Analyses of Dynamical Noise in Chaos by Temporal Fluctuation of Singular Values

Masaru Todoriki, Hiroyuki Nagayoshi and Atsuyuki Suzuki

The University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo, Japan
Phone:+81-3-5841-6997, Fax:+81-3-3818-3455
Email: todoriki@q.t.u-tokyo.ac.jp, naga@lyman.q.t.u-tokyo.ac.jp, suzuki@q.t.u-tokyo.ac.jp

1. Introduction

It is commonplace, that there exists no physical system that is not subject to noise in the real world. In general, there are two types of noise in any physical system, namely, measurement noise and dynamical noise. The latter type of noise is said to be realistically intrinsic to a physical system, different to the former type, and yielding an extremely complicated mechanism accompanied by feedback. Accordingly, it is quite difficult to analyze both on theoretical and experimental levels.

On the other hand, as chaos system particularly has a strong nonlinearity and sensitivity to the initial condition, dynamical noise may cause a remarkable and fatal influence on chaos systems. Recently, some studies relating to dynamical noise in chaos have appeared[1][2][3]. In this analysis, we have also investigated the influence of dynamical noise on chaos data numerically, and shown one possible methodology to qualitatively distinguish the influence of dynamical noise and that of measurement noise in a way different to that of other studies. We use the Singular Value Decomposition (SVD)-based method. The SVD method, which is frequently used as a principal component analysis tool, is, however, applied to underline temporal fluctuation of the chaos system caused by noise, by detecting the temporal fluctuation of singular values. The temporal fluctuation is estimated by two indices: the correlation coefficient (CC) and the normalized root mean square error (E), which are frequently used in estimating the prediction of the time series data. From our previous work[6], it was indicated that the influence of dynamical noise on chaos can be estimated and distinguished from that of measurement noise by using the indices.

Accordingly, in this study, we investigate the possibility of extracting the influence of dynamical noise in noise-mixed data including both dynamical noise and measurement noise corresponding to the real world, as the next stage. Finally, we apply our method to noisy experimental data in Chua's electronic circuit, and indicate the effects of our approach on real world data.

2. Noise

It is generally said that there are two types of noise in any physical system, they being dynamical noise and measurement noise. The following identifies each,

$$\mathbf{x_{n+1}} = \mathbf{f}(\mathbf{x_n}) + \xi^{(\mathbf{D})}_{\mathbf{n+1}}, \tag{1}$$

$$\mathbf{y_{n+1}} = \mathbf{g}(\mathbf{x_{n+1}}) + \xi^{(\mathbf{M})}_{\mathbf{n+1}}, \tag{2}$$

where $\mathbf{x_n}$ and $\mathbf{y_n}$ are original times series vector and observed time series vector respectively, $\mathbf{f}$ is a governing function of the chaos system, $\mathbf{g}$ is an observation function by which multi-dimensional data in the real world is usually transformed into 1-dimensional data, and $\xi^{(\mathbf{D})}$ and $\xi^{(\mathbf{M})}$ expresses independent identically distributed Gaussian dynamical and measurement noise with the mean 0, respectively.

3. The SVD Based-Method

Singular values can be obtained from scalar chaotic time series data, using the SVD method, which is often utilized in the presumption of the degree of unknown dimension of physical systems, including noise in the real world[4]. How noise influences singular values can be explained to some extent, by analyzing the behavior of elements of the variance-covariance matrix emergeing from the SVD method.

3.1. Influence of Measurement Noise

If measurement noise is added to a system, the influence of noise appears as the increase of singular values in the whole range of singular values as explained in Ref.[4]. The reason is that all off-diagonal elements of the variance-covariance matrix are realistically not subject to measurement noise. That is, even if measurement noise is added, those elements, each of which is composed of cross-correlations, becomes almost zero. On the other hand, the diagonal elements, each of which is composed of auto-correlations increase in proportion to the noise level. As a result, raised singular values are obtained.

3.2. Influence of Dynamical Noise

When dynamical noise is added to a system, singular values are also influenced by the noise. However, there seems to be a difference from the case of measurement noise, because dynamical noise is closely related to the original system and is thought to be inseparable from the system. This is different to measurement noise, which is independent of the system. As a result, it is expected that the influence of dynamical noise can be observed as a temporal fluctuation of the system, which cannot be expected in the case of measurement noise. The reason can be explained by the behavior of the elements of the variance-covariance matrix. All off-diagonal elements of the variance-covariance matrix are subject to dynamical noise, because those elements, each of which is composed of cross-correlation, in turn, have non-zero values, which change with time. Accordingly, it might be said that the influence of dynamical noise on chaos can be estimated by investigating the temporal fluctuation of singular values calculated from the SVD method. The possibility has been already investigated in our previous study[6]. As a result, we have a possibility to distinguish the influence of dynamical noise with that of measurement noise on chaos data by detecting the fluctuation of singular values using the SVD method. Consequently, in this study, we investigate the possibility of applying our method to more realistic situations.

3.3. Estimate of Temporal Fluctuation of Singular Values

We introduce the following two indices for the temporal fluctuation of singular values in order to analyze the results in detail: $Correlation\ Coefficient$ (CC) and $Normalized\ Root\ Mean\ Square\ Error$ (E) as follows:

$$CC = \frac{\sum_{i=1}^{N}(s(i)-\overline{s})(a(i)-\overline{a})}{\sqrt{\sum_{i=1}^{N}(s(i)-\overline{s})^2}\sqrt{\sum_{i=1}^{N}(a(i)-\overline{a})^2}}, \quad (3)$$

$$E = \frac{\sqrt{\sum_{i=1}^{N}(s(i)-a(i))^2}}{\sqrt{\sum_{i=1}^{N}(a(i)-\overline{a})^2}}, \quad (4)$$

where $s(i)$ is singular values at the $i-th\ index$ of singular value for each trial and $a(i)$ is an average of $s(i)$ over all trials at each $i-th\ index$ of singular value, respectively, and $\overline{s}$ and $\overline{a}$ is an average of $s(i)$ and an average of $a(i)$ over all $index\ i$ of singular value. In this study, the maximum number N of singular values is 50 and the maximum number of trials is 200. That is, 50 pieces of CC and E is obtained for each data and 200 sets of those pieces of CC and E are acquired. CC and E are estimators to investigate the difference between singular values $s(i)$ in each data and the averaged ones $a(i)$. CC changes in the range from 1 to 0 and indicates 1 in the case where there is no difference. As the amount of increase of the difference is larger, CC decreases to 0. On the other hand, E changes in the range from 0 to $+\infty$ and indicates 0 in the case where there is no difference. As the amount of difference increase is larger, E increases into $+\infty$. Namely, it is possible to estimate the influence of dynamical noise by investigating the alternation of CC and E corresponding to a temporal fluctuation of singular values. However, in fact, instead of CC and E, *Averaged* CC and *Averaged* E calculated by averaging CC and E over all trials are used for simplification.

4. Numerical Analysis

4.1. Noisy Time Series Data

Chua's electronic circuit, which is described by three-dimensional ordinary differential equations, is used[5].

$$C_1\frac{dV_{C_1}}{dt} = \frac{1}{R}(V_{C_2}-V_{C_1}) - f_{N_R}(V_{C_1}), \quad (5)$$

$$C_2\frac{dV_{C_2}}{dt} = \frac{1}{R}(V_{C_1}-V_{C_2}) + i_L, \quad (6)$$

$$L\frac{di_L}{dt} = -V_{C_2}, \quad (7)$$

where $f_{N_R}(V_{C_1})$ denotes the 3-segment odd-symmetric voltage-current characteristic of the nonlinear resistor N_R. This is because the system exhibits some varieties of typical chaotic behaviors such as double scroll chaos, and it is also easy to obtain realistic data through experiments. Here we use value of V_{C_1} as 1-dimensional time series data. The iid Gaussian noise is added to right sides of eq.(5), eq.(6), and eq.(7) with certain strength. We prepare 200 temporally sequential data sets, each of which has $25,000$ data points for the purpose of finding the temporal fluctuation of the singular values obtained from the SVD method, and calculate 50 numbers of singular values, for each data. Time delay in embedding is determined to be 15 time steps with use of mutual information and phase diagram. These conditions are determined through trial and error so that the statistically and relatively stable results can be obtained. The numbers of calculated singular values is a sufficiently larger number than 3, which is the dimension of Chua's circuit.

Here, three kinds of time series data, which indicate double-scroll chaos, are prepared, these being noise-free

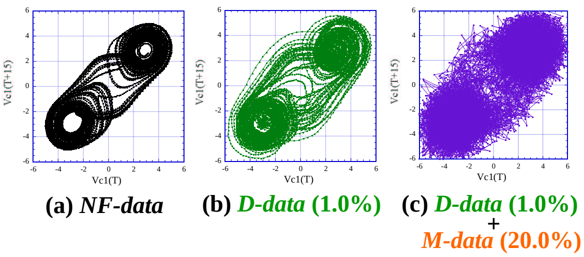

Figure 1: Phase Diagram for (a) *NF-data*, (b) *D-data*(1.0%), and (c) *D-data*(1.0%) + *M-data*(20.0%).

time series data (*NF-data*), dynamical noise induced data (*D-data*), and measurement noise induced data (*M-data*). In addition, we set some noise levels for *D-data* and *M-data*. Noise levels are 0.01, 0.05, 0.1, 0.5 and 1.0 % in the case of *D-data* and 20.0% in the case of *M-data* as a ratio of standard deviation of noise data to that of *NF-data*. We prepare 5 cases, where *M-data* (20.0%) is added to all *D-data* and compare results with our previous work. Here, *the 4th-order Runge-Kutta method* is used for the numerical method where time step and calculation points are 0.000005 and over 5, 000, 000, respectively. Phase diagrams are illustrated in fig.1

4.1.1. Results of the SVD Method

Fig.2 shows calculated singular values of some data on behalf of each type of data as a histogram. A similar trend can be seen among all types of data. The differences among them can be recognized especially at dominant singular values on the left-hand side. Singular values in (b) and (c) are relatively lower than the ones in (a). That is, it can be said that singular values in (b) and (c) are dispersed in comparison with the ones in (a) and are fluctuating with time. Accordingly, such fluctuation is investigated next.

4.1.2. Averaged CC and Averaged E

Figure 3 indicates the trends of both *Averaged CC* and *Averaged E* are quite similar to the ones in the previous results, independently of measurement noise induced. From this result, it is found that our method is effective in estimating dynamical noise, even under circumstances where measurement noise exists.

5. Experimental Test

We investigate the possibility of applying our method to experimental data. Gaussian noise is added to the circuit as dynamical noise, as shown in fig.4, where

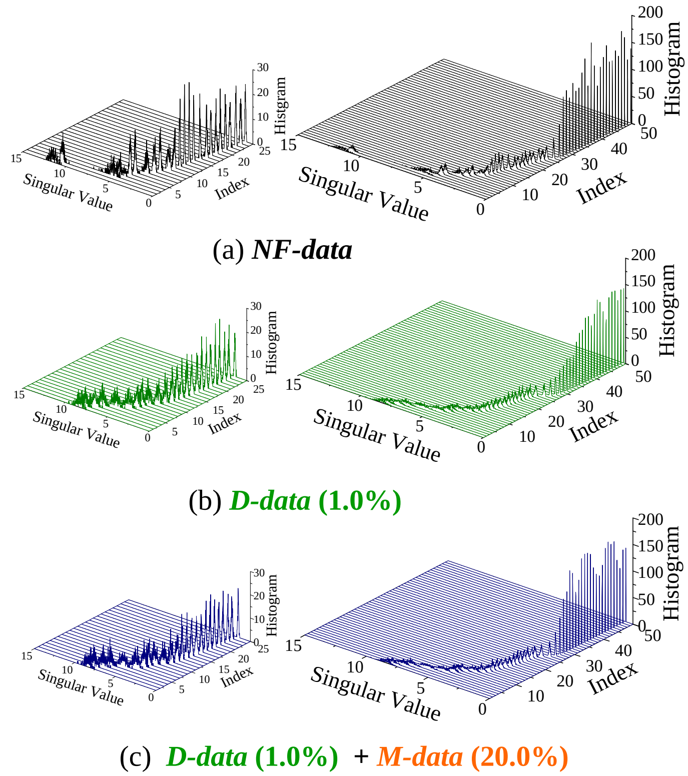

Figure 2: Histogram of singular values in descending order for 200 trials in some chaos data. (a) *NF-data*, (b) *D-data*(1.0%), and (c) Noise-mixed data with *D-data*(1.0%) and *M-data*(20.0%). Appearance of dominant singular values in the range of small *index* from 1 to 25 is illustrated on the left hand side of an each result.

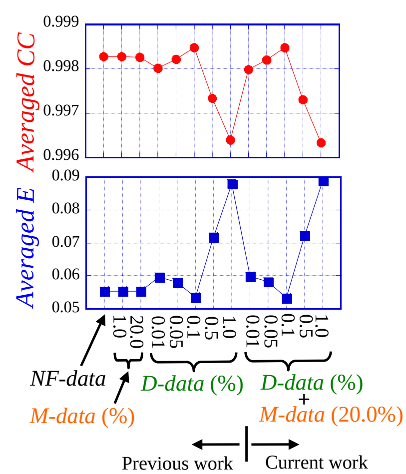

Figure 3: *Averaged CC* and *Averaged E* for all noise-mixed data. Some previous results are also illustrated for comparison. The trends of the current work is quite similar to the previous ones in spite of the existence of measurement noise.

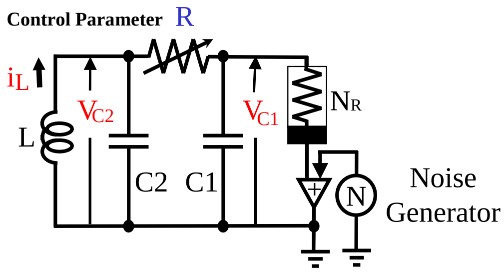

Figure 4: Chua's circuit with dynamical noise. The generated Gaussian noise is added to the line of Nonlinear Resister N_R.

the function synthesizer WF1946 of NF *Corporation* is used as a noise generator. Three types of noise level are prepared: low, middle, and high. Accordingly, we measure 4 types of experimental data, that is, (a) no noise data, (b) low-level noise data, (c) middle-level noise data, and (d) high-level noise data. Besides, an additional Gaussian noise is numerically added to the entire output signal measured at experiment as artificial measurement noise. Consequently, we call the additional data as (e), (f), (g), and (h) in order and totally obtain 8 types of data. *Averaged CC* and *Averaged E* are calculated. According to fig.5, it is found that the differences can be observed between dynamical noise data (b), (c) and (d) and no noise data (a), with regard to the values of *Averaged CC* and *Averaged E*. Namely, it is indicated that the influence of dynamical noise at experiment can be recognized. Furthermore, in (f), (g), and (h) the influence of dynamical noise can be also extracted in spite of the existence of numerical measurement noise. Consequently, it can be said that our approach can be also applied to real world data.

6. Conclusions

It is found that our SVD based-method can be also applied to an estimate of the influence of dynamical noise in noise-mixed chaos data, including dynamical noise and measurement noise. The estimate can be undertaken by calculating the values of the two indices CC and E as estimators of the temporal fluctuation of singular values. The approach can be applied to experimental data and the effects of our approach on real world data are illustrated.

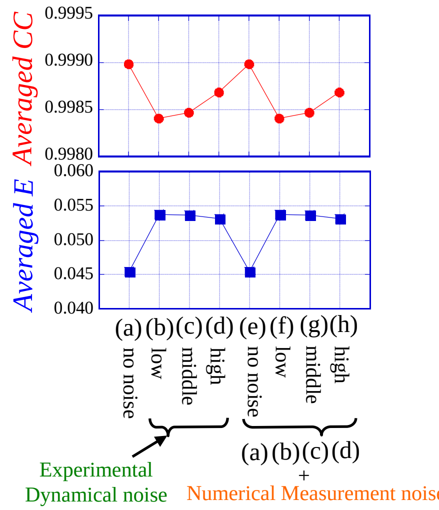

Figure 5: *Averaged CC* and *Averaged E* for experimental data without noise (a), with dynamical noise (b)(c)(d), and with additional numerical measurement noise (e)(f)(g)(h). The influence of dynamical noise can be recognized from the difference of values of *Averaged CC* and *Averaged E*, independently of the existence of measurement noise.

Acknowledgments

We would like to thank Professor K. Aihara of the University of Tokyo for his fruitful comments and Professor T. Ikeguchi of Saitama University for stimulating discussions and for his useful comments.

References

[1] G. G. Szpiro, "Measuring Dynamical Noise in Dynamical Systems," *Physica D*, vol. 65, pp.289–299, 1993.

[2] J.-Y. Ko, J.-S. Lih, M.-C. Ho, C.-C. Tsai, and J.-L. Chern, "Determinism Test, Noise Estimate and Hidden Frequency Recognition," Chinese J. Phys., 37-5, pp.449-465, 1999.

[3] J.P.M. Heald and J. Stark, "Estimation of Noise Levels for Models of Chaotic Dynamical Systems," *Phys. Rev. Lett.*, vol. 84–11, pp.2366–2369, 2000.

[4] D.S. Broomhead and G.P. King, "Extracting Qualitative Dynamics from Experimental Data," *Physica D*, vol.20, pp.217–236, 1986.

[5] R. N. Madan, "Chua's Circuit: A Paradigm for Chaos," *World Scientific Nonlinear Science Ser. B* vol.1, 1993.

[6] M. Todoriki "Estimation of the Influence of Dynamical Noise on Chaos Data using SVD Method," *Proc. NOLTA2001* vol.1, pp.135–138 2001.