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A Discrete-time Chaotic Neuron Model with Delayed Output

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1. Introduction

A lot of mathematical neuron models have been proposed and investigated. We proposed a high-dimensional discrete-time chaotic neuron model[3] by generalizing Aihara's one[1]. The high-dimensional model is derived by temporal extension with the internal state of the neuron. In the two-dimensional model, we showed bursting, which cannot be observed in one-dimensional models, when a Hopf bifurcation with a 2-periodic orbit occurs.

Most of neuron models usually neglect the existence of delay from the internal state to the output in the neuron. However, it is more general to take the delay into consideration[4]. In a continuous-time neuron model, Nakajima and Hayakawa proposed a neuron model with such a delay called an inverse delayed model[2]. In the model, it is assumed that there exists the first-order lag instead of the delay. Nakajima and Hayakawa introduced an energy function in order to discuss behavior as a neural network composed by the inverse delayed model, and they applied the neural network to an N-queen problem.

However, there are few discrete-time neuron models with the delay. Therefore, in this paper, we will propose such a discrete-time model with delayed output by generalizing Aihara's chaotic neuron model, and show coexistence of attractors, which cannot be observed in one-dimensional models.

This paper is organized as follows: In Sec. 2, we propose a discrete-time chaotic neuron model with delayed output, and derive a symmetric property with respect a parameter. In Sec. 3, we analyze In Sec. 4, we conclude this paper.

2. A Discrete-time Model with Delayed Output

2.1. Derivation of the delayed model

In this subsection, a discrete-time neuron model will be derived by generalizing a chaotic neuron model proposed by Aihara et al.[1], which is described by the following first-order difference equation:

$$y(t+1) = ky(t) - \alpha x(t) + c, \tag{1}$$

where t is the discrete time, $y(t)$ and $x(t)$ are the internal state and the output of the neuron at the time t, respectively, k is the damping factor of the refractoriness, and c is a parameter between 0 and 1. The output $x(t)$ is a *static* variable because the delay is neglected in Aihara's model, and it is determined by $x(t) = f(y(t))$, where f is the logistic function with the steepness parameter ε:

$$f(u) = \frac{1}{1+\exp(-u/\varepsilon)}. \tag{2}$$

In this paper, the delay between the internal state and the output is taken into account. In other words, we assume that the output $x(t)$ is a *dynamic* variable determined by

$$\beta'\Big\{x(t+1) - x(t)\Big\} = f(y(t+1)) - x(t+1), \tag{3}$$

where β' is a small parameter. The above equation can be rewritten as follows:

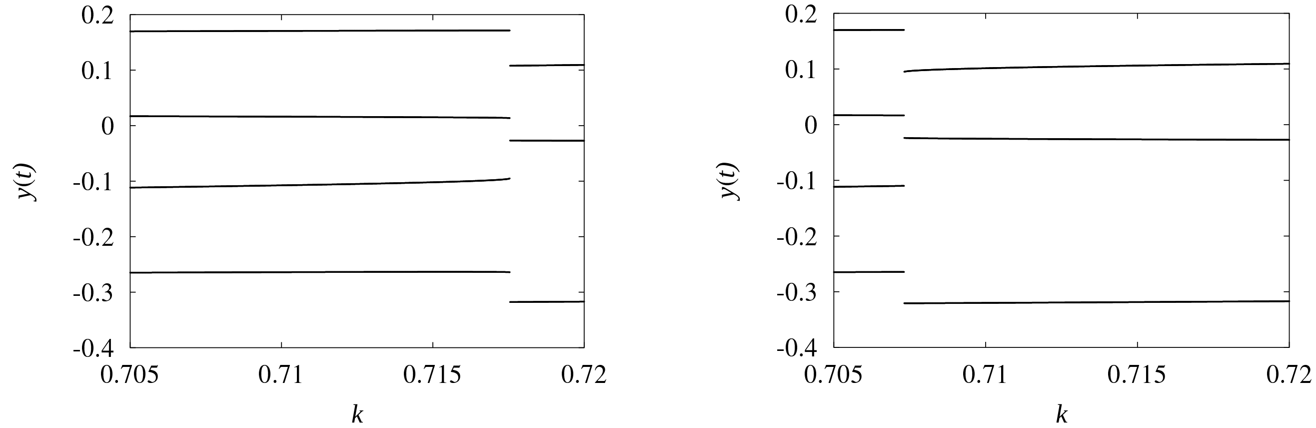
$$x(t+1) = \beta x(t) + (1-\beta) f(y(t+1)), \tag{4}$$

where $\beta = \beta'/(1+\beta')$. In the case $\beta = 0$, Eq. (4) implies $x(t+1) = f(y(t+1))$ so that the model is reduced to Aihara's one.

Note that Nakajima and Hayakawa[2] assumed that an output function is hysteresis-type. Hence, in order to avoid difficulty of analysis, they introduced an inverse function of the output one because the inverse one is single-valued. However, we do not use an inverse one since we use the logistic function as an output one in this paper.

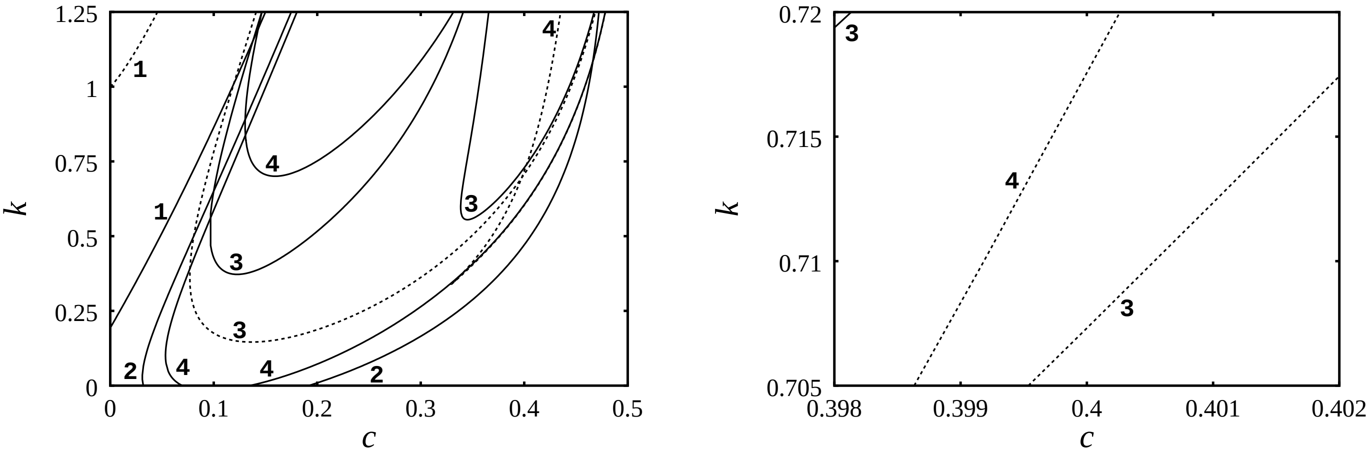
In summary, we propose and analyze the following discrete-time chaotic neuron model with delayed output in this paper:

$$\begin{cases} y(t+1) = ky(t) - \alpha x(t) + c, \\ x(t+1) = \beta x(t) + (1-\beta) f(y(t+1)). \end{cases} \tag{5}$$



(i): Increasing k from 0.705 to 0.720. (ii): Decreasing k from 0.720 to 0.705.

Figure 1: Bifurcation diagrams. β and c are set to be 0.25 and 0.40, respectively.



(i): A bifurcation set β is set to be 0.25. (ii): Enlargement of Fig. 2(i).

Figure 2: Bifurcation sets. Solid and dashed lines illustrate a period-doubling and a saddle-node bifurcation, respectively. Numbers in the figures show the period of the corresponding periodic orbits.

2.2. Symmetry with respect to parameter c

We consider that the case that $\alpha = 1$ in this paper. Note that the logistic function (2) has the relation that $f(u) = 1 - f(-u)$. From the relation and $\alpha = 1$, we can rewrite Eq. (5) as follows:

$$\left\{\begin{array}{l} -y(t+1) = -ky(t) - \alpha(1 - x(t)) + (1 - c), \\ 1 - x(t+1) = \beta(1 - x(t)) + (1 - \beta)f(-y(t+1)). \end{array}\right. \tag{6}$$

From Eq. (6), Eq. (5) has central symmetry about $(y,\ x) = (0,\ 0.5)$. From the symmetry, it is sufficient to assume that the parameter c lies between 0 and 0.5 without loss of generality because it is usually set to be between 0 and 1. Therefore, in this paper, the parameter c is set to be between 0 and 0.5.

3. Analysis

In the following, the parameters α and ε is set to be 1.0 and $0.03125 (= 2^{-5})$, respectively.

Figs. 1(i) and (ii) show bifurcation diagrams, where β and c are set to be 0.25 and 0.40, respectively, and k is increased and decreased between 0.705 and 0.720. Discontinuous changes are observed when k is about 0.718 and 0.707 in Fig. 1(i) and (ii), respectively.

In order to consider the discontinuous changes, Fig. 2(i) shows a bifurcation set on c-k plane, where $\beta = 0.25$.

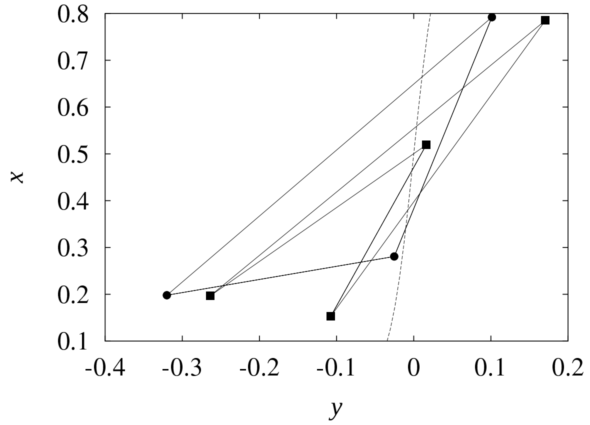


Figure 3: Coexistence of attractors where β, c and k are set to be 0.25, 0.40, and 0.71, respectively. The dashed line shows the logistic function (2). Squares and circles illustrate 3-periodic and 4-periodic points, respectively.

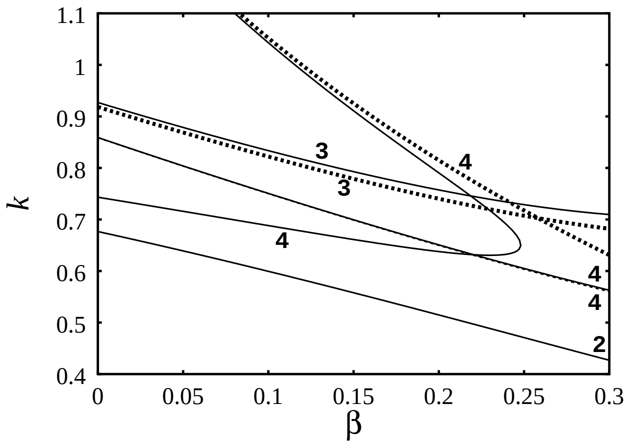


Figure 4: Another bifurcation set. c is set to be 0.40.

Figure 2(ii) also shows enlargement of Fig. 2(i). In both figures, solid and dashed lines show a period-doubling and a saddle-node bifurcation, respectively. Numbers in the figures show the period of the corresponding periodic orbits. Note that no Hopf bifurcation occurs because the absolute value of the product of both eigenvalues of any n-periodic points is $(k\beta)^n$, that is, less than unity generally. Figure 2 shows that saddle-node bifurcations with 3-period and 4-period cause the discontinuous changes shown in Fig. 1(i) and (ii), respectively. Hence, there coexist 3-periodic and 4-periodic attractors in the range of k between 0.707 and 0.718. In order to understand the coexistence, Fig. 3 shows the coexistence of the 3-periodic and 4-periodic attractors. In the figure, Squares and circles illustrate 3-periodic and 4-periodic points, respectively, and the dashed line shows the logistic function (2).

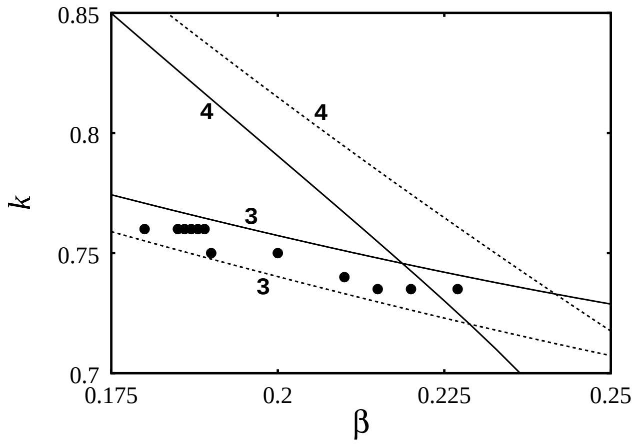


Figure 5: Enlargement of Fig. 4. Numerical simulation for coexistence of attractors is applied for sets of parameters illustrated by circles.

On the other hand, coexistence of attractors cannot be observed in Aihara's model[1], i.e., the proposed model where $\beta = 0.00$. In order to discuss the relation between β and the coexistence of attractors, Fig. 4 shows a bifurcation set on k-β plane, where $c = 0.40$. Thick dashed lines in Fig. 4 illustrate bifurcation sets which cause the coexistence of attractors discussed above. We choose some parameters illustrated by circles shown in Fig. 5 in order to check for the coexistence numerically. The numerical simulation shows that there coexist chaotic and 3-periodic attractors when β and k are set to be 0.186 and 0.740, and that there exists only a 3-periodic attractor when β and k are set to be 0.185 and 0.740. Therefore, one of the coexistence of attractors disappears in the range β between 0.185 and 0.186 where $k = 0.740$.

4. Concluding Remarks

In this paper, we have proposed a second-order discrete-time neuron model with delayed output, and analyzed the model. We have shown discontinuous changes in bifurcation diagrams, and derived bifurcations set to discuss the phenomena. Moreover, we have shown coexistence of attractors. The coexistence of attractors is a remarkable characteristic in the proposed model. Numerical simulation have shown that there coexist attractors if β is larger. However, we have not shown a greatest lower bound of β for the coexistence. Hence, we will investigate the bound numerically and theoretically, Another future study is analysis of a neural network composed by the proposed model.

References

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