Analysis of Hetero Associative Memory with Hysteresis

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Abstract

Associative memory is one of the most important applications of artificial neural networks. The associative memories can be classified into two categories: auto association model and hetero association model. In this article, we consider the ability of a continuous-time hetero associative memory by using hysteresis neural networks. This system can retrieve some desired memories as a cyclic output sequence. We consider the condition that all desired memories can be stored as cyclic output sequence.

1. Introduction

The artificial neural network serves as an associative memory by retrieving an associated output pattern given some input pattern. The association can be classified into auto-associative or hetero-associative, depending on whether or not the input and output patterns belong to the same set of patterns[1]. For the hetero associative memory, if all desired memories can be stored each other, the stored pattern recalls as a cyclic output sequence which consists with some desired patterns.

In this article, we propose and analyze a hetero associative memory which can retrieve some desired memories as a cyclic output sequence. This system contains hysteresis neuron[2] whose activation function represents piecewise linear bipolar hysteresis. The output of the system takes only two values, and the state variable takes real number. Namely, the system is said to be a hybrid dynamical system[3]. This system is continuous-time system, so the dynamics of the system describes a piecewise linear differential equation.

Many of previous studies about hetero associative memory, treats a discrete-time system[4][5][6]. The feature of the dynamics of discrete-time system depends on a synchronous update of the state. In generally, the continuous-time system corresponds to a asynchronous update mode. Therefore, some kinds of attractor in discrete-time system cannot observe in continuous-time system[7]. The simultaneously switching of output, however, can be observed in hysteresis neural networks[8]. The switching future plays an important role in a hetero associative memory. In order to consider the difference between a continuous-time associative memory and a discrete-time system, we analyze the proposed hysteresis neural network.

In this research, we aim at the strict analysis of the dynamics of cyclic output sequence which contains some desired memories. There is the case where the system cannot retrieve the desired output sequence[9].We will clarify its cause, and we will give the necessary condition of stability for the hetero associative memory.

2. System

The objective system is described as the following differential equation[9].

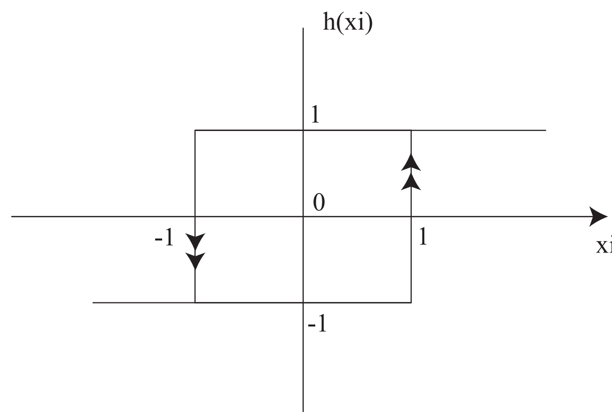

Figure 1: A normalized bipolar hysteresis. The output is switched from $+1$ to -1 if x_i reaches the left threshold "-1", and vice versa.

$$\left\{\begin{array}{l} \gamma_i \dfrac{dx_i}{dt} = -x_i + \displaystyle\sum_{j=0}^{N-1} w_{ij} h(x_j), \\ \\ h(x_i) = \left\{\begin{array}{ll} +1 & \text{for } x_i > -1, \\ -1 & \text{for } x_i < +1, \end{array}\right. \end{array}\right. \tag{1}$$

where $x_i \in \Re$ is a state variable of i-th neuron, $\gamma_i \in \Re$ is a time constant, $w_{ij} \in \Re$ is a connection coefficient between i-th neuron and j-th neuron, and N denotes the number of neurons. $f(x_i)$ has a hysteresis property as shown in Fig.1. In this article, the positive output state is said to be "action state", and the negative output state is said to be "resting state". For simplification of the analysis, we assume that all time constants have the same value, namely we set $\gamma_i = 1$ for $\forall i$.

In order to memorize some desired memories as a cyclic sequence, the connection coefficient is determined by the cross-correlation learning[10] as the follows:

$$w_{ij} = \frac{1}{M}\left(\sum_{l=1}^{M-1} p_i^{l+1} p_j^l + p_i^1 p_j^M\right), \tag{2}$$

where, $p_i^l \in \{+1, -1\}$ is i-th element of l-th desired memory pattern, and M is number of desired memories.

By using the above equation, the system can be regarded as "Hetero Associative Memory." If the system memorizes all desired memories, the desired memories can be retrieved by the appearance order such as: $p^1 \to p^2 \to p^3 \to \cdots \to p^M \to p^1$. In this article, we argue what kind of pattern can be memorized.

3. Simulation

In this section, we carry out two experiments to confirm the fundamental dynamics of this system.

3.1. Simulation I

In order to confirm fundamental property of the system, we conduct a simplest experiment as the follow. The initial configuration is set as the following:
The number of neurons N is 16, and the number of desired memories M is 4. The desired memories are given as

$$\begin{aligned}
p^1 &\equiv (+,+,+,+,+,+,+,+,-,-,-,-,-,-,-,-),\\
p^2 &\equiv (-,-,-,-,+,+,+,+,+,+,+,+,-,-,-,-),\\
p^3 &\equiv (-,-,-,-,-,-,-,-,+,+,+,+,+,+,+,+),\\
p^4 &\equiv (+,+,+,+,-,-,-,-,-,-,-,-,+,+,+,+).
\end{aligned}$$

Note that p^k and p^{k+1} are orthogonality relation. Also, p^k and p^{k+2} are inverted relation.

By using the first desired memory pattern, the initial value of the inner state of each neuron is set as

$$x_i(0) = p_i^1. \tag{3}$$

By using above initial configuration, a numerical simulation is carried out. In this case, we confirm that the system exhibits a periodic output sequence whose appearance order of pattern obeys the learning order as:

$$p^1 \to p^2 \to p^3 \to p^4 \to p^1 \cdots.$$

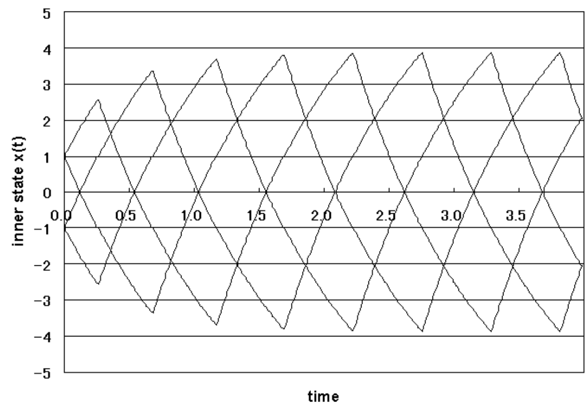

Figure 2: Time wave form of the inner state in the case where 4 desired memories are stored. The horizontal axis denotes time evolution, and the vertical axis denotes the inner state. The system consist with 16 neurons, then 16 inner states are overlapped in this figure. Since the inner states become 4 clusters, only 4 wave form can be observed.

The time evolution of the inner state is shown in Fig.2 The desired memories can be classified four clusters: for example one cluster compose of 1, 2, 3 and 4-th neuron whose corresponding desired memories have the same sign. The inner state of these exhibits in-phase synchronization. Therefore, Fig.2 shows the inner states exhibit four phase synchronization. This figure tell us the system exhibits a simple periodic attractor which consists with four clusters.

3.2. Simulation II

Next, we simulate the other case which the number of desired memories is eight. Other parameters are applied the same as the previous simulation. The desired memories are given as

$$\begin{aligned}
p^1 &\equiv (+,+,+,+,+,+,+,+,-,-,-,-,-,-,-,-),\\
p^2 &\equiv (-,-,+,+,+,+,+,+,+,+,-,-,-,-,-,-),\\
p^3 &\equiv (-,-,-,-,+,+,+,+,+,+,+,+,-,-,-,-),\\
p^4 &\equiv (-,-,-,-,-,-,+,+,+,+,+,+,+,+,-,-),\\
p^5 &\equiv (-,-,-,-,-,-,-,-,+,+,+,+,+,+,+,+),\\
p^6 &\equiv (+,+,-,-,-,-,-,-,-,-,+,+,+,+,+,+),\\
p^7 &\equiv (+,+,+,+,-,-,-,-,-,-,-,-,+,+,+,+),\\
p^8 &\equiv (+,+,+,+,+,+,-,-,-,-,-,-,-,-,+,+).
\end{aligned}$$

Note that p^k and p^{k+4} are inverted relation.

In this case, we confirm that the system exhibits a periodic output sequence whose appearance order of pattern obeys the learning order. The time evolution of the inner state is shown in Fig.3 In this case, the system exhibits eight clusters that each cluster consists of two neurons. The reason why eight

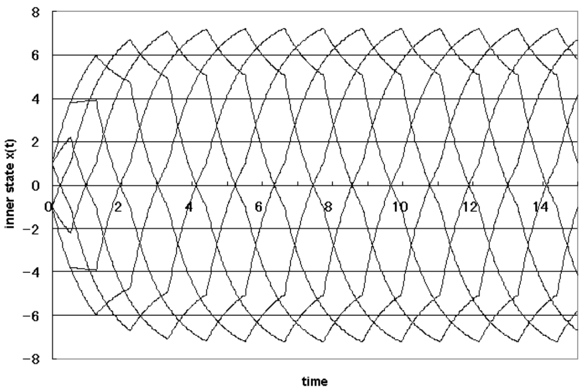

Figure 3: Time wave form of the inner state in the case where 8 desired memories are stored into 16 neurons system. Since the inner states become 8 clusters, only 8 wave form can be observed.

clusters are generated, that the desired memories can be classified into eight categories. Therefore, the system exhibits a periodic attractor with eight clusters.

4. Discussion

Based on the result of simulations, we consider the necessary condition of stability for the hetero associative memory.

4.1. Stability condition for hetero association

In order to retrieve all desired memories for proposed system, we consider the property of desired memories. Here, we assume that p_i^k is a current output which corresponds to k-th desired memory pattern. e_i denotes the equilibrium point of the neuron. The transition of the output vector can be classified into the following three cases:

(i) The output of neuron changes from resting state to action state.
This is the case where the desired memories have a structure such that $p_i^k = -1$ and $p_i^{k+1} = +1$. In order to retrieve this relationship, the following condition has to be satisfied.

$$e_i > +1 \text{ for } p_i^k = -1 \text{ and } p_i^{k+1} = +1 \tag{4}$$

(ii) The output of neuron changes from action state to resting state.
This is the case where the desired memories have a structure as $p_i^k = +1$ and $p_i^{k+1} = -1$. In this case, the following condition has to be satisfied.

$$e_i < -1 \text{ for } p_i^k = +1 \text{ and } p_i^{k+1} = -1 \tag{5}$$

(iii) The output does not change its state.
In this case, the inner state of neuron does not reach the threshold, and its output keeps the same value. In the case where the output of neuron takes resting state, the following condition has to be satisfied

$$e_i < +1 \text{ for } p_i^k = -1 \text{ and } p_i^{k+1} = -1 \tag{6}$$

On the other hand, in the case where the output of neuron takes action state, the following condition has to be satisfied

$$e_i > -1 \text{ for } p_i^k = +1 \text{ and } p_i^{k+1} = +1 \tag{7}$$

Referring to Eqs.(4),(5),(6), and (7), we can obtain the following conditions.

$$\begin{cases} e_i p_i^k < -1 \text{ for } p_i^k p_i^{k+1} = -1, \\ e_i p_i^k > -1 \text{ for } p_i^k p_i^{k+1} = +1. \end{cases} \tag{8}$$

where,

$$e_i = \sum_{j=1}^{N} w_{ij} p_i^k \tag{9}$$

is an equilibrium point when the output vector corresponds to the k-th desired memory pattern.

If the condition (8) is satisfied for all desired memories, the system can can be retrieved by the order of desired memories. Figure 4 shows the relationship between the equilibrium point and the output.

The equilibrium point depends on the memorized desired memory patterns and the current output vector. Now, we assume that the output vector corresponds to k-th desired memory pattern. In this case, the equilibrium point is given by the following equation.

$$e_i = \sum_{l=1}^{M-1} p_i^{l+1} \langle p^l, p^k \rangle + p_i^1 \langle p^M, p^k \rangle \tag{10}$$

where, $\langle a, b \rangle$ means the inner product between vector a and b.

4.2. Analysis of Simulation I

Here, we analyze why the patterns of Simulation I can be retrieved successfully. The desired memories have the following relation:

$$\begin{cases} \langle p^{\,k \ mod \ M+1}, p^{\,(k+1) \ mod \ M+1} \rangle = 0 \\ p^{\,k \ mod \ M+1} = \overline{p}^{\,(k+2) \ mod \ M+1} \end{cases} \tag{11}$$

where $\overline{a}$ denotes the inverse vector of a.

Substitute Eqs.(11) for Eq.(10), we obtain the following equation.

$$e_i = \frac{N}{M} (p_i^{\,k \ mod \ M+1} - p_i^{\,(k+2) \ mod \ M+1}) \tag{12}$$

The equilibrium point expressed in Eq.(12) satisfies the condition (8). Therefore, the desired memory patterns of Simulation I can be retrieved successfully.

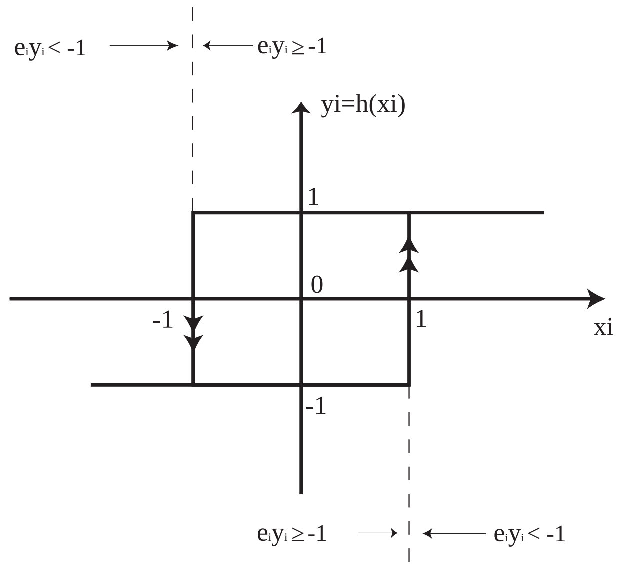

Figure 4: The relationship between the equilibrium point and the output. If the equilibrium point exists outside of threshold of hysteresis, the output may change its sign. Contrarily, if the equilibrium point exists on the hysteresis branch, the output cannot change its sign.

4.3. Analysis of Simulation II

The desired memory patterns of Simulation II have the following relation:

$$\begin{cases} \langle p^{\,k \; mod \; M+1}, p^{\,(k+1) \; mod \; M+1} \rangle = \dfrac{N}{2} \\ \langle p^{\,k \; mod \; M+1}, p^{\,(k+2) \; mod \; M+1} \rangle = 0 \\ \langle p^{\,k \; mod \; M+1}, p^{\,(k+3) \; mod \; M+1} \rangle = -\dfrac{N}{2} \\ p^{\,k \; mod \; M+1} = \overline{p}^{\,(k+4) \; mod \; M+1} \end{cases} \tag{13}$$

Substitute Eqs.(13) for Eq.(10), the equilibrium point is given as.

$$e_i = \frac{N}{M} \sum_{j=1}^{M} p_i^{\,j \; mod \; M+1} U\left(\left| (\, k \; mod \; M+1) - j \right| \; mod \; 5 \right), \tag{14}$$

where, $U(\cdot)$ is a function as

$$U(v) = \begin{cases} 1 & \text{for } v = 0, \\ \frac{1}{2} & \text{for } v = 1, \\ 0 & \text{for } v = 2, \\ -\frac{1}{2} & \text{for } v = 3, \\ -1 & \text{for } v = 4. \end{cases} \tag{15}$$

The equilibrium point expressed in Eq.(15) satisfies the condition (8). Therefore, the desired memory patterns of Simulation II can be retrieved successfully.

5. Conclusions

This article proposed the hetero associative memory which consists with hysteresis neuron. The system evoke stored desired memories as a cyclic output sequence. And, we gave the condition that the system recalls all desired memories as successive output sequence.

In this article, we confirmed the desired memory patterns satisfy the condition. The development of the synthesis procedure of the connection coefficient matrix which satisfies the condition (8) is one of our future problems.

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