_{sat} is chosen as the secant slope at the points whose voltages are 0.95 and 1.05 times of the rated voltage; Also, k is chosen as the secant slope at the point whose voltage is knee point's voltage; β is defined in Eq.(1) which depends on the knee point of the saturation curve. The calculation results for the above parameters are shown in Table 2(a)-(b). Substituting parameters k_{sat}, k, E, λ_k, and β into Eqs (3)-(5) and Eq.(6), we can obtain for various orders harmonics for the magnetization current, as shown in Table 2(a)-(b).

Table 2. Theoretical predictions of the harmonic components of the magnetization current

k	k_{sat}	E	λ_k	β	Fundamental
26.5	2.58	1.0364 pu	0.8 pu	0.882 rad	0.0849 pu (100%)
3^{rd} harmonic	5^{th} harmonic	7^{th} harmonic	-	-	-
0.0305pu (36%)	0.0108pu (12.707%)	0.0017pu (2.0358%)	-	-	-

From Table 1 and Table 2(a)-(b), it can be observed that the 3^{rd}, 5^{th} and 7^{th} harmonic contents predicted by Eqs.(3)-(6) are very close to, and lower than, the experimental results and but only have 8% errors. This difference was due to the negligence of the hysteresis effects of the iron core in the experiment. This omission will led to the differences in the estimation for non-linear area slope k_{sat} and phase angle β.

5. Conclusions

In the third and fourth section, we have shown that the derived equations Eqs.(3)-(6) can be applied to estimate the low order harmonic components of magnetization currents in a single-phase transformer. However, the bias in the estimation of parameters k, k_{sat}, and β will has certain influence on high order harmonics. In Eq.(2), we have proved that there are no even harmonic components of the magnetization current. As for the odd harmonic components, third, fifth and seventh-harmonic components are our major concerns. The merits of Eqs.(3)-(6) are that the harmonics can be easily calculated without the huge computational efforts. Especially, the errors between the experimental results and theoretical predictions are very minor.

On the other hand, we have pointed out that when considering the harmonic components of the magnetization current, if α, the ratio between the saturated and the non-saturated iunductance, is very small in the magnetization curve, the voltage rating

V_{max} should satisfies $V_{max} \le V_k$, where V_k is the saturated voltage defined in Eq.(10); otherwise, it will result in large amounts of harmonic components of magnetization currents. Furthermore, one can apply Eq.(10) and Eq.(11) to select appropriate voltage rating for a transformer.

6. References

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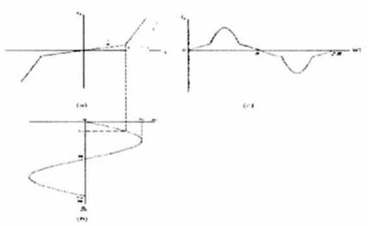

Figure 1: (a) Describing function of saturation curve (b) Flux linkage $\lambda(t)$ (c) Exciting current $i_e(t)$ which was obtained by describing function

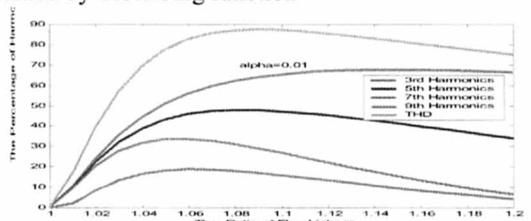

Figure 2: The percentage of the third, fifth, seventh, ninth-harmonic component and THD of the magnetization current versus the ratio of flux linkage (E/λ_k), where $\alpha = 0.01$. Note that if the frequency ω is fixed, then $E/\lambda_k = V/V_k$, where V_K is the saturated voltage defined in Eq.(19).

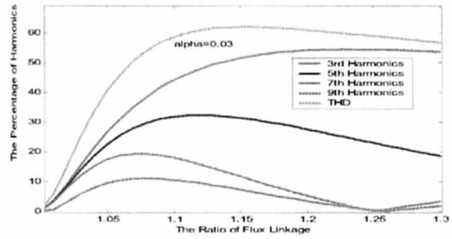

Figure 3: The percentage of the third, fifth, seventh, ninth-harmonic components and THD of the magnetization current versus the ratio of flux linkage (E/λ_k),where $\alpha = 0.03$

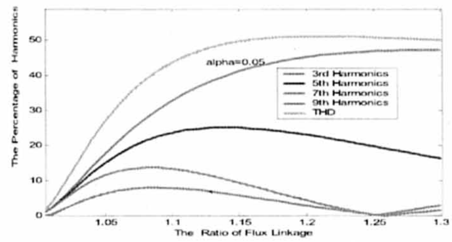

Figure 4: The percentage of the third, fifth, seventh, ninth-harmonic component and THD of the magnetization current versus the ratio of flux linkage (E/λ_k), where $\alpha = 0.05$

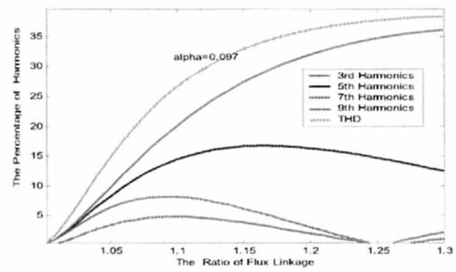

Figure 5: The percentage of the third, fifth, seventh, ninth-harmonic component and THD of the magnetization current versus the ratio of flux linkage (E/λ_k), where $\alpha = 0.097$

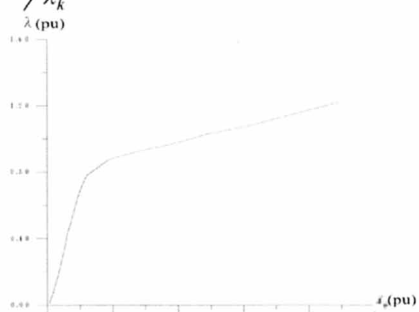

Figure 6: Experimental saturation curve