A Shape Analysis Method by Unifying Active Contour Model and Fourier Description

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Abstract: A difficult problem is shape presentation of object in computer vision. We can learn, match, restructure, utilize to interesting object by the shape presentation. A shape analysis method by unifying active contour model and Fourier description is proposed in the paper. We analyze shape of object using the method efficiently and extract shape character of object. It is propitious further to recognize, understand object. Active contour model (snake) was first proposed by M.Kass et al in 1987, have been used more and more widely in applications of image analysis and computer vision. Active contour search minimum value of energy function under action of internal energy and external energy. Minimum value of energy function is obtained, i.e. contour of object is obtained. Next we express shape character of object using Fourier description. We may distinguish shape-utilizing points on the contour after contour of object has obtained. Because active contour overcome shortcoming of traditional edge detection or segmentation, so next we obtain shape character of object using Fourier description efficiently. At last, we prove reliability and practicability of the method by some experiments.
Keywords: Shape Analysis, Active Contour Models, Fourier Description

1. Introduction

A difficult problem is shape presentation of object in computer vision. We can learn, match, restructure, utilize to interesting object by the shape presentation. Commonly we need extract contour of object for shape presentation. Contour extraction is one of the fundamental tasks in computer vision and image processing. Many methods have been proposed for various types of images. Even though the edges of objects in images can be easily shown by Canny or Laplacian methods, correct contour detection requires high-level knowledge and this calls for the active contour method. How to convert high-level knowledge into exerting energy (or force) in the contour detection has become one of the most important tasks within the active contour model research in recent years. The active contour model, also termed "snake" because of the nature of its evolution, is a sophisticated approach to contour extraction and image interpretation. The determination of the presence of an object contour depends not only on the local image force at a specific point, but also on the properties of a contour's shape.

A shape analysis method by unifying active contour model and Fourier description is proposed in the paper. We analyze shape of object using the method efficiently and extract shape character of object. It is propitious further to recognize, understand object.

2. Active Contour Models

Active contour model (snake) was first proposed by M.Kass[1] et al in 1987, have been used more and more widely in applications of image analysis and computer vision. It is a close or unclose elasticity curve and is expressed by set of some controlled-points. When model search character contour, it move gradually from initial position to character position by distortion and movement of elasticity curve. In this approach, an energy-minimizing contour is controlled by a combination of the following two components: one controls the smoothness of the contour; the other attracts the contour toward the object boundary. Although the implementation of this approach is sensitive to its initial position and is vulnerable to image noise, it provides a powerful interactive tool for image segmentation and has been investigated extensively among the model-based techniques.

2.1. Energy Formulation

Let Ω be a bounded open subset of R^2 , with $\partial\Omega$ its boundary. Let $u_0:\bar{\Omega}\to R$ be a given image, and $C(s):[0,1]\to R^2$ be a parameterized curve.

Active contour model is defined as follow:

$$\inf_C J(C)=\int_0^1[J_{\text{int}}(C(s))+J_{image}(C(s))+J_{ext}(C(s))ds$$

Where J_{int} is internal energy of active contour, J_{image} is gives rise to the image forces, and J_{ext} is external energy of active contour. Energy minimizing model of active contour is composed of internal energy, external energy and image force. Its action is controlled by internal energy and external energy. Internal energy controls the smoothness of the contour. External energy attracts the contour toward the object in the image. Active contour search minimum value of energy function under action of internal energy and external energy. Minimum value of energy function is obtained, i.e. contour of object is obtained.

The internal energy can be written

$$J_{\text{int}} = (\alpha(s) \mid C_s(s) \mid^2 + \beta(s) \mid C_{ss}(s) \mid^2)/2$$

The spline energy is composed of a first-order term controlled by $\alpha(s)$ and a second-order term controlled by $\beta(s)$. The first-order term makes the snake act like a membrane and the second-order term makes it act like a thin plate. Adjusting the weights $\alpha(s)$ and $\beta(s)$ controls the relative importance of the membrane and thin-plate terms. Setting $\beta(s)$ to zero at a point allows the snake to become second-order discontinuous and develop a corner. The controlled continuity spline is a generalization of a Tikonov stabilizer and can formally be regarded as regularizing the problem.

In order to make snakes useful for early vision we need energy functionals that attract them to salient features in images. Kass presented three different energy functionals that attract a snake to lines, edges, and terminations. The total image energy can be expressed as a weighted combination of the three energy functionals

$$J_{image} = w_{line} J_{line} + w_{edge} J_{edge} + w_{term} J_{term}$$

By adjusting the weights, a wide range of snake behavior can be created.

The simplest useful image functional is the image intensity itself. If we set

$$J_{line} = I(x, y)$$

then depending on the sign of w_{line}, the snake will be attracted either to light lines or dark lines.

We may set

$$J_{edge} = -\mid \nabla I(x, y) \mid^2$$

then the snake is attracted to contours with large image gradients.

2.2. Numerical Solution

We discretize the equation by finite differences.

If $E(v) = (E_1(v), E_2(v))$ is the sum of image and external forces, we have Euler equation:

$$\begin{cases} -(\alpha(s)C_s(s))' + (\beta(s)C_{ss}(s))'' = E(v) \\ C(s) \in c'[0,1], and\ C(0) = C_0, C'(0) = C'_0, \\ C(1) = C_1, C'(1) = C'_1 \end{cases}$$

becomes after finite differences in space (step h):

$$\frac{1}{h}(a_i(C_i - C_{i-1}) - a_{i+1}(C_{i+1} - C_i)) + \frac{b_{i-1}}{h^2}(C_{i-2} - 2C_{i-1} + C_i)$$
$$-2\frac{b_i}{h^2}(C_{i-1} - 2C_i + C_{i+1}) + \frac{b_{i+1}}{h^2}(C_{i+2} - 2C_{i+1} + C_i)$$
$$-(E_1(v), E_2(v)) = 0$$

where we defined

$$C_i = C(ih);\ a_i = \frac{\alpha(ih)}{h};\ b_i = \frac{\beta(ih)}{h^2}$$

This can be written in the matrix form:

AC=E

where A is pentadiagonal and V and F denote the vectors of positions C_i and forces at these points $E(C_i)$.

We solve the equation, and then minimum of J is obtained.

2.3. Contour Extraction Result

We choose one image to validate above algorithm. Fig.1 is a handwritten numeral seven. First we draw an initial curve, see Fig.2. Then active contour will converge, see Fig.3. Result of contour extraction sees Fig.4.

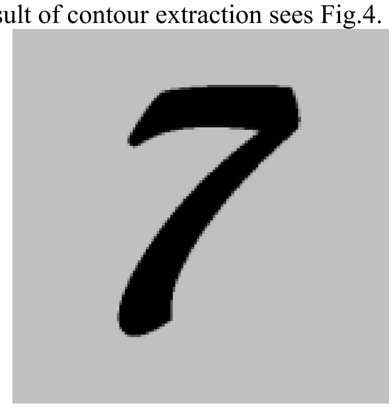

Fig. 1 Handwritten numeral seven

Fig. 2 Initial curve of active contour models

Fig. 3. Active contour is converging

Fig. 4. Result of contour extraction

Next we express shape character of object using Fourier description.

3. Fourier description

We may distinguish shape-utilizing points on the contour after contour of object has obtained. If M points may utilize on the contour, contour can be look upon as in complex plane. We track circuit along contour from any point on the contour, so we may obtain complex sequence. Fourier transform of the complex sequence is called Fourier description[3][4].

3.1. Algorithm Introduction

Set γ is simply close clockwise curve. Its parametric form is

$$(x(l), y(l)) = z(l)$$

where l is arc-length of curve, $0 \le l \le L$, L is full length of γ curve.

Set $\theta(l)$ is tangent direction of point that arc-length is l on the curve. $\delta_0 = \theta(0)$ is tangent direction of initial point $z(0)$. $\phi(l)$ present angel cumulation function. It is circumrotatory angel from tangent direction of initial point to tangent direction of point that arc-length is l along curve. See Fig.5.

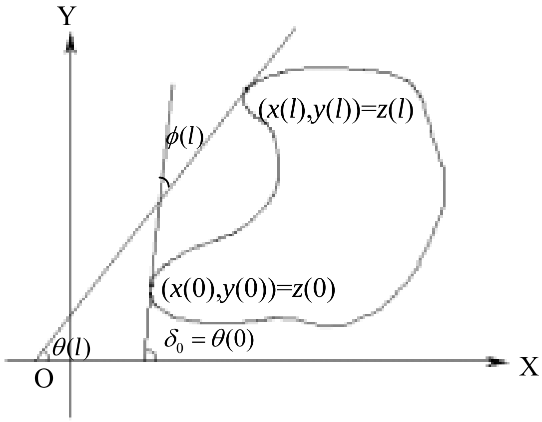

Fig.5. Several parameters of Fourier Description

Change rule of $\phi(l)$ may describe shape of close curve.

Next we change $\phi(l)$ to periodic function in $[0, 2\pi]$, and then make Fourier outspread. We describe it using Fourier coefficient.

Induct new variable t, set

$$l = \frac{Lt}{2\pi}$$

where $l \in [0, L]$, $t \in [0, 2\pi]$. Define

$$\phi^*(t) = \phi(\frac{Lt}{2\pi}) + t, \ t \in [0, 2\pi] \qquad (1)$$

$\phi^*(t)$ is periodic function in $[0, 2\pi]$, and

$$\phi^*(0) = \phi^*(2\pi) = 0$$

$\phi^*(t)$ is invariable under translate and rotate of curve γ. Presentation form of curve γ is discrete line.

We outspread $\phi^*(t)$ to Fourier series:

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty} (a_k \cos kt + b_k \sin kt) \qquad (2)$$

where

$$\mu_0 = \frac{1}{2\pi} \int_0^{2\pi} \phi^*(t) dt$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \phi^*(t) \cos ntdt$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \phi^*(t) \sin ntdt$$

$$n = 1, 2, \ldots$$

According to (1) and (2), set $\lambda = Lt / 2\pi$, we have

$$\mu_0 = \frac{1}{L} \int_0^{L} \phi(\lambda) d\lambda + \pi$$

$$a_n = \frac{2}{L} \int_0^{L} \left[\phi(\lambda) + \frac{2\pi\lambda}{L} \right] \cos \frac{2\pi n \lambda}{L} d\lambda$$

$$b_n = \frac{2}{L}\int_0^L \left[\phi(\lambda) + \frac{2\pi\lambda}{L}\right]\sin\frac{2\pi n\lambda}{L}d\lambda$$
$$n = 1, 2, ...$$

We know $\phi(l)$ is step function. So have

$$\mu_0 = -\pi - \frac{1}{L}\sum_{k=1}^{m} l_k \Delta\phi_k$$
$$a_n = -\frac{1}{n\pi}\sum_{k=1}^{m}\Delta\phi_k \sin\frac{2\pi n l_k}{L}$$
$$b_n = -\frac{1}{n\pi}\sum_{k=1}^{m}\Delta\phi_k \operatorname{cosin}\frac{2\pi n l_k}{L}$$
$$n = 1, 2, ... \qquad (3)$$

where $l_k = \sum_{i=1}^{k}\Delta l_i$.

Swing-phase angle form of equation (2) is

$$\phi^*(t) = \mu_0 + \sum_{n=1}^{\infty} A_n \cos(nt - \alpha_n) \qquad (4)$$

where

$$A_n = (a_n^2 + b_n^2)^{1/2}$$
$$\alpha_n = tg^{-1}(b_n / a_n)$$

(A_n, α_n) is polar coordinate of (a_n, b_n). A_n is called regularity swing, α_n is called *n*th regularity phase angle.

$\{\mu_0, a_n, b_n\}$ in equation (3) and $\{A_n, \alpha_n\}$ in (4) is shape feature that area edge is extracted by Fourier description.

3.2. Shape Analysis result

We compute μ_0, A_n, α_n for above example. We choose n from 1 to 10.

Result:

$\mu_0 = -2.9013$

i	A_i	α_i
1	0.529	1.524
2	0.187	1.983
3	1.045	4.327
4	0.695	2.361
5	0.621	5.214
6	0.207	3.159
7	0.912	0.156
8	0.015	3.018
9	0.725	4.215
10	0.406	2.384

We may reconstruct contour of object using above data. See Fig.6.

Fig.6. Reconstruction result

Because of $n \to \infty$ in (4), but we compute only n from 1 to 10, so reconstruction result is not close.

4.conclusions

In this paper, we proposed a shape analysis method unifying active contour model and Fourier description. By our method, we can extract shape feature of object and reconstruct object. Experimental results indicate that the method can be implemented easily and is efficient.

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