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Basins of Attraction of Phase Patterns in Star-Coupled Wien-Bridge Oscillators Stimulated by Pulse

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1. Introduction

There have been many investigations of mutual synchronization and multimode oscillation in coupled oscillators [1]–[4]. Endo et al. have reported the multimode oscillations in various types of the coupled oscillators networks[1]. The synchronization phenomena in resistively coupled oscillators have been reported[2]–[4]. In particular, we have reported synchronization phenomena observed from N oscillators with the same natural frequency mutually coupled by one resistor [3, 4]. In these systems, various synchronization phenomena can be observed. Especially, in LC oscillators systems, we have confirmed experimentally that N-phase oscillation can be stably excited when each oscillator has strong nonlinearity [3]. In this case, there exist $(N-1)!$ stable phase states according to the initial states. Moreover, we have investigated the coupled system with RC Wien-bridge oscillators. This system is suitable for VLSI implementation because the system does not include any inductors. They also exhibits the "phase-shift synchronization" and we can get 3^{N-1} different stable phase patterns[4]. Because these "star-coupled" oscillators exhibit a large number of different steady states, they will be used as a structural element of large scale memories and neural networks.

On the other hand, recently, many researches on the networks of the nonlinear elements have been reported [5, 6], because they are important not only as a model for nonlinear systems but also from the viewpoint of biological information processing and their possible engineering applications. Chua et al. have proposed the network of the sparsely interconnected nonlinear elements called cellular neural network (CNN)[5]. Kaneko has suggested the possibility of the information processing using the network of the nonlinear elements by investigating the globally coupled chaotic elements from various aspects[6].

When we use the coupled oscillators systems as neural networks and large scale memories, it should be an important problem how to control the systems to get the appropriate phase patterns. To achieve the phase pattern control, as a first step, we have proposed the star-coupled system of Wien-bridge oscillators driven by the periodic pulse train and confirmed that the stimulation of the pulse train can cause the phase pattern switching [9].

If we use such pulse stimulation to control the coupled oscillators systems, we have to know how the phase patterns vary in detail after the pulses are added to the system. In this study, we calculate the basins of attraction of the phase patterns in the star-coupled Wien-bridge oscillators system stimulated by pulse using SPICE. We pay attention to three parameters, width, magnitude and timing of pulse, and calculate the basins of attraction of the phase patterns when these parameters are varied. To derive such basin structures can be the preparation for controlling the phase patterns in coupled oscillators systems by using these parameters as the control parameters.

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2. Circuit Models

The proposed circuit model is shown in Fig. 1. In this system, four identical Wien-bridge oscillators are coupled by one linear coupling resistor r. The DC voltage source V_C are connected to *Osc* 1 through the switch controlled by the signal $sw(t)$ as shown in Fig. 1 (c). $sw(t)$ is the periodic signal with period T and the switch closes Δt sec in every T sec. Therefore this unit can generate the periodic pulse train to stimulate *Osc* 1. Assume that $\Delta t < \tau < T$, where τ is the period of the oscillators in steady states. In particular, T should be sufficiently larger than τ because the synchronization has to be achieved within T.

Without the pulse train stimulation, the system exhibits the synchronization phenomena with 0° or $\pm 120^\circ$ phase shift as shown in [4].

3. Phase Pattern Switching

In this section, we show the example of simulation results of the proposed system using the circuit simulator package SPICE. In the following simulations, we fix $T = 30$msec and τ is measured as approximately 1.08msec. We have carried out the simulations with $\Delta t = 10\mu$sec. We take $V_C = 15$V, which is the same voltage as the positive source voltage of the op amps.

We show the results when $\Delta t = 10\mu$sec in Fig. 2. Figure (a) shows the time waveforms of the oscillators, and (b) shows the lissajours' figure of v_1 and v_2 in whole ranges. Figures (c)–(e) show the magnified lissajours' figures. During $t = 0.5$–0.6sec, three pulses are added to the system. The transition diagram of the phase pattern switching when the pulses are added is shown in Fig. 3. In this figure, the notations A, B and C mean the phase states (see Ref. [4]) when the reference oscillator whose phase state is A is $Osc\ 4$.

From these results, notice the following remarks:

- In Fig. 2 (a), we can see that the stimulation occurs at $t = 0.515$sec and the phase difference between v_1 and v_2 is changed after the stimulation.
- From lissajours' figures Fig. 2 (b), (c) and (d), three kinds of the closed curves are seen in one figure. Therefore, the phase difference between v_1 and v_2 (or v_3) switches among $-120°$, $+120°$ and $0°$ by the stimulations by the pulse train.
- There is some cases in which the phase patterns are not changed after the stimulation as shown in Fig. 3. Before and after the second stimulation, the phase pattern is not switched.

4. Basins of Attraction of Phase Patterns

Next, we show the 2 parameter phase pattern diagram when $\Delta t = 30\mu$sec in Table 1. In this table, we show the the stable phase state of $Osc\ 1$ depending on both the phase when the pulse is added and the voltage of the source V_C. It is important that we know which phase states are seen according to both the timing of adding pulse and magnitude of pulse because we are going to control the network. More detailed analysis will be shown in near future.

From these results, the pulse train stimulation can switch the phase pattern of the system. Because the phase states of one oscillator are three kinds (i.e. ternary coded), the system is expected to be used as a structural element of multi-valued logic system. Moreover it is important that the phase of only the oscillator directly connected to the switch can be changed but that ones of the others are not affected. It means that we can choose the oscillator arbitrarily to switch the phase states. Therefore, if we want to switch the phase patterns of the multiple oscillators, the switch units are connected to each oscillator which we want to switch the phase states. Although we do not know which phase pattern will appear after the stimulation at present, the results will be quite useful for designing

Table 1: 2 parameter phase states diagram.

	Phase									
V_C	0	$\frac{1}{5}\pi$	$\frac{2}{5}\pi$	$\frac{3}{5}\pi$	$\frac{4}{5}\pi$	π	$\frac{6}{5}\pi$	$\frac{7}{5}\pi$	$\frac{8}{5}\pi$	$\frac{9}{5}\pi$
15	B	B	B	B	B	B	A	C	C	C
14	B	B	B	B	B	B	A	C	C	C
13	B	B	B	B	B	B	A	C	C	C
12	B	C	B	B	B	B	A	C	C	C
11	B	C	B	B	B	B	A	C	C	C
10	C	C	C	B	B	B	A	A	C	C
9	C	C	C	B	B	B	A	A	C	C
8	C	C	C	B	B	B	A	A	C	C
7	C	C	C	C	B	B	A	A	A	C
6	C	C	C	C	B	B	A	A	C	C
5	C	C	C	C	C	B	A	A	C	C
4	C	C	C	C	C	B	A	A	C	C
3	C	C	C	C	C	B	A	A	C	C
2	C	C	C	C	C	C	A	A	C	C
1	C	C	A	C	C	C	A	A	C	C
0	C	A	A	C	C	C	A	A	C	C

the control system of the coupled oscillators systems in the future. When such control systems are achieved, the coupled oscillators systems will be quite suitable for a structural element of CNNs, large scale memories and multi-valued logic systems.

5. Conclusions

In this paper, we have shown the basins of attraction of the phase patterns in the star-coupled systems of Wien-bridge oscillators driven by the periodic pulse train. The pulses stimulate the system and the phase patterns are switched after stimulations.These results will be quite useful for designing the control system of the coupled oscillators systems in the future. When such control systems are achieved, the coupled oscillators systems will be quite suitable for a structural element of CNNs, large scale memories and multi-valued logic systems.

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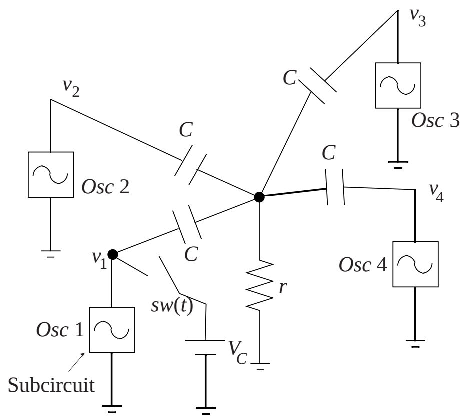
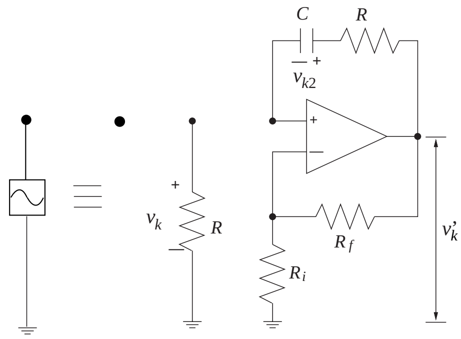


(a)



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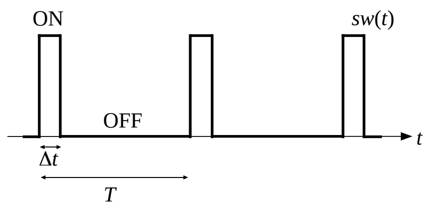

(c)

Figure 1: (a) Circuit model. (b) Constriction of subcircuits. (c) Control signal of the switch.

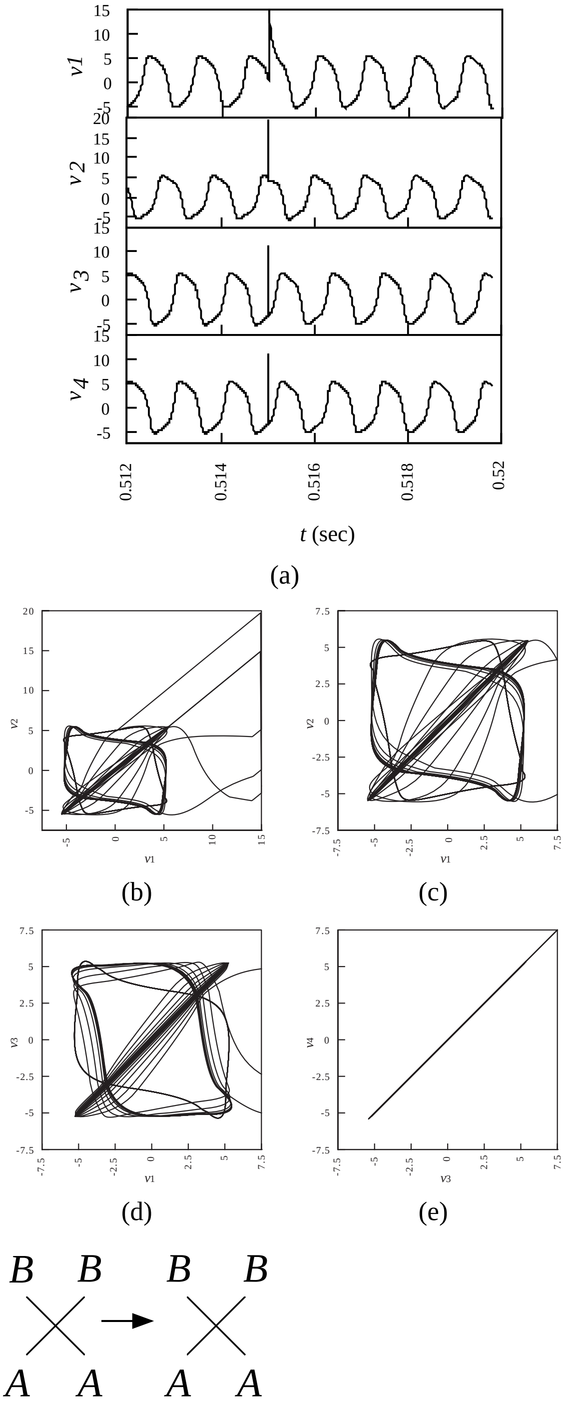

Figure 2: (a) Time waveforms during $t = 0.512$–0.52sec, and lissajours' figures during $t = 0.5$–0.6sec (b) when $\Delta t = 10\mu$sec. (c) v_1 vs. v_2, (d) v_1 vs. v_3, (e) v_3 vs. v_4. (f) Phase pattern transition during $t = 0.512$–0.52sec. ($R = 10$kΩ, $C = 0.015\mu$F, $R_f = 14.7$kΩ, $R_i = 4.7$kΩ)

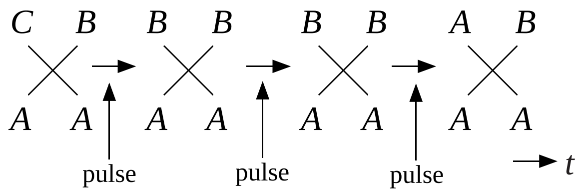

Figure 3: Phase pattern transition after pulse stimulations during $t = 0.5$–0.6sec when $\Delta t = 10\mu$sec.