Scaling Property of Foreign Exchange Market and Its Relation to Turbulence

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Abstract

A strong similarity between foreign exchange rates and turbulence, which apparently belong to completely different kind of phenomena, is a highly challenging problem to look into. We have studied this question quantitatively based on substantial amount of statistical analysis of real data, such as tick data collecting all the quotes for one year and the experimental data on highly developed three dimensional uniform, isotropic turbulence newly taken in the facility of our University. We have identified the scaling regions of both data and its relation to the inertial range of the turbulence, indices of Lévy stable distribution, as well as Kolmogorov scaling of the n-th moments.

Key words Price Fluctuation, Turbulence, Kolmogorov Scaling, Lévy Stable Distribution

1. Introduction

Finding laws that are valid for a wide range of seemingly different subjects is a challenging mission [1]. In most cases, however, it seems rare to see quantitative analysis based on detailed data. Though it has been expected that the financial time series behave similarly to turbulence, data are beyond reach for most computer scientists. Recently due to rapidly increasing interest on using computers for financial transactions, high-frequency data (called tick data) in finance has gradually been accumulated and released to research community.

Gashghaie et.al., by using HFDF93 as an example of high frequency data on foreign currency exchange market, and used tow different data of turbulence, showed quantitatively that those two time series are indeed similar [2]. They demonstrated that the scaling law in probability density distributions of price changes Δx of foreign exchange rates taken at different time delays Δt and that of velocity differences Δv of fully developed, isotropic, homogeneous, three dimensional turbulence at different spatial scales Δr are similar, and the corresponding n-th moments follow the same scaling law known as Kolmogorov scaling. This has attracted much attention.

They have also argued the possible correspondence between turbulence and foreign exchange by comparing the cascade structure in energy flow for turbulence and in information flow for foreign exchange market. In turbulence the energy is given at the macroscopic level to the fluid, which diffuses from large-sized eddies to smaller-sized eddies then finally dissipates into the heat energy of individual molecular motions inside tiny eddies of the size smaller than the Kolmogorov scale.

Fully-developed isotropic 3 dimensional turbulence	Foreign currency exchange rate
Velocity difference Δv	Price difference Δx
Energy	Information
Spatial distance (Δr)	Time delay (Δt)
$\left\langle (\Delta v)^n \right\rangle \propto (\Delta r)^{n/3}$	$\left\langle (\Delta x)^n \right\rangle \propto (\Delta t)^{\xi_n}$

Table 1 Correspondence between turbulence and foreign currency exchange (from Ref.[2])

Similarly in currency exchange market, macroscopic information given into the market diffuses into the price movement in much finer time scales that influences decision makings of investors and is eventually absorbed into the motion of individual price movements. Indeed the motion of prices at very fine resolution of time is very close to the pure random motion.

We compare in this report the same tick data used in Ref. [2] and the result obtained in the recent experiment of three dimensional turbulence done in the multi-fan turbulent wind tunnel in our University, by means of detailed numerical analysis, in order to find out properties commonly observed in both of the two apparently distant phenomena.

The financial data we used for this analysis is the exchange rates of US Dollar and German Mark from October 1, 1992 to September 30, 1993. The total number of ticks amounts to 1,472,241 [3].

2. Probability Density Distribution

We first plot the probability density distribution of velocity difference in our newly taken experimental data of turbulence at various values of special resolution Δr [4]. We used 250,000 data points obtained in our 15 meters wind tunnel located in Miyazaki University. The Reynolds number is $Re_\lambda = 376$ and sampling frequency is 10kHz. We denote the velocity at r be v(r), and the difference between r and $r+\Delta r$ be $\Delta v = v(r+\Delta r) - v(r)$

The result is shown in Figure 1 for the range from $\Delta r = 1$ to $\Delta r = 1000$ (in unit of 6.8×10^{-4} meters). Three lines correspond to $\Delta r = 1$, 10, 100. We also check that there is a solid inertial range in our turbulence data. Inertial range is often talked in the context of power spectrum being a straight line in log-log plot with slope -5/3. Figure 2 shows that our data exhibit this property for the region 7<f<700 [Hz].

Next we compute the probability density for foreign currency exchange rates. Here we used the first 250,000 points in US dollar to German Mark exchange rates in HFDF93. We denote the price at time be x(t), and the price difference $\Delta x = x(t+\Delta t) - x(t)$ for $\Delta t = 1$-100 [tick]. Figure 2 shows the probability distribution for $\Delta t = 10$, 50, 100 [tick].

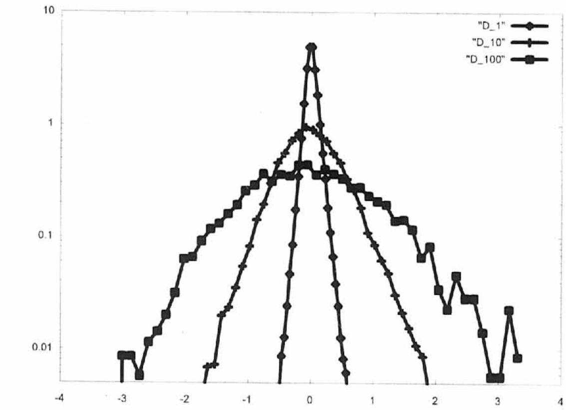

Figure 1 Probability density of velocity difference of turbulence, obtained in the Multi-fan turbulence wind tunnel in Miyazaki University. Experiments were done in June 2002, for $Re_\lambda = 376$, and the sampling frequency 10kHz. $P(\Delta v)$ versus Δv. The three lines correspond to $\Delta r = 1$, 10, 100 (in unit of 6.8×10^{-4} meters) from inner to outer.

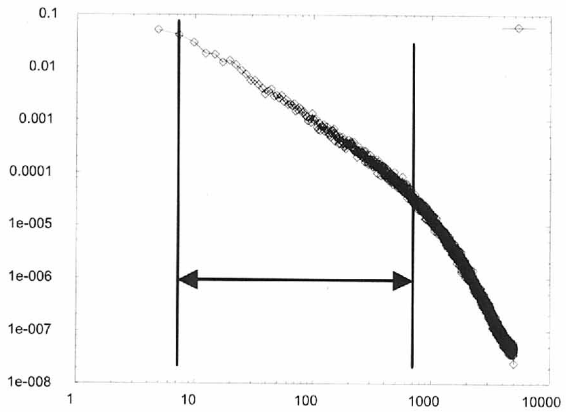

Figure 2 Power spectrum S(f) of turbulence data as a function of frequency f in log-log plot. The slope of the center part is -5/3 and can be regarded as the inertial range. This range extends from f=7 to 700 [Hz].

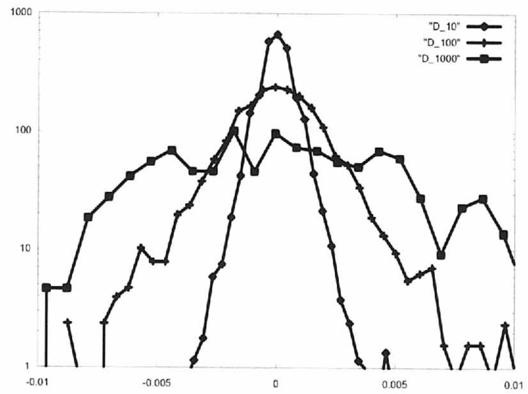

Figure 3 Probability density distribution of US Dollar vs. German Mark exchange rates from October 1, 1992 to September 30, 1993. We have used the first 250,000 data points. $P(\Delta x)$ versus Δx are plotted. Three lines denote $\Delta t = 10$, 100, 1000 [tick] from inner to outer.

3. Scale Invariant Region

Lévy stable distribution

$$L_{\alpha,\beta}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\beta|k|^{\alpha}+ikx}\,dk \qquad (1)$$

is self-similar under the following scale redefinition

$$L_{\alpha,\lambda\beta}(x) = \lambda^{-\frac{1}{\alpha}} L_{\alpha,\beta}(\lambda^{-\frac{1}{\alpha}} x) \qquad (2)$$

Using this, we can view the scale dependence of the distribution in Figure 1 and Figure 2 by means of probability of return to the origin $L_{\alpha,\beta}(0)$ such as

$$L_{\alpha,\lambda\beta}(0) = \lambda^{-\frac{1}{\alpha}} L_{\alpha,\beta}(0) \qquad (3)$$

Taking logarithm of both hand sides, the rate of return to the origin vs. λ becomes a straight line within the range of resolution in which the scaling property holds. In such a case the index α that characterizes Lévy distribution can be obtained from the slope as $-1/\alpha$ [5]

Figure 4 shows the result of foreign exchange rate, and Figure 5 shows the result of turbulence. Scale invariant region for foreign currency exchange rate is $10<\Delta t<150$, where the slope of the straight graph is approximately equal to -0.462 and the corresponding value of index α is approximately 2.16. For turbulence scale invariant region is $10<\Delta r<100$, where the slope of the straight line is approximately -0.376 and the corresponding value of index α is approximately 2.66.

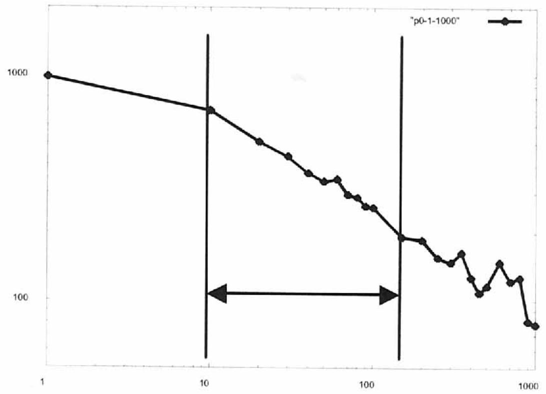

Figure 4 Probability of return to the origin P(0) for foreign currency exchange rates as a function of Δt. In log-log plot, the slope at Δt=10-150 is approximately –0.462 and the corresponding value of index α is approximately 2.16.

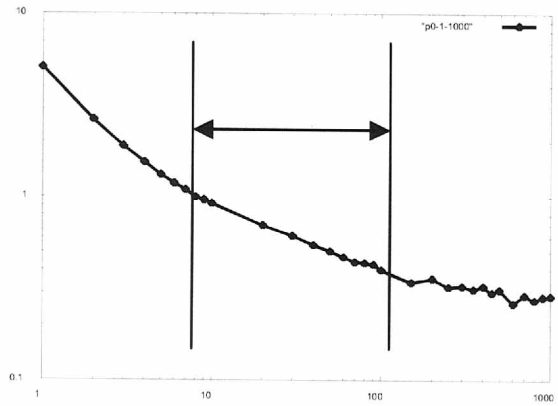

Figure 5 Probability of return to the origin P(0) for turbulence as a function of Δr. In log-log plot, the slope at Δr =10-100 is approximately -0.376 and the corresponding value of index α is approximately 2.66.

The index α in Figures 4 and 5 resulted in the numbers larger than 2. We have estimated this index in two different methods, by fitting the whole probability density by means of Lévy stable distribution in order to find the best fitting parameters, and by means of exponential distribution function defined as follows:

$$f(x) = \frac{\alpha\beta^{1/\alpha}}{2\Gamma(1/\alpha)} \exp(-\beta|x|^{\alpha}) \qquad (4)$$

As Figure 6 shows, the values of index α lies between one and two as expected. Although Figures 4 and 5 successfully extracted the scaling regions, the values of α estimated in this method may not be suitable to determine absolute values of the index.

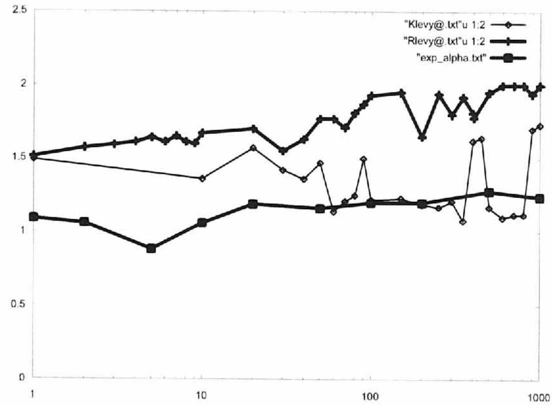

Figure 6 The index α as a function of scale parameters. Foreign exchange data fitted by the Lévy distribution is represented by a fine line, and turbulence data fitted by the Lévy distribution in the upper thick line and fitted by the exponential distribution in the lower thick line. All cases are consistent with $1<\alpha<2$.

4. Comparing Turbulence and Foreign Exchange

We can compare the two time series, turbulence and foreign exchange rate in each of the scale invariant region. It is known that the n-th moment of three-dimensional, isotropic, homogeneous turbulence follow Kolmogorov's scaling law within the inertial region :

$$\left\langle (\Delta v)^n \right\rangle \propto (\Delta r)^{n/3} \qquad (5)$$

In Figure 7, the left hand side of Equation (5) vs. Δr are plotted in log-log scale. For n=2, 4, 6, the slope in the range of 10< Δr <100 as a function of n increases monotonically. The center part of the region 1<Δr<1000 corresponds to the scaling region in Figure 5.

To be more quantitative, we compute the slopes by the least square fit. The result for the range Δr =10-100 almost coincides with the Kolmogorov exponent, n/3.

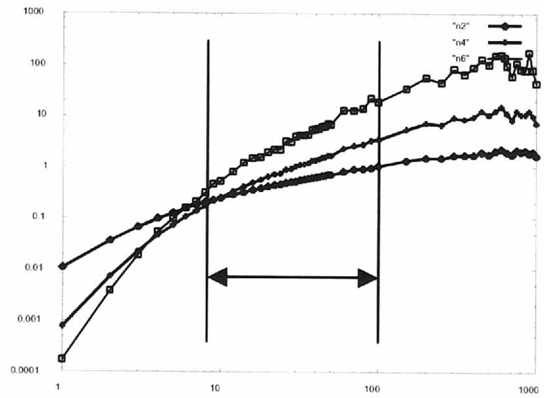

Figure 7 Log-log plot of Equation (5) for turbulence experiment. The three lines corresponds to n=2, 4, 6 in the order of increasing slope.

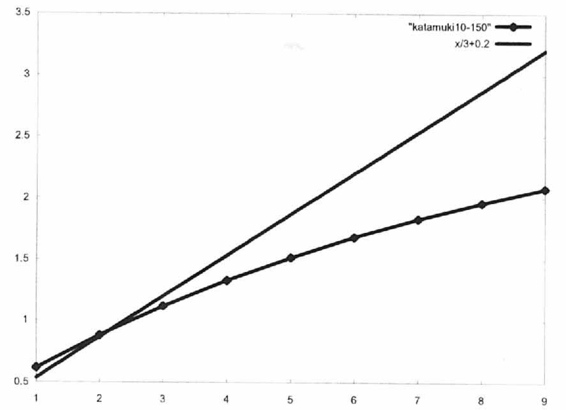

Figure 8 The index for the case of turbulence, in Eq. (5) as a function of n for the region Δr=10-100 (in unit of 6.8×10^{-4} meters). The straight line is Kolmogorov's value n/3.

For exchange rate, on the other hand, we expect to observe the moment relation:

$$\left\langle (\Delta x)^n \right\rangle \propto (\Delta t)^{\xi_n} \qquad (6)$$

If the index ξ_n indeed satisfies $\xi_n = n/3$, Eq. (5) and Eq. (6) are practically the same under the substitutions

$$\Delta x \rightarrow \Delta v \quad \text{and} \quad \Delta t \rightarrow \Delta r$$

as in Table 1.

Comparing Figures 7, 8 for turbulence and Figures 9, 10 for foreign exchange market, those two time series are indeed very similar within the limit of resolution in our analysis.

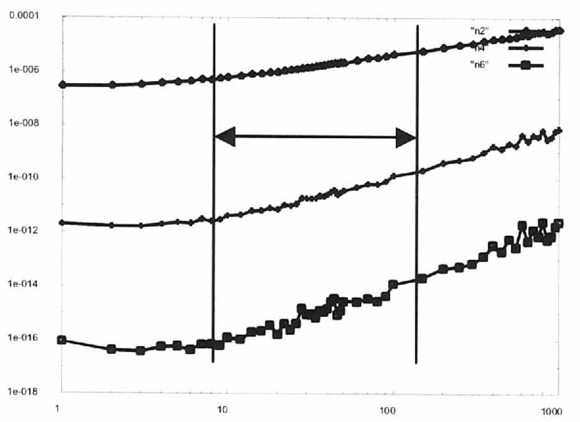

Figure 9 The index ξ_n in Eq.(6) for foreign exchange market as a function of n.

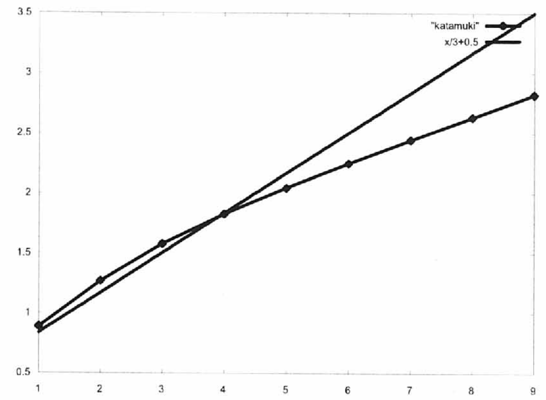

Figure 10 The index ξ_n in Eq.(6) for foreign exchange rate as a function of n, compared with ξ_n=n/3. For n=1-5 good overlap is observed.

5. Conclusion

We have examined the similarity between foreign exchange market and three-dimensional turbulence by means of direct analysis of large size data. For turbulence we have taken new set of data in our facility for Re_λ =376. These data generated in our 15 meters long multi-fan wind tunnel exhibit a clear signal of -5/3 rule for the inertial range extending to two digits of interval. We compared the probability density for various scale parameters, the center value of the probability density, which is equal to the probability of return to the origin in random walk and the scaling laws of moments. So far turbulence appears very similar to foreign exchange market.

References

[1] Manfred Shroeder, "Fracral, Chaos, Power Laws: Minuites From Infinite Paradise", W.H.Freeman and Co., N.Y., 1991.

[2] S. Gashghaie, W. Breymann, J. Peinke, P. Talkner and Y. Dodge, NATURE, vol. 381, pp.767-770, 1996.

[3] HFDF is a set of refined tick data prepared for academic use in order to promote competitive analysis. Professor Tamura of Institute of Mathematical Statistics, Japan made us use a part. In this report we used the first 250K data points in the exchange rate of US Dollar and German Mark from October 1, 1992 to September 30, 1993.

[4] These data were taken at Multi-fan Wind Tunnel (15m) in Miyazaki University by H. Miyagi and S. Ozono on May 22, 2002.

[5] Rosario N. Mantegna and H. Eugene Stanley, NATURE, vol. 376, pp.46-49, 1995.