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Spiral Waves in Non-Regular Two Dimensional Arrays of Elements

Hisa-Aki Tanaka and Kousuke Moriya

University of Electro-Communications,
1-5-1, Chofugaoka, Chofu-Shi, Tokyo, 182-8585, Japan.
Phone:+81-424-43-5174, Fax:+81-424-43-5174
Email: htan@ee.uec.ac.jp, mori@synchro2.ee.uec.ac.jp

1. Introduction

Networks of interconnected oscillators emerge in a wide range of engineering issues. Examples are known in millimeter-wave power-combining and beam-scanning control systems [1,2,3], a novel VLSI clocking [4], and Josephson junction arrays, where implementation of two-dimensional (2D) synchronous arrays has been a technical challenge because a number of oscillators are packed in a limited space and are required to oscillate in unison. As opposed to one-dimensional (1D) linear arrays of oscillators, planar arrays of oscillators can sometimes exhibit 2D phase-lagged stable synchronous patterns called modelock or travelling (spiral) waves [5]. This modelock has been a notorious hazard because it hampers the desired in-phase synchronization of the oscillator arrays.

In a 2D array of voltage-controlled oscillators (VCOs) for VLSI clocking [4], modelock is avoided by adding a special phase detector (PD) whose response decreases monotonically beyond a phase difference of $\pi/2$ to each VCO. However, in solid-state circuits such as MESFET oscillators for millimeter-wave generation, the interaction between oscillators is due to the 'injection-locking' mechanism and the resulting synchronizing characteristics (corresponding to the PD response) come from the intrinsic nonlinearity of the oscillator. Thus, in such cases avoiding modelock by tuning the synchronizing characteristics may not be straightforward (if not impossible), as opposed to the case of the VLSI clocking circuit.

This study proposes an alternative synchronizing method that avoids modelock by introducing dynamic coupling with only on-off switches to the array (see Fig.1). The basic idea comes from the observation that nonregular (percolation-like) 2D networks of oscillators attain the in-phase synchronization state by destruction of the 'core' of the spiral wave pattern (center of the modelock) [6]. Systematic numerical simulations are carried out to consider the effectiveness of the method for possible applications to millimeter-wave power-combining and beam-scanning control systems. The limitations and robustness of the method are also investigated.

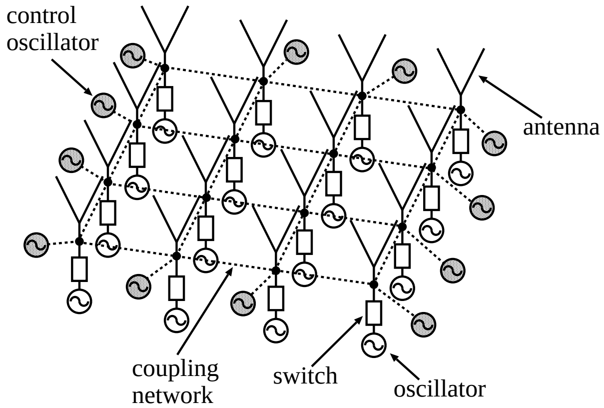

Fig.1 2D square array oscillators with switched couplings

After we have presented the above results [7], we found some connection between our result and 'spiral breakup' on a non-regular array of excitable medium (; models of cardiac tissue) [8]. Investigation on this connection is also presented briefly.

2. Phase dynamics in 2D oscillator arrays

We assume here a weak coupling between adjacent oscillators, and sufficiently uniform oscillator characteristics. Under such conditions, a systematic derivation of the phase equation for oscillators can be made (e.g., see [1]), which eventually takes the following form:

$$\dot{\theta}_i = \omega_i + \Delta\omega_m \sum_j \sin(\Phi + \theta_j - \theta_i), \qquad (1)$$

where θ_i and ω_i represent the oscillation phase and

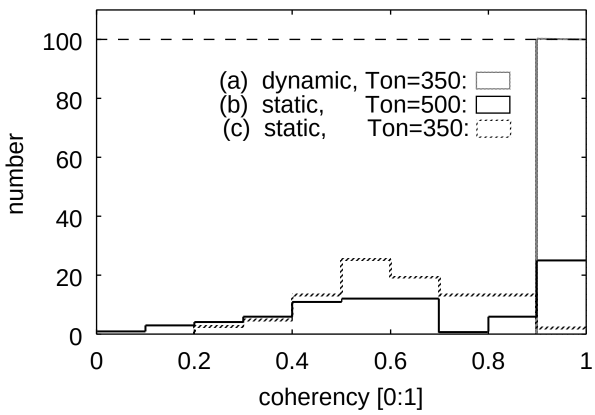

Fig.2 Coherency distribution of σ after 12000-cycle oscillations
(a) dynamic network, (b,c) static network

free running frequency of the i^{th} oscillator, respectively. $\Delta\omega_m$ is interpreted as the locking range of each oscillator which is assumed to be small and the same for all oscillators. The phase lag Φ reflects signal delay, which cannot be neglected for the case of radiative coupling. However, if the coupling is made by one-wavelength waveguides, Φ is assumed to be 0 and we focus on this case. The above discussion is valid for any network topology. For the 2D oscillator array with switched coupling as shown in Fig.1, Eq. (1) can be modified to the following phase equation

$$\dot{\theta}_i = \omega_i + \Delta\omega_m \sum_j S_i S_j \sin(\Phi + \theta_j - \theta_i), \qquad (2)$$

where the additional parameter S_i takes 1/0 depending on the on/off states of the switch between the i^{th} oscillator and the coupling network. The summation $\sum_j$ is taken for adjacent (j^{th}) oscillators to the i^{th} oscillator. Here, we assume each switch independently takes the on/off states alternatively for a time span T_{on}/T_{off} respectively. It is clear that if all switches are on, Eq. (2) reduces to Eq. (1). On the other hand, if all switches are off, Eq. (2) becomes $\dot{\theta}_i = \omega_i$, which implies that all oscillators are free-running. Between these two extremes, we have intermediate states where any two oscillators with $S_i = 1$ are connected by the network and certain amount of oscillators with $S_i = 0$ are disconnected from the network, forming a percolation-like, global network of interacting oscillators.

We performed numerical simulations of Eq. (2) for the following cases;
(a) dynamically switched networks with $(T_{on}, T_{off}) = (350, 150)$,
(b) static networks with $(T_{on}, T_{off}) = (500, 0)$, i.e., all switches are on, and
(c) static networks with $(T_{on}, T_{off}) = (350, 150)$, in which network topology is fixed at random.

For each case, 100 trials are made for 30×30-oscillator array networks with random initial oscillation phases and switching states. The natural frequencies ω_i are chosen uniformly from $[1.0, 1.05]$ at random. To measure the degree of phase synchronization, the phase coherency σ is introduced as follows,

$$\sigma = \sum_{j=1}^{N} S_j \exp(i\theta_j)|/\sum_{j=1}^{N} S_j. \qquad (3)$$

It is noted that $\sigma = 1$ corresponds to the in-phase synchronized state and the lower coherency σ indicates loss of in-phase synchrony. Figure 2 shows the result of this simulation (after 12000-cycle oscillations). A clear difference is observed between the case (a) with $(T_{on}, T_{off}) = (350, 150)$ and the case (b) with $(T_{on}, T_{off}) = (500, 0)$. The latter has a wide-spread coherency distribution, which is due to the generation of modelock (sometimes several spiral waves coexist in the array.). On the other hand, the former has a sharp distribution below $\sigma = 1$, where no modelock remains and all (connected) oscillators exhibit in-phase synchronization.

Such avoidance of modelock in switched array networks can be explained as follows. Figure 3 shows typical examples for cases (a) without control oscillators, and (b) with control oscillators attached to the outside of the array. (For the case (b), driving freqencies ω_i of the control oscillators are gradually set to be low to high from the top to the bottom of the array shown in Fig. 3(b).) In both cases of Fig. 3(a) and (b), it is observed that (1) initially, (multiple) stable spiral waves exist (at this stage all switches are on.), (2) shortly after the switching is initiated, some cores of the spiral waves are destroyed or moved, (3) the spiral waves continue to be destroyed, (4) finally they are lost and a coherent pattern is stabilized, and (5) at this stage, we obtain completely coherent patterns by making all switches on again.

A comparison between the coherency distribution in case (b) and case (c) (Fig. 2) shows an interesting nature of the 'core' dynamics of spiral waves. It is observed that in case (b) some spiral waves can escape to the boundary of the array or collide each other and disappear because the array is regular. On the other hand, in case (c), static 'defects' (; off switches) in the array bother the cores of spiral waves to move freely and almost all spiral waves are trapped in the array. We consider this observation gives the reason why some instances in Fig. 2(b) lead to coherent, while almost all instances in Fig. 2(c) lead to incoherent.

Such trapping phenomena of spiral waves is also found recently in non-regular arrays of excitable media [8]. A connection between our results and the corresponding phenomena in [8] is under investigation.

In our simulations, each swich is assumed to be periodically switched just for simplicity. This periodicity is found to lead to a long-term survival of spiral waves in some rare instances. Such a situation is shown in Fig. 4. In Fig. 4, positions of cores of spiral waves are traced in space and time, and two spiral waves are observed to be meandering with period $T = 500 = T_{on} + T_{off}$i. However, if we introduce some stochasiticity into 'switches', such somewhat artificial periodic meandering is observed to be lost.

3. Discussion

In the above simulations of Eq. (2), the case of $\Phi = 0$ is considered. We also considered cases of $\Phi = 0.01\pi$ and 0.1π, modelling mismatches to one-wavelength interconnections. For the $\Phi = 0.01\pi$ case with 30×30-arrays, several percent of the trials did not converge to the completely coherent state (at least for 12000-cycle oscillations). 10×10 and 20×20-arrays are also simulated under the same conditions, and 100 percent convergence is obtained for $\Phi = 0.01\pi$. This implies that smaller arrays have better in-phase synchrony, and this is what the simulation in [3] has reported. In the case of $\Phi = 0.1\pi$, none of 10×10, 20×20, and 30×30-arrays converged to the in-phase state, suggesting a certain upper limit of Φ beyond which the presented method is no longer effective.

For more effective synchronous network topologies, we performed simulations of Eq. (2) on triangular lattice networks under the same simulation protocol as that mentioned above. As opposed to the above square lattice cases, triangular lattices show better in-phase synchrony. For example, 10×10, 20×20, and 30×30-oscillator arrays with a $\Phi = 0.1\pi$ phase delay have shown 100 percent in-phase synchrony.

4. Conclusions

Although a theoretical study has not been completed, systematic numerical simulations support the efficacy of the presented method for avoiding modelock. The required switches are expected to be realized using simple circuitry. Also, a connection between our results and phenomena in non-regular arrays of excitable media deserves further investigation.

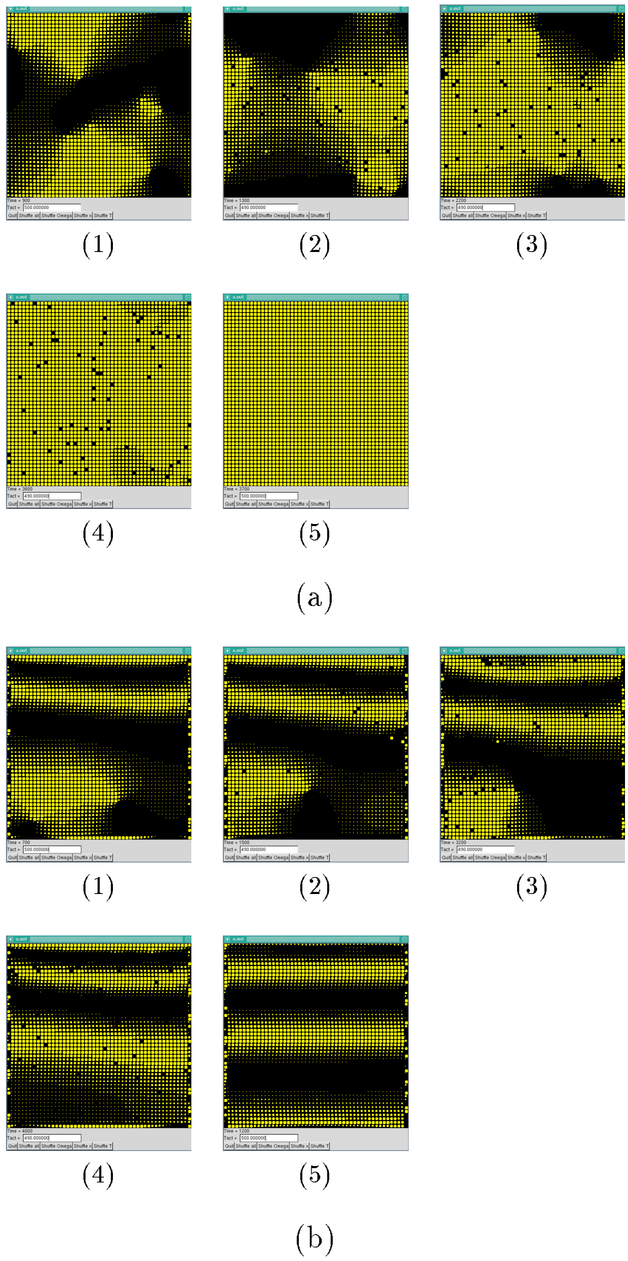

Fig.3 Synchronization process to coherent patterns in 2D array oscillators, (a) without control oscillators, (b) with control oscillators.

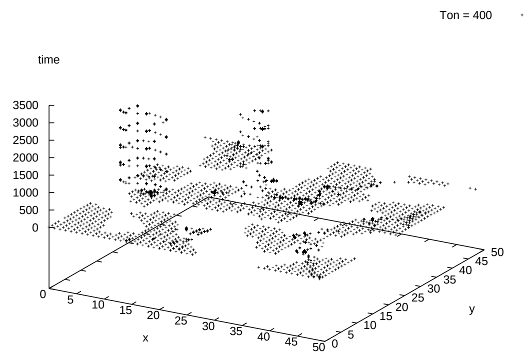

Fig.4 Trace of cores of spiral waves. 50×50-oscillator arrays with $(T_{on}, T_{off}) = (400, 100)$ is considered.

Acknowledgments

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