Synchronization Phenomena in Hysteresis Neural Networks

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1. Introduction

Recently, a system that can treat dynamical information, is attracted to great attention[1]-[3]. The reason why such system receives great attention, is that such dynamical information processing function can be found in biological neural networks. Especially, we think that synchronization phenomenon plays an important role for signal processing in brain. In our previous works, we proposed and analyzed hysteresis neural networks (abbr. HNN)[4]. By using the HNN, we proposed an associative memory that its stable equilibrium point is regarded as the information. The SHN, however, cannot generate desired output sequence. On the other hand, we proposed a novel dynamical associative memory[5]. The each information in the dynamical associative memory is represented by phase different.[5] Such dynamical associative memory can be regarded as a coupled oscillators system. In this article, we propose an oscillatory hysteresis neural network, and analyze it. Also, we propose an application of the oscillatory hysteresis neural network.

2. Hysteresis Neuron

First, we consider a hysteresis neuron[6][7]. The objective hysteresis neuron is described as

$$\begin{cases} \dfrac{dx(t)}{dt} = -x(t) + e(y(t)), \\ \\ y(t) = h(x(t)) = \begin{cases} +1 & \text{for } x > -1, \\ -1 & \text{for } x < +1, \end{cases} \end{cases} \tag{1}$$

where $x(t) \in \Re$ is a state variable, $y(t) \in \{+1, -1\}$ is an output, and $e(y(t)) \in \Re$ corresponds to a feedback parameter. $h(x(t))$ is a bipolar piecewise linear hysteresis as shown in Figure 1.

The state variable of (1) varies toward the equilibrium that denotes $e(y(t))$. Since the equilibrium point is only depends on its output, the equilibrium point becomes a constant while the output does not change. If the equilibrium point does not exist on the hysteresis branch, the trajectory reaches the threshold of the hysteresis before it converges to the equilibrium point. When the trajectory hits the threshold of the hysteresis, the output changes its sign, and the corresponding equilibrium point changes too. This hysteresis neuron can be regarded as a relaxation oscillator whose oscillation frequency can be controlled in the feedback parameter. Figure 2 shows the wave form of the state variable in a hysteresis neuron. The different point is only the self feedback parameter: the feedback parameter is given as $e(y(t)) = -2y(t)$ in Fig.2(a), and it given as $e(y(t)) = -4y(t)$ in Fig.2(b). As these wave forms indicate, the feedback parameter controls its oscillation frequency.

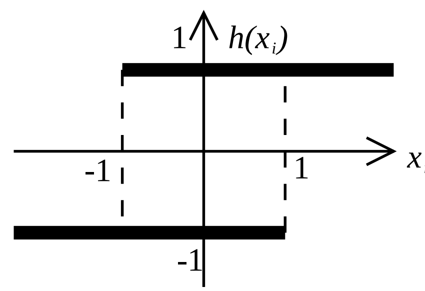
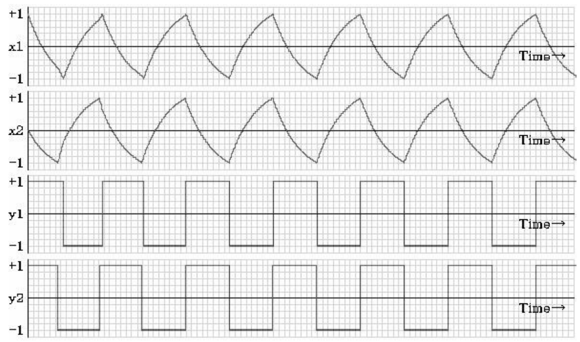


Figure 1: A normalized bipolar hysteresis. The output is switched from $+1$ to -1 if x_i reaches the left threshold "-1", and vice versa.

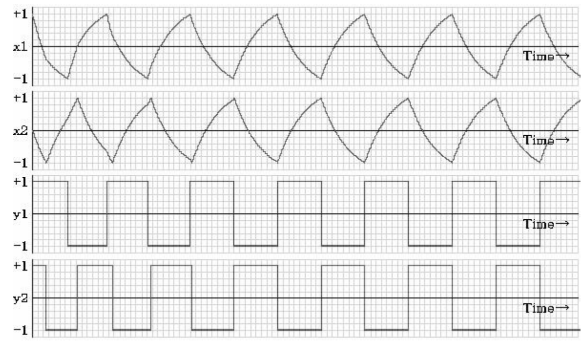
Next, we consider the case where some hysteresis neurons are coupled as

$$\tau_i \frac{dx_i(t)}{dt} = -x_i(t) + \sum_{j=1}^{N} w_{ij} y_j(t), \tag{2}$$

where $\tau_i > 0$ is a time constant, and N is the number of neurons. $w_{ij} \in \Re (i \neq j)$ is a coupling coefficient, w_{ii} is a self feedback parameter. For simplicity, we consider all time constants have the same value, hereafter.

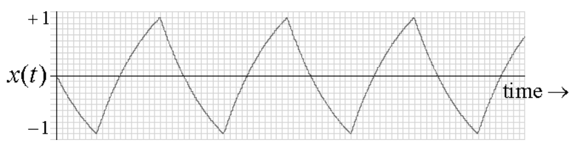


(a) in-phase synchronization

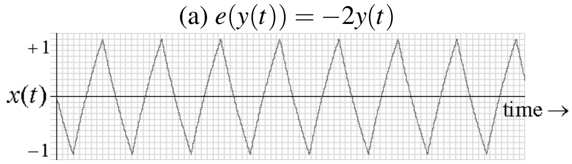


(b) opposite-phase synchronization

Figure 3: The wave form from two hysteresis neuron system. The wave form of (a) is observed in the case where the system has an excitatory connection, and (b) is observed in the case where the system has an inhibitory connection.



(a) $e(y(t)) = -2y(t)$



(b) $e(y(t)) = -4(t)$

Figure 2: The wave forms from a hysteresis neuron. The upper low frequency wave form is observed in the case where the feedback parameter is given as $e(y(t)) = -2y(t)$. The lower high frequency wave form is observed in the case where the feedback parameter is given as $e(y(t)) = -4y(t)$.

The equilibrium point $p_i(t)$ is given from Eqn.(2) by

$$p_i(t) = \sum_{j=1}^{N} w_{ij} y_j(t). \tag{3}$$

If

$$p_i(t) y_i(t) \leq 0, \text{ for some} i \tag{4}$$

is satisfied, a stable output vector does not exist in the system and the output must oscillate[4].

We suppose that the connection coefficient has the following relation:

$$\begin{cases} |w_{ii}| > \displaystyle\sum_{j=1, j\neq i}^{N} |w_{ij}|, \\ w_{ii} + \sum_{j=1, j\neq i}^{N} |w_{ij}| < -1, \forall i. \end{cases} \tag{5}$$

Namely, the connection coefficient matrix has diagonally dominant. If the system adopts such assumption (5), the condition (4) is satisfied. Therefore, this system must exhibit oscillatory state.

For what has been discussed above, the fundamental frequency is influenced by the feedback parameter. The coupling parameters play a role as frequency modulation. Figure 3 shows typical wave forms from the two neuron system. In this figure, the upper two wave forms correspond to the state variable, x_1 and x_2, respectively. The lower two wave forms correspond to the output, y_1 and y_2, respectively.

Both results are observed into the following initial configurations. The initial value is adjusted as

$$\begin{array}{l} (x_1(0), y_1(0)) = (+1, +1) \\ (x_2(0), y_2(0)) = (0, +1) \end{array} \tag{6}$$

In the case of Fig.3(a), the parameters configuration is

$$\begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ 1 & -4 \end{pmatrix}. \tag{7}$$

In this case, the cross-connection coefficients have a positive value. Namely, these coefficients can be regarded as excitatory connections. In this case, the system exhibits in-phase synchronization. On the other hand, the parameters configuration of Fig.3(b) is

$$\begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} = \begin{pmatrix} -4 & -1 \\ -1 & -4 \end{pmatrix}. \tag{8}$$

In this case, the cross-connection coefficient has a negative connection coefficient, that is an inhibitory connection. In this case, the system exhibits opposite-phase synchronization.

3. Analysis

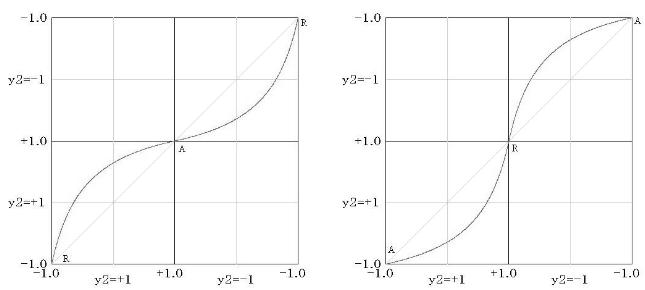
In order to analyze these synchronization phenomena, we define a return map. Since this system always satisfies (4), the range of the state variable is $[-1,+1]$. Let Φ be a one dimensional domain such that

$$\Phi \equiv \{x_2(T) || x_2(T)| \geq 1, x_1(T) = 1\} \tag{9}$$

We can define a return map from Φ to itself:

$$\mathscr{F} : \Phi \mapsto \Phi \tag{10}$$

Note this mapping can be easily calculate by using the fast calculate algorithm[4]. Figure 4 shows the return map $\mathscr{F}$. The parameter configuration of Figure 4(a) obeys (7), and (b) obeys (8). These return maps contain two fixed points that are



(a) (b)

Figure 4: Return map $\mathscr{F}$ of the two neuron system. (a) and (b) correspond to the case of Figures 3(a) and (b), respectively. "A" denotes an attractor, and "R" denotes a repeller.

shown as "R" and "A", respectively. The point "R" denotes repellent fixed point, and "A" denotes attractive fixed point. Then all initial value converges to the "A" fixed point. In the case of Fig.4(a), the attractive fixed point is $x_2 = 1$. Based on the definition of this return map, this attractor represents in-phase synchronization. When the parameters set as (7), all initial value converges to the in-phase synchronization attractor such as Fig.3(a).

On the other hand, in the case of Fig.4(b), the attractive fixed point is $x_2 = -1.0$. Based on the definition of this return map, this attractor represents opposite-phase synchronization such as Fig.3(b).

It follows from what has been said thus far that the excitatory connection leads to in-phase synchronization, and the inhibitory connection leads to opposite-phase phase synchronization.

4. Application

In this section, we investigate graph coloring problems to solve by using the oscillatory state of the hysteresis neural networks. The description of the graph coloring problems is that a vertex coloring of an undirected graph is an assignment of a label to each node. It is required that the labels on the pair of nodes incident to any edge be different. A minimum

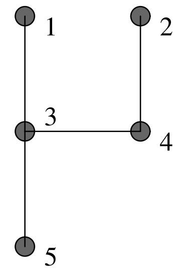


Figure 5: Example of 2-colorable graph with 5 vertices[8].

coloring of a graph is a coloring that uses as few different labels as possible.

Dr.Wu studied the possibility of coloring graphs by means of synchronized coupled oscillators[8]. We apply the oscillatory hysteresis neural network to the graph coloring problems. In order to solve the graph coloring problems, the hysteresis neurons are assigned to each vertex. Figure 5 shows a simplest example of 2-colorable graph with 5 vertices which is introduced in Ref.[8]. From Fig.5, the adjacent matrix A of the graph is given as

$$A \equiv \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \tag{11}$$

where $a_{ij} = 1$ represents connected between i-th vertex and j-th vertex, and $a_{ij} = 0$ represents disconnected state.

Based on this adjacent matrix, we determine the non-diagonal elements of the connection matrix of the oscillatory hysteresis neural networks.

$$w_{ij} = \alpha \frac{a_{ij}}{\sum_{k=1}^{N} a_{ik}} \text{ ,for } i \neq j. \tag{12}$$

where a_{ij} is a component element of the adjacent matrix, and α is a coefficient constant.

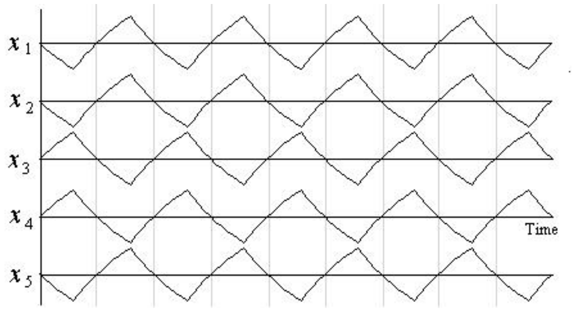
The diagonal element of the connection matrix is determined by Eq.(5). Consequently, the connection coefficient matrix W is given as

$$W \equiv \begin{pmatrix} -5 & 0 & -1.2 & 0 & 0 \\ 0 & -5 & 0 & -1.2 & 0 \\ -0.4 & 0 & -5 & -0.4 & -0.4 \\ 0 & -0.6 & -0.6 & -5 & 0 \\ 0 & 0 & -1.2 & 0 & -5 \end{pmatrix}. \tag{13}$$

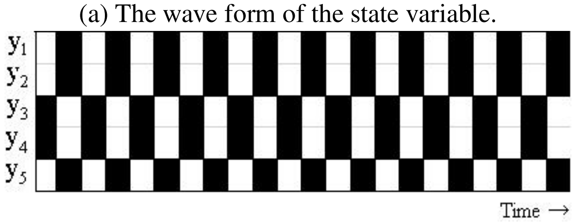
Note that this matrix is an example.

In this case, the system exhibits periodic attractor as shown in Fig.6. Figure 6(a) shows the wave form of the state of each neuron, and Fig.6 shows the corresponding output sequence.

In Figure 6(b), the black box indicates the output is negative, and the white box indicates the output is positive.



(a) The wave form of the state variable.



(b) The output sequence which corresponds to (a).

Figure 6: The oscillatory state corresponds to the solution of 2-colorable graph with 5 vertices. The horizontal axis corresponds to the time evolution, and the vertical axis corresponds to the neuron.

As these wave form indicate, the wave form can be classified into two clusters by using its phase. Also, Fig.7 shows two dimensional phase space projection. A slanted line of an upward slant to the right denotes these two neurons have in-phase synchronization relation, and the other denotes these have opposite-phase synchronization. For example, it is in-phase synchronization between 1st and 5-th neuron, and it is opposite-phase synchronization between 1st and 2nd neuron. As this figure indicates, the neurons can be classified into two clusters. One cluster consists with x_1, x_4, and x_5, these exhibits in-phase synchronization. On the other cluster contains 3rd and 4th neuron, then these vertices is assigned the same color. Namely, the phase difference of each neuron corresponds to the solution of the graph coloring problem.

5. Conclusions

In this article, we analyzed the synchronization phenomena in the oscillatory hysteresis neural networks. We clarified that the system exhibits in-phase synchronization oscillation when the cross-connection coefficient denotes excitatory connection. Also, the system exhibits opposite-phase synchronization when the cross-connection coefficient denotes inhibitory connection. Based on this results, we proposed an application which dyes 2-colorable graphs. To solve k-colorable graph by the hysteresis neural network is one of our future problems.

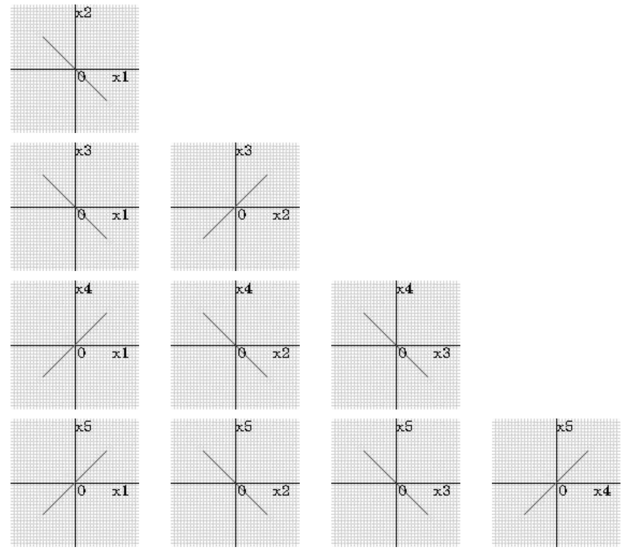


Figure 7: The phase space projection of each neuron. This figure shows the relationship between each neuron.

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