International Symposium on Nonlinear Theory and its Applications
Xi'an, PRC, October 7–11, 2002

Circle Detection using Directional Derivatives of Gaussian

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Key words: Directional Derivatives of Gaussian, circle detection, Hough Transform.

1. Introduction

This paper proposes an alternative system for circle detection. The detection of circular shapes inside an input image is accomplished with the use of Directional Derivatives of a Gaussian Function. The system presents insensibility to translation and scaling of the circular shapes. The authors have already proposed concentric shape detectors using directional derivatives [1]. Also, a detector of concentric shapes using an Heuristic function and its implementation in a Cellular Neural Network was discussed in [2]. Through all this works the proposed operator was able to detect only concentric shapes, i.e., shapes that have a common central point. In this work we have tried to overcome this problem.

The well-known Hough transform is used as a reference to determine the positive and negative points of our proposed system. For instance, while the Hough transform needs a binary-edge-detected image as input, we will see that the proposed system performs edge detection while is searching for the presence of circular shapes. Another difference is that the Hough Transform needs to know the radius of the circle to be searched, while our system does present insensibility to scaling problems.

Our system has two principal process: an edge detection process and a searching process. The edge detection process is accomplished using directional convolution masks that perform an orientation-based edge detection. Thus, convolution masks that are able to detect *only* circular shapes are generated and applied to the whole image. The searching process is achieved with the use of a simple image scanning. Although, scanning the whole image is a time-consuming process, some improvements can be made in order to reduce the amount of calculations. Some of the improvements include: a) the generation of fewer convolution masks and, b) an alternative scanning method that is based on information extracted from the image. The next two sections cover briefly the theoretical aspects of the system. Conditions for simulation and the preliminary results we have obtained are shown in Sec.4. Section 5 covers the conclusions and some suggestions for a future work.

2. Directional derivatives

For this research, the well-known expression for the nth-order directional derivative ∂_β^n of a function $F(u,v)$ in any direction β is used. This expression is defined as:

$$\partial_\beta^n F(u,v) = (cos\beta\ \partial_u + sin\beta\ \partial_v)^n F(u,v). \qquad (1)$$

Using Eq.(1) we will be able to express a directional derivative of any function $F(u,v)$ in any direction ($cos\beta, sin\beta$).

3. Circle Detection System

As it was described in the last section, the proposed detection system is based in two well defined processes: the edge detection process and the scanning process.

3.1. Edge Detection Process

Conventional edge detection methods allows the detection of all edges in a given image without regarding its orientation. Thus, a selective edge detection is not possible. For our system to be able to detect circles, a selective edge detection method was used. This method is based in the use of directional convolution kernels or **DCKs** for short. These **DCKs** are square matrices able to detect or erase edges with a given orientation, performing a Directional Edge Detection (**DED**) or an Directional Edge Erasing (**DEE**). **DED** and **DEE** can improve the extraction of important data from an input image, discarding image data that doesn't need to be post-processed.

Both, **DED** and **DEE**, can be performed with the help of **DCKs**.

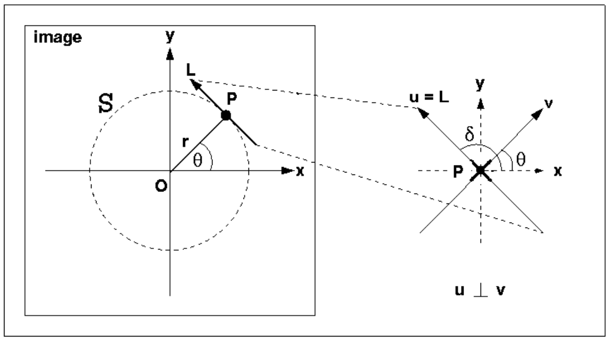

Figure 1: Determination of the edge orientation.

3.1.1. Generation of DCKs

DCKs are square matrices to be applied to an image using standard convolution process.

Consider the circular shape S present in a given input image $I(x, y)$ as shown in Fig.1, with a reference point $O(0, 0)$. The shape S can be expressed as a set of pixels $P(x, y)$ that fulfill a condition with respect to the reference point $O(0, 0)$. Therefore, the circular shape will be detected if, at a given pixel $P(x, y)$, only edges with orientation δ can be selected.

Analyzing well-known edge detection operators that do not perform rotation invariant edge detection (i.e., Sobel, Roberts, etc.), give us an start point to determine what kind of convolution masks we must produce. Two conditions were obtained: a) the matrix must have full symmetry with respect to one of the axis and, b) the matrix must have the same number of negative values and positive values.

An **DCK**$_\delta(m, n)$ is defined as "A $M \times N$ matrix capable of detecting edges with orientation δ" (refer to Fig.1). To produce a matrix with these characteristics, we proposed the method of sampling a function $F(u, v)$ that satisfies the above conditions. Following is a description of a function $F(u, v)$ that is suitable for our purposes.

3.1.2. Directional Derivatives of a Gaussian Function

In past works, approximation methods have been used to produce a suitable function. For this work, a more robust function was used. The functions that best suited our requirements are the first-order partial derivatives of a Gaussian function. These are defined as:

$$\frac{\partial G}{\partial u} = G'_u = \frac{e^{-\frac{u^2+v^2}{2\sigma^2}}}{\sigma^3\sqrt{2\pi}} u \, , \qquad \frac{\partial G}{\partial v} = G'_v = \frac{e^{-\frac{u^2+v^2}{2\sigma^2}}}{\sigma^3\sqrt{2\pi}} v \, . \tag{2}$$

Figure 2 shows the representation of G'_u. This function

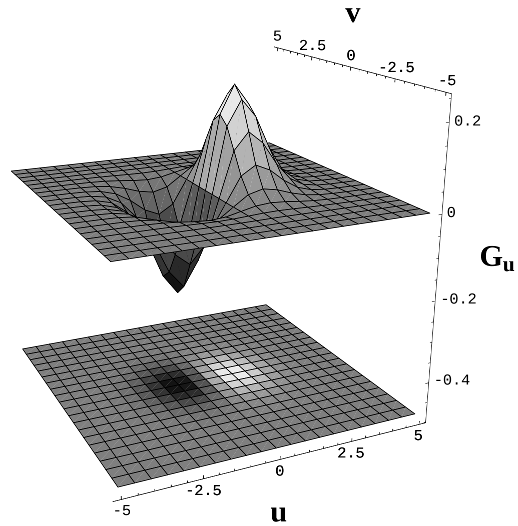

Figure 2: Derivatives of Gaussian: First-order partial derivative G'_u.

was used to obtain the values for the convolution masks.

To obtain the values for a convolution kernel **DCK**$_\delta(m, n)$, a directional derivative $\partial^n_\delta G(u, v)$ is obtained from Eq.(1)as:

$$\partial_\delta G(u, v) = (cos(\delta - \phi)\, \partial_u + sin(\delta - \phi)\, \partial_v) G(u, v), \tag{3}$$

with the order of the derivative n equal to 1 and the value of the orientation angle $\beta = \delta - \phi$. ϕ is a rotational control value that can be equal to 0 or $\frac{\pi}{2}$. If $\phi = \frac{\pi}{2}$, the masks produced will be able to detect borders with an orientation δ. On the other hand, if $\phi = 0$, the masks produced will be able to erase borders with an orientation δ. A detailed explanation can be found in any of the given references.

As a next step, the values of the **DCK**$_\delta(m, n)$ are obtained sampling the function $\partial_\delta G(u, v)$:

$$\mathbf{DCK}_\delta(m, n) = \sum_m \sum_n \partial_\delta G(u, v) h_s(u - m\Delta u, v - n\Delta v), \tag{4}$$

where h_s is a limited impulse function, Δu and Δv are the sampling intervals. If M and N express the size of the resultant matrix, m and n are defined as $m = [-\frac{M}{2}, \frac{M}{2}]$ and $n = [-\frac{N}{2}, \frac{N}{2}]$.

For example, let's suppose that we want to produce a 5×5 convolution kernel to detect edges with vertical orientation (i.e., $\phi = \frac{\pi}{2}$ and $\delta = \frac{\pi}{2}$). According to Eq.(3), the function $\partial_{\delta=\frac{\pi}{2}} G(u, v)$ is reduced to:

$$\partial_{\delta=\frac{\pi}{2}} G(u, v) = \partial_u G(u, v) = G'_u. \tag{5}$$

Then, using Eq.(4) , a 5×5 matrix is obtained. This sampling process is represented by Fig.3.

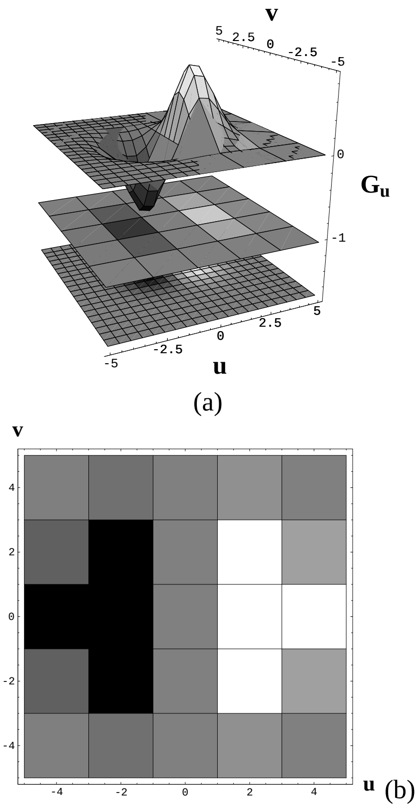

Figure 3: Generation of the $\mathbf{DCK}_{\delta=\frac{\pi}{2}}(m,n)$ (a) sampling process of function $G_u^{'}$. (b) obtained mask.

That means that, ideally, there is a need to generate one convolution mask for every angle (i.e. 360 masks). Due to image resolution, there is no need to generate all the masks.

3.2. Scanning Process

Once we have obtained a function to detect (or erase) edges with a given orientation, the next step is to analyze each pixel in the image to determine the orientation of the convolution mask to be applied. Let's explain this step with more detail.

As a first step a reference point (x_0, y_0) is selected inside the image. The determination of a reference point is necessary since it will be considered as the center of possible circles. In our past works, we have determined this reference point to be *only* in the center of the image, thus, allowing the system to detect only those circles whose center was the center of the image. In this work, many reference points can be selected according to a *selection criteria* to be explained later.

Next, for each pixel (x, y) inside the image, we determine its angular position with respect to the origin. This can be observed in Fig.4. Using the angular position δ, the directional derivative $\partial_\delta^n G(u, v)$ is obtained. Sampling this function, a $\mathbf{DCK}_\delta(m, n)$ is obtained. We use this matrix and perform a convolution with pixel (x, y). Note that, as it was discussed before, we have not generated one masks with every single value of δ. The image is divided in sectors with a fixed angular position. Thus, the problem is reduced to determine in which sector the pixel is, and generate the same convolution masks for all the pixels inside the same sector. This process is repeated for the entire image. After a complete pass is finished, the reference point is move to its next position.

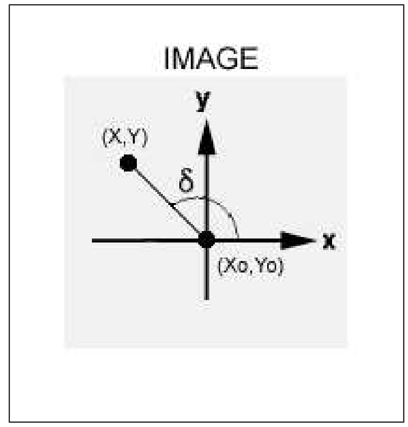

Figure 4: Scanning Process. Determination of angular position.

The *selection criteria* for the next reference point depends on many aspects. For this work the following points were taken into consideration: (a) A given circle is completely inside the image and, (b) The maximum and minimum radius of the circles are known. Using the mentioned considerations a reduced group of reference points is determined. Given a group of R reference points, R output images will be obtained.

The last step is to compose output images to produce a final response of the system. For this research the output images were added and a threshold was applied to the resultant image. A method that we consider really easy. Better methods will be explore in the future.

4. Simulation and Results

Figures 5 and 6 display the most representative results obtained using our system. In Figures 5(a) through (f), an input image composed by two circles is used. On the other hand, Figs.6(a) through (f), show the system response when a image composed by a circle and a square is used as an input. Figure 5(a) is the input image of 255×255 pixels. Figures 5(b) through (e) are some of the intermediate images obtained with different reference points. For example, Fig.5(c) is the resultant image when the reference point is located exactly in the center of the bigger circle. The differ-

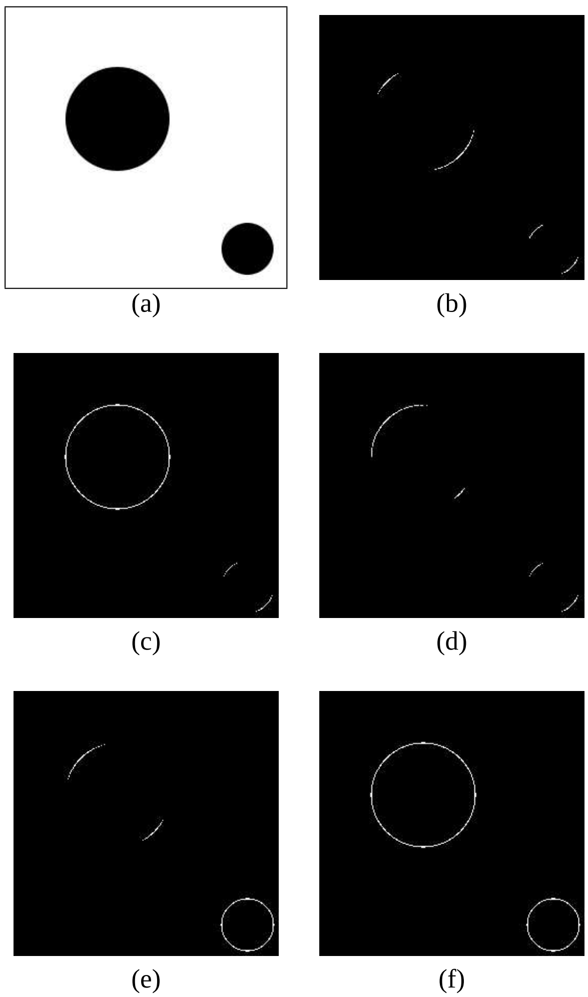

Figure 5: Simulation results. (a) Input Image. (b)-(e) output images with reference point located at (b) the upper left corner, (c) the center of the big circle, (d) the center of the image, (e) the center of the small circle. (f) output image.

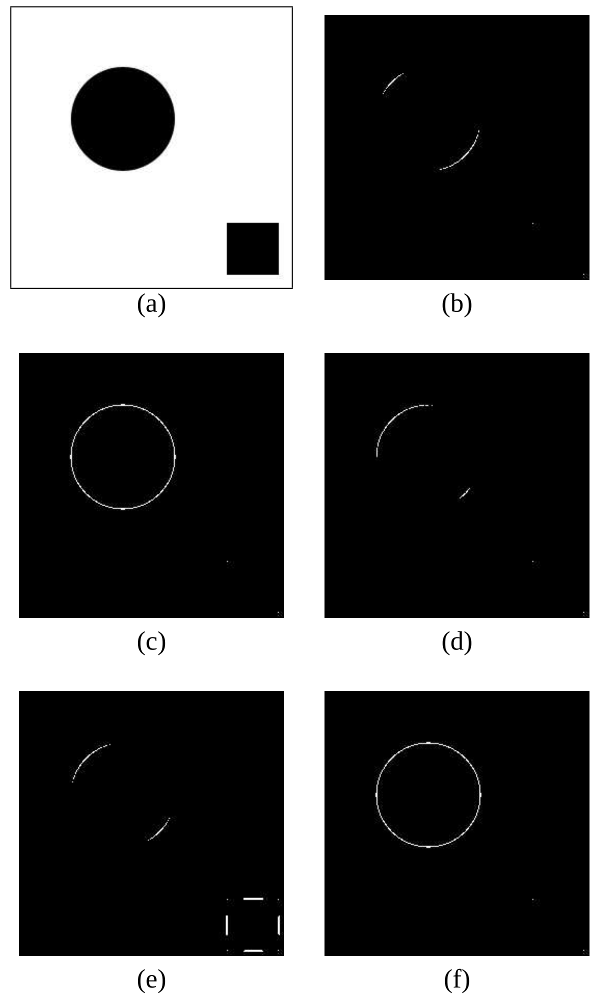

Figure 6: Simulation results. (a) Input image. (b)-(e) output images with reference point located at (b) the upper left corner, (c) the center of the big circle, (d) the center of the image, (e) the center of the small square. (f) output image.

ence with Fig.5(b) or (d) can be easily appreciated. In these figures, the reference point is located neither in the center of the big circle, nor in the center of the small one. Figure 5(f) is the response of the system. The same explanation applies for Fig.6(a) through (f). But as can be appreciated in Fig.results2(e), a low response is obtained since the figure is an square and not a circle.

5. Conclusions and Future Work

In this work, an alternative circle detector system has been described. The results obtained are encouraging to continue improving the system. There are a lot of points to improve including (as the critical ones): (a) the generation of the output image and, (b) how a reference point is selected. Any future improvement will be announced.

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