# Application of Chaos in Measurement

Wenbo Liu and Shenglin Yü

College of Automation Engineering
Nanjing University of Aeronautics & Astronautics , Nanjing 210016, P. R. China
Phone:86-25-4892766, Fax: 86-25-4892300, Email: WenboLiu@nuaa.edu.cn

**Abstract:** Using the high sensitivity to initial values of chaotic systems, this paper describes an application of chaos in the field of measurement. A general method for signal coding based on symbolic sequences and the relationship between the variable (to be measured) and its symbolic sequence is presented. Some performances of the chaos-based measurement system are also discussed. Theoretical analysis and simulation and experiment results show that chaotic systems are potentially attractive in the field of measurement.

## 1. Introduction

In recent years, there has been growing interest in applying chaotic systems to a variety of engineering technologies[1]~[4]. Not only the random appearance of a chaotic signal makes it an attractive candidate for many signal processing and communication applications, such as pattern recognition, spread spectrum and secure communications, but also the high sensitivity of chaos to system parameters and initial conditions of chaotic systems have been used for improving measurement quality and quantization of signals.

In this paper, we focus our attention on the application of the sensitivity of chaos to initial values in measurement. The main idea for realizing measurement using a chaotic system is that the chaotic trajectories are different when they start from slightly different initial values, even the initial difference is extremely small. In the mean time, a trajectory and an initial value of a chaotic system is a rigorously one-to-one correspondence. So, if the chaotic trajectory can be exactly known, then the initial value corresponding to the change of variables (to be measured) should also be known, and the purpose of measuring the unknown variables can be well achieved. In this paper, we choose a simple first-order discrete-time chaotic system as the measurement circuit for an initial value sensor, where the system's dynamics are governed by a piecewise linear map. A method for signal coding of this system and the relationship between the variable and its symbolic sequence are presented. Finally, we also analyze some performance of the chaos-based measuring system via experiments.

## 2. Chaotic Signal Coding and Estimation of Measured Signals

A chaotic system has the prominent property of sensitivity to initial conditions, but not every chaotic system can be used for the purpose of measurement. Generally, if a chaotic system satisfies the following conditions, one can choose it to design a measuring system:

- The phase space $B$ of trajectories and the space (set) $X$ of the initial values must be one-to-one. So, every initial value $x_0$ as an element in space $X$ corresponds uniquely to a trajectory $\{x_0, x_1, x_2, \ldots\ldots, x_i, \ldots\ldots\}$ in space $B$.
- The chaotic system has only one attractor. The system should have no other attractors, limit cycles, and fixed points.
- Trajectories can be coded and the mathematical relationship between the generated symbolic sequence and the initial value corresponding to the trajectory can be determined.

Many 1-D chaotic systems satisfy the above conditions. In this paper, we choose the following chaotic model to design a measurement system:

$$x_n = f(x_{(n-1)}) \tag{1}$$

with

$$f(x) = \begin{cases} x/s_1 & 0 \le x \le s_1 \\ (x-s_1)/s_2 & s_1 < x \le 1 \end{cases}$$

where $f: I \to I, I = [0,1]$ is a chaotic map, $s_1 > 0$, $s_2 > 0$, $s_1 + s_2 = 1$, and $x \in [0,1]$. The map and the process of iteration are shown in Fig.1.

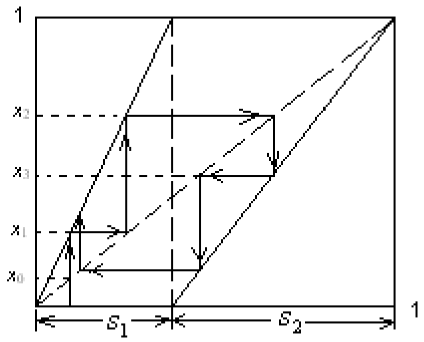


Fig. 1 The chaotic map

Here, let $x_0 = v(n)$ be an initial value of the chaotic system, where $v(n)$ is the signal to be measured at time $n$, and $f^k$ denotes the $kth$ composition of the map $f$. Then, $\{f^k(x_0)\}_{k=0}^{N}$ is a chaotic signal, named the trajectory with initial value $x_0$. Assuming that at iteration $i$, the phase point is $x_i = f^i(x_0)$, and we assign it a symbol $a_i = m$ ( $m \in \{0,1\}$ ). Then, any $x_0 = v(n)$ can be iterated as an orbit $X = \{x_0, x_1, ..., x_i, ....\}$, and any orbit can be encoded as a string $A = \{a_0, a_1, ..., a_i, ...\}$, which is called a symbolic sequence. The above coding determines a map ϕ from $X$ to $A$ as:

$$\Phi(A) = x_0 = v(n)$$

Because the map ϕ is one-to-one for generating partitions, $x_0 = v(n)$ can be determined uniquely by a string $A = \{a_0, a_1, ..., a_i, ...\}$ as $i \to \infty$.

For chaotic system (1), The symbolic sequence is obtained according to

$$a_i = \begin{cases} 0, & when \quad x_i \le s_1 \\ 1, & when \quad x_i > s_1 \end{cases}$$

The symbolic sequence $A = \{a_0, a_1, ..., a_i, ...\}$ is composed of "0" and "1", and any initial value $x_0 = v(n)$ can be estimated (denoted by $\hat{x}_0$) by $A$ when the number of iteration is finite, as follows:

$$\hat{x}_0 = \hat{v}(n) = \sum_{i=0}^{N} s_1^{N_i^0+1} s_2^{N_i^1} a_i \tag{2}$$

where $N < \infty$ is the number of iterations, $N_i^0$ is the number of "0" before $a_i$, $N_i^1$ is the number of "1" before $a_i$. A proof of formula (2) can be found in [9].

**Example 1** For system (1), let $s_1 = 0.8, s_2 = 0.2$, $x_0 = 0.899$, $N = 19$. Then, the orbit

$B = \{x_0, x_1, ......, x_{19}\} =$

{0.89900000000000, 0.49500000000000, 0.61875000000000, 0.77343750000000, 0.96679687500000, 0.83398437500000, 0.16992187500000, 0.21240234375000, 0.26550292968750, 0.33187866210937, 0.41484832763672, 0.51856040954590, 0.64820051193237, 0.81025063991546, 0.05125319957731, 0.06406649947164, 0.08008312433955, 0.10010390542444, 0.12512988178055, 0.15641235222569 },

and the symbolic sequence $A = \{a_0, a_1, ..., a_{19}\} =$ {1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}

The estimate of $x_0 = v(n)$ is:

$$\hat{x}_0 = \hat{v}(n) = 0.8 + 0.8^4 0.2 + 0.8^4 0.2^2 + 0.8^{11} 0.2^3$$
$$= 0.89899119476736$$

In order to increase the accuracy of the measurement, in practice, we generally judge the last code, either $a_N = 0$ or $a_N = 1$. If $a_N = 0$, we will add one half of the value $a_{N+1} = 1$ to the estimated result. Then, formula (2) will be written as

$$\hat{x}_0 = \hat{v}(n) = \sum_{i=0}^{N-1} s_1^{N_i^0+1} s_2^{N_i^1} a_i + \frac{1}{2} s_1^{N_N^0+2} s_2^{N_N^1} \tag{3}$$

So, in Example 1, the estimate of $x_0 = v(n)$ can be calculated as :

$$\hat{x}_0 = \hat{v}(n) = 0.8 + 0.8^4 0.2 + 0.8^4 0.2^2 + 0.8^{11} 0.2^3 + \frac{1}{2} 0.8^{17} 0.2^4$$
$$= 0.89900920916587$$

## 3. Computer Simulation and Analysis

In order to further investigate the performance of measurement using formula (2) to estimate the initial value (the time variables $V(n)$ to be measured), we have carried out some numerical simulations, with results described below.

**Simulation 1**: Parameters $s_1$=0.28 and $s_2$=0.72 are fixed, while the number of iterations $N$ is varie from 1 to 20 for different variables $x_0$. The estimated values $\hat{x}_0$ are shown in Fig .2, where. $x_0$=[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0].

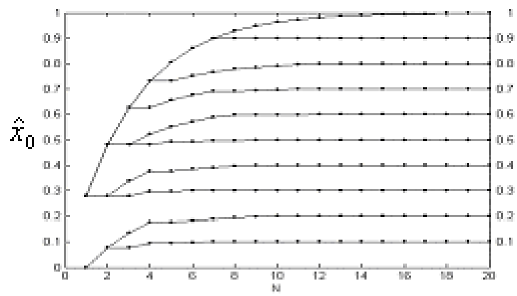

Fig. 2 Relationship between $N$ and $\hat{x}_0$

**Simulation 2**: The number of iterations, $N$=20, is fixed, while the parameter $s_1$ is varied for different initial values

$x_0$, with the results as shown in Fig.3. where $x_0$=[0.03, 0.15, 0.3, 0.45, 0.6, 0.75, 0.9].

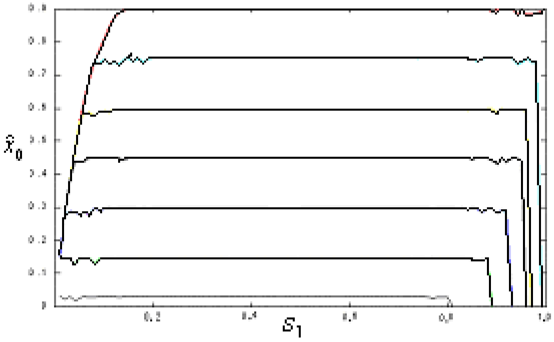


Fig. 3 Relationship between $s_1$ and $\hat{x}_0$

**Simulation 3**: The number of iteration, $N$=20, is fixed, for different parameter $s_1$, simulations of the relationship between the initial values $x_0$ and its relative error $E_r\%$ are shown in Fig. 4. Where $E_r = |(x_0 - \hat{x}_0)| / x_0$, the real line corresponds to $S_1$=0.67, the dished corresponds to $S_1$=0.16.

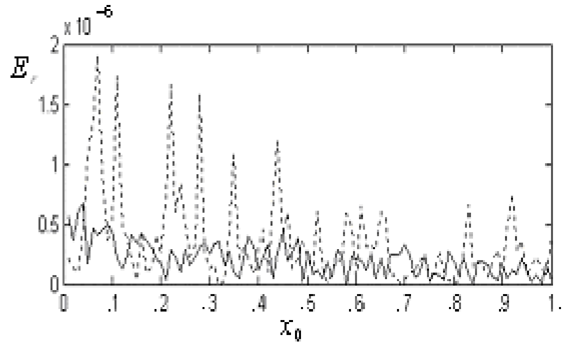


Fig. 4 Relationship between $x_0$ and its relative error

From the above simulations, we can conclude:

1. When the chaos-based measurement system parameters $s_1$ and $s_2$ are fixed, for any initial value $x_0$ or measured variable $v(n)$, its estimation $\hat{x}_0$ is more accurate with the increase of the iteration numbers $N$ (see Fig. 2).
2. When the value $x_0$ is small, one must also select small system parameters $s_1$ in order to assure the measurement precision (refer to absolute error), and vice versa (see Fig. 3). But relative error is not same as the absolute error (see Fig.4). It can be proved by losing information and simulated that at $s_1$=0.5, the relative error of the whole measurement system is the most small[9].

## 4. Circuit and Experimentation Results

In this section, an electronic circuit is designed to realize the chaotic system (1). Here, we select parameters as $s_1 = 0.3, s_2 = 0.7$ and measured variable(that is $x_0$) is denoted by voltage $v_{in}$. The block-diagram of the designed circuit for system (1) is presented in Fig. 5, and the control signals which implement the iterations of system (1) are shown in Fig. 6. The output (point Q) of D trigger is the symbolic sequence $\{a_0, a_1, ...., a_{N-1}\}$ of the initial value $v_{in}$.

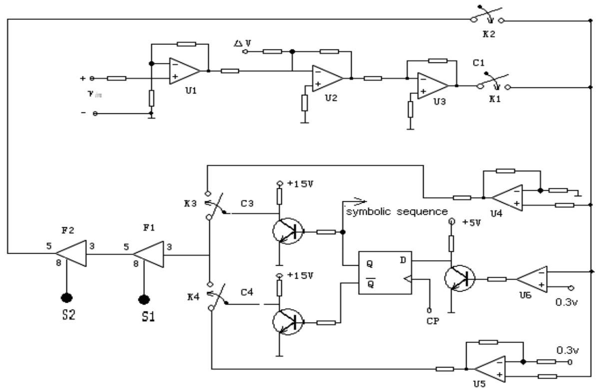


Fig. 5 Block diagram of the circuit

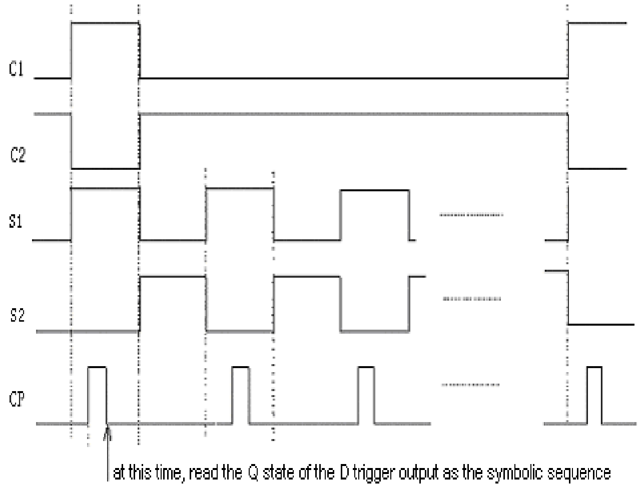


Fig. 6 The control signals

In Fig. 5, the circuit consists of operational amplifier ICL7650($U_1$~$U_6$), analog switching CD4066($k_1$~$k_4$), sampler and holder LF398($F_1$~$F_2$), D trigger, voltage reference MC1403(provides 0.3V voltage ), etc. Where, $U_2$ acts as an adder , add $\Delta v$ to $v_{in}$ when signal $v_{in}$ is very small to improve the measuring results; $U_4$ and $U_5$ implement operation of $x_{n+1} = x_n / s_1$ and $x_{n+1} = (x_n - s_1)/s_2$ respectively; $U_6$ acts as a comparator, implements the signal coding and selects this time iterating value. $F_1$ and $F_2$ hold the iterating value($x_n$) and $F_2$ also has the role of delay. In Fig. 6, cp is the clock , its periodic is the time of iteration ;and the impulse number of cp in the interval of $C_1$ periodic is the number $N$ of iteration. By computing the value of the sequence output from D trigger, one can gains the result of the measured voltage $v_{in}$ . The experimental results are shown as table 1.

Table 1. The experimental results (N=20, unit: *m*v)

| Real voltage values | Measured values | Absolute error |
|---|---|---|
| 0.05 | 0.00 | 0.05 |
| 5.01 | 3.21 | 1.8 |
| 10.12 | 8.64 | 1.48 |
| 15.03 | 14.09 | 0.94 |
| 20.05 | 19.56 | 0.49 |
| 25.15 | 24.98 | 0.17 |
| 30.34 | 30.01 | 0.33 |
| 40.08 | 39.89 | 0.18 |
| 70.23 | 69.93 | 0.30 |
| 100.00 | 100.01 | 0.01 |
| 200.45 | 200.23 | 0.22 |
| 245.09 | 242.98 | 0.11 |
| 260.21 | 260.00 | 0.21 |
| 300.01 | 299.45 | 0.46 |
| 400.05 | 400.01 | 0.04 |
| 500.68 | 499.96 | 0.72 |
| 600.23 | 599.89 | 0.34 |
| 700.12 | 699.69 | 0.43 |
| 800.09 | 800.11 | 0.03 |
| 900.01 | 899.91 | 0.1 |
| 940.23 | 938.09 | 2.14 |
| 960.12 | 959.23 | 0.89 |
| 970.01 | 968.47 | 2.43 |
| 980.23 | 979.12 | 1.11 |
| 998.91 | 997.10 | 1.91 |

## 5. Conclusions

An application of chaotic systems in the field of measurement is reported in this paper. The process using a chaotic system to realize measurement is discussed in detail. Although the precision of the chaos-based measurement system is not much higher than the other linear measurement systems, our research indicates that nonlinear and instable systems such as chaotic systems can indeed be used in the field of measurement and their potential utilization in other applications are quite attractive.

**Acknowledgments**

The author wish thank chair professor Guanrong Chen in city university of Hongkong for his revision to this paper and helpful suggestions.

## References

[1] G. Parodi,S.Ridella, and R. Zunious, "Using chaos to generate keys for associative noise-like coding memories ," *Neural networks*, Vol. 6,pp.559-572,1993.

[2] S. Papadimitriou, A. Bezerianos, and T. Bountis, "Secure communication with chaotic system of difference equation ," *IEEE Trans. Comput*.,Vol.46,pp. 27-38,Jan. 1997.

[3] E. Chernukha, "Utilization of chaotic system properties to improve measurement quality, " *IEEE Instrumentation and Measurement Technology Conference, Belgium*, June 4-6,pp.1408-1411,1996.

[4 ] Humin Yu, "Chaotic coding based on symbolic sequence and its applications in A/D " *Circuits and Systems, IEEE APCCAS 2000. The 2000 IEEE Asia-Pacific Conference on* ,2000, pp. 85-88

[5] S.kay and V.Nagesha, "Methods for chaotic signal estimation," *IEEE Trans. Signal Processing*, vol.43, pp.2013-2016, Aug.1995.

[6] H.papadopoulous and G.Wornell, "Maximum likelihood estimation of a class of chaotic signals," *IEEE Trans. Inform. Theory*, vol.41, pp.312-317, Feb.1995.

[7] Sichum Wang, Patrick C. Yip, and Herry Leung, " Estimation initial condition of noisy chaotic Signals Generated by piece-wise linear Markov maps using itineraries ," *IEEE Transactions on signal processing,* Vol. 47, pp.3289_3302, 1999

[8] Wenbo Liu, "The chaotic sensors ", Science and Technology report of aeronautics and astronautics, 2000

[9] Wenbo Liu, "The basic research of chaotic application in engineering," A dissertation for the application of doctorate, Nanjing university of aeronautics and astronautics, November,2001