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Associative Pattern Retrieval Produced by Stochastic Resonance in Excitable Neural Network

Shin Mizutani†, Kenichi Arai‡ and Kazuyuki Yoshimura§

NTT Communication Science Laboratories, NTT Corporation
2-4 Hikaridai, Seika-cho, Soraku-gun, Kyoto, 619-0237 Japan
Phone:+81-774-93-5214, Fax:+81-774-93-5155
{†shin, ‡ken, §kazuyuki}@cslab.kecl.ntt.co.jp

1. Introduction

Stochastic resonance (SR) [1] has been used in information processing, particularly for signal detection in neural systems, and experimental results have been reported [2, 3, 4]. With regard to sensory systems, these studies insist that SR is the principle behind subthreshold signal detection in a real noisy environment. With a view to explaining experimental results, studies have mainly concentrated on SR using neuron models [5, 6]. These model studies focus on subthreshold signal detection in sensory systems. By contrast, some high-level functions of human vision, audition and memory retrieval are considered to be the results of SR [7, 8, 9]. These functions caused by SR may be related to coupled systems and spatiotemporal signals because the neuron activities are represented as firing patterns. SR has also been reported in actual neural networks that should correspond to these high-level function results [10, 11]. However, these SR studies on actual neural networks were related to a simple situation that included a sinusoidal signal in noise.

Here we focus on an associative memory network [12, 13] consisting of a FitzHugh-Nagumo (FHN) neuron model [14, 15] provided to deal with SR in excitable dynamics caused by a spatiotemporal signal. The weights are learned by a modified Hebbian rule using orthogonal unbiased patterns. The network with Gaussian white noise is modulated by a temporally sinusoidal pattern. We consider that this modulation pattern is an input signal and the noise is a fluctuation in the environment. We previously used one of the learning patterns as the modulation pattern, and reported the results of spatiotemporal SR in this system [16]. From those results, when we used the overlap between the network output and the learning pattern as the network response, we found the SR properties of pattern oscillatory retrieval in the overlap timeseries synchronized with the sinusoidal modulation. As found in previous SR studies, the noise variance maximized the resonance response and the resonance curve could be fitted by the theoretical SR function of the excitable dynamics. The network and neuron resonance frequency was similar. We determined for the first time that, in a heterogeneous system, the coupling strength maximized the resonance response. This resonance mechanism was explained by a following scenario. The parameter crosses a bifurcation point to a limit cycle corresponding to the modulation pattern. In other words, the state crosses a threshold to an excitable state corresponding to the modulation pattern. In this work, we use a randomly flipped pattern from a learning pattern as a modulation pattern. This approach is provided to study oscillatory retrieval properties by spatiotemporal SR modulated by a distorted learning pattern. We evaluate the robustness of retrieval when we decrease the similarity between this modulation pattern and the learning pattern. Section 2 shows the model and the modulation pattern and Sec. 3 provides the numerical results. Finally, we summarize our conclusions related to the oscillatory retrieval properties produced by spatiotemporal SR and speculate on the meaning of spatiotemporal SR in actual neural systems and their functions.

2. Model

To study pattern oscillatory retrieval by spatiotemporal SR, we focus on an associative memory network that is formed of coupled FHN neurons. The neuron model has a fast variable u representing the internal state of a neuron and a slow variable v corresponding to the refractory period. The constant input I_0 biases the neuron toward the threshold so that it oscillates. A network modulated by a sinusoid with a small amplitude I_i and a frequency f_m and Gaussian white noise $\eta_i(t)$ with mean 0 and variance D, can be represented by the following equations.

$$\tau \frac{du_i}{dt} = u_i - \frac{u_i^3}{3} - v_i$$

$$+\sum_{j=1}^{N} w_{i,j}(u_j - u_{\text{eq}}) + \eta_i(t)$$
$$+ I_0 + I_i \sin(2\pi f_m t), \qquad (1)$$
$$\frac{dv_i}{dt} = u_i - \beta v_i + \gamma, \qquad (2)$$

where the parameters are $\tau = 0.1$, $\beta = 0.8$, $\gamma = 0.7$, and $I_0 = 0.0$. The value of u_{eq} for a parameter set is determined by the equilibrium state in the absence of a sinusoid and noise in a single neuron model. Here, we fix $u_{\text{eq}} = -1.2$ as $I_0 = 0.0$ and the number N of neurons in the network is 256. The noise is uncorrelated both within the neurons and in time.

The weight $w_{i,j}$ is learned by the following modified Hebb rule.

$$w_{i,j} = \frac{w}{Na(1-a)} \sum_{\mu=1}^{p} \xi_i^{\mu}(\xi_j^{\mu} - a). \qquad (3)$$

w represents the coupling strength and a is the mean activity of the learning patterns. We use this asymmetrical learning rule because the neuron experiences a rebound effect [12, 13] by which it is fired through an inhibitory input. The learning patterns ξ_i^{μ} $(i = 1, \ldots, N$, $\mu = 1, \ldots, p)$ with a mean activity $a = 0.5$ are the orthogonal $p = 3$ patterns coded with 0 and 1 values.

The modulation amplitude $\vec{I} = (I_1, \ldots, I_i, \ldots, I_N)$ is constructed from a modulation pattern ξ_i^{mod} with a small value ϵ as follows.

$$I_i = \epsilon \xi_i^{\text{mod}}. \qquad (4)$$

We fix $\epsilon = 0.05$ as a small value against input noise variance D. This modulation pattern ξ_i^{mod} is constructed by random flipping from the ν-th learning pattern. Random flipping replaces 1 with 0 or 0 with 1 in randomly chosen neurons. However ξ_i^{mod} maintains its orthogonality with other learning patterns and mean activity a. The overlap r between the modulation pattern ξ_i^{mod} and the ν-th learning pattern is

$$r = \frac{1}{Na(1-a)} \sum_{i=1}^{N} (\xi_i^{\nu} - a)(\xi_i^{\text{mod}} - a). \qquad (5)$$

r represents the similarity between the modulation pattern ξ_i^{mod} and the ν-th learning pattern. Here we call this overlap r the modulation pattern similarity. We study the retrieval properties produced by SR with decreases in the modulation pattern similarity r.

Here we introduce an overlap $m_{\mu}(t)$ as an order parameter in the network behavior. Since we are often interested in resonance with binary signals, we quantize the internal state u timeseries into a binary output of 1 or 0 reflecting whether or not the neurons are firing. To define the overlap, timeseries $u_i(t)$ is transformed into binary output $x_i(t)$ as follows.

$$x_i = \begin{cases} 1, & \text{if } u_i > u_{\text{th}}, \\ 0, & \text{otherwise.} \end{cases} \qquad (6)$$

The overlap $m_{\mu}(t)$ is defined as follows.

$$m_{\mu}(t) = \frac{1}{Na(1-a)} \sum_{i=1}^{N} (\xi_i^{\mu} - a)(x_i(t) - a). \quad (7)$$

This overlap $m_{\mu}(t)$ shows the similarity between the network output $x_i(t)$ at time t and the μ-th learning pattern. We call $m_{\mu}(t)$ the learning pattern overlap. In the same way, we define overlap $m_{\text{mod}}(t)$ for the modulation pattern and call $m_{\text{mod}}(t)$ the modulation pattern overlap.

We calculate the mean peak powers $P_{\nu}(f_m)$ and $P_{\text{mod}}(f_m)$ to evaluate the resonance response of $m_{\nu}(t)$ and $m_{\text{mod}}(t)$ at modulation frequency f_m. We also consider the power ratio $P_R(f_m) = P_{\nu}(f_m)/P_{\text{mod}}(f_m)$ to compare the modulation frequency powers for the oscillatory retrieval. This ratio is expressed in decibels (dB). When $P_R(f_m) \geq 1$ $(10\log_{10} P_R(f_m) \geq 0)$ and the network shows clear spatiotemporal SR, we regard as successful retrieval of the ν-th learning pattern by SR.

3. Numerical Results

Figure 1 shows the peak powers when the noise variance was varied. In a range of noise variance D, $P_{\nu}(f_m)$ is larger than $P_{\text{mod}}(f_m)$. The amplitude of $m_{\nu}(t)$ is larger than that of $m_{\text{mod}}(t)$ at the modulation frequency f_m. This means that the network oscillatory output is more similar to the learning pattern than the modulation pattern. This retrieval effect is large when the weight is large.

Next, we fix noise variance D and calculate $P_{\nu}(f_m)$ and $P_{\text{mod}}(f_m)$ as a function of modulation pattern similarity r. Figure 2 shows $P_{\nu}(f_m)$ and $P_{\text{mod}}(f_m)$. When the modulation pattern similarity r is larger than a critical value, $P_{\nu}(f_m) \geq P_{\text{mod}}(f_m)$. This shows that when the modulation pattern is a slightly distortion of the learning pattern, the network can retrieve the learning pattern, not the modulation pattern. This is caused by the retrieval function of the associative memory model, which is provided by a cooperative effect through interaction. With a larger coupling strength w, the result is similar and the retrieval effect become larger than that with a small w.

Lastly, we consider the power ratio $P_R(f_m)$ in order to compare the modulation frequency powers for oscillatory retrieval synchronized with the sinusoidal pattern

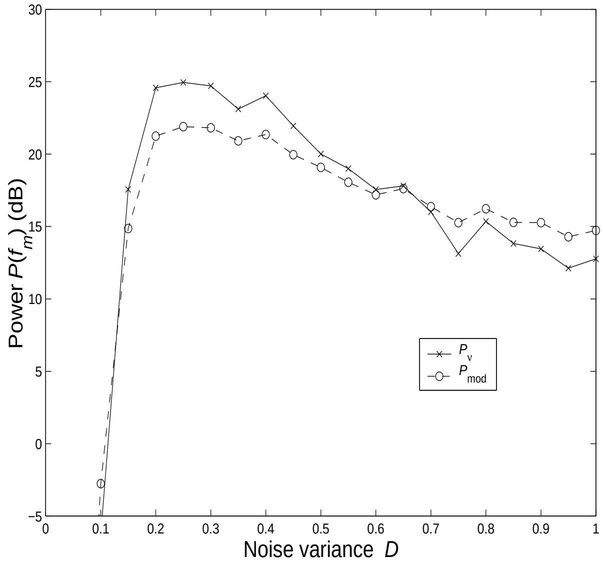

Figure 1: Powers $P_\nu(f_m)$ and $P_{\mathrm{mod}}(f_m)$ as a function of noise variance D. The parameters are $w = 0.2$, $f_m = 0.2$, and $r = 0.5$.

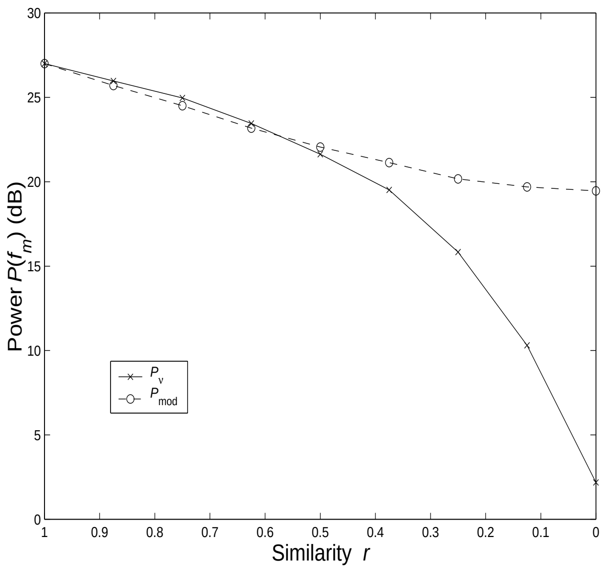

Figure 2: Powers $P_\nu(f_m)$ and $P_{\mathrm{mod}}(f_m)$ as a function of modulation pattern similarity r. The parameters are $w = 0.1$, $f_m = 0.25$, and $D = 0.6$.

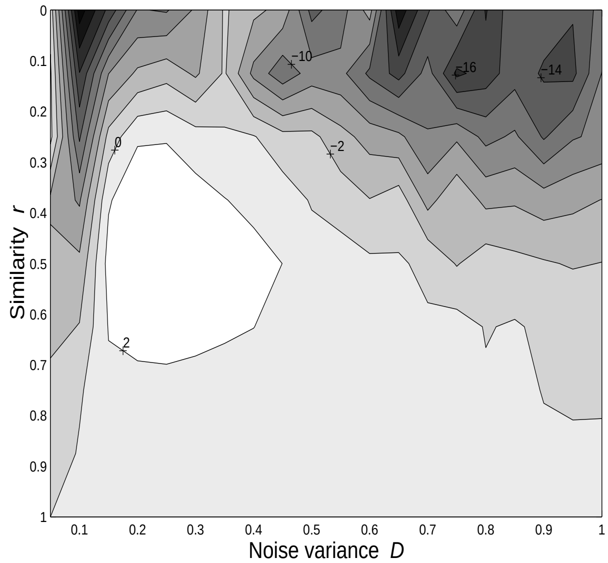

Figure 3: Power ratio $P_R(f_m)$. The numbers represent the ratio in dB. The parameters are $w = 0.2$ and $f_m = 0.2$.

modulation. Figure 3 shows $P_R(f_m)$. $P_R(f_m) \geq 1$, when the modulation pattern similarity r is larger than a critical value and noise variance D is in a certain range. The region of $P_R(f_m) \geq 1$ is extended in the direction of r and reduced in the direction of D as the coupling strength w increases. Here we note for a further large w that when w becomes too large, clear pattern SR properties are not obtained in the network even when using a learning pattern as a modulation pattern [16].

4. Conclusion and Discussion

We studied the properties of pattern oscillatory retrieval produced by SR in an associative memory network coupled FHN neuron model. The modulation pattern was constructed from one of the learning patterns by random flipping. We undertook a numerical study of stochastic synchronized retrieval with the sinusoidal pattern modulation when the similarity between the modulation pattern and the original learning pattern was reduced. The network stochastically and periodically retrieved the learning pattern rather than the modulation pattern when the similarity between two patterns was larger than a critical value and the noise variance was in a certain range. This means that the network oscillatory output was more similar to the learning pattern than the modulation pattern. This retrieval effect was large when the weight was large. The range of the learning pattern retrieval for a large weight was extended in the similarity direction and decreased in the noise variance direction. These responses show that the network retrieves the learning pattern, not the modulation pattern even when the modulation pattern is only a slight distortion of the learning pattern. This is caused by the retrieval function of the associative memory model. The results show an effective function whereby spatiotemporal SR can robustly detect subthreshold spatiotemporal signals even when the pattern signal is slightly distorted.

Here we consider the importance of spatiotemporal SR for the high-level functions of neural systems. Actual neural systems that have high-level functions, code information by firing patterns and not by firing each neuron. We can assume that the firing pattern behavior is important for information processing and transmission. Suppose that a certain firing pattern is evoked by a spatiotemporal input. Even when the spatiotemporal input is sightly distorted, it may be able to activate the same firing pattern. In a sensory SR situation, we did not consider the distortion of the input signal. Spatiotemporal SR in an associative memory network can resonate with the distorted spatiotemporal input and retrieve a learning pattern by cooperative resonance between resonators. If the behavior and formation of firing patterns in actual neural systems are important for high-level information processing, spatiotemporal SR is more suitable for neural systems than ordinary sensory SR. These functions are necessary for the high-level functions of neural systems. This spatiotemporal SR may become a simple model of high-level information processing by SR in neural systems.

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