Speed Control using Mamdani Fuzzy PI+PD Controller

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Abstract: This paper proposes a Mamdani fuzzy PI+PD controller for controlling the speed of two-inertia system. The simplicity in designing the proposed controller is described. System performances and stability of the Mamdani fuzzy PI+PD fuzzy control system are shown.

1. Introduction

It is known that the conventional proportional-integral (PI) and proportional-integral-derivative (PID) controller with well-tuned parameters will give good system performance [1]. However the controllers are not proper for the nonlinear or uncertain system. On the other hand, it is also known that the fuzzy controller both in Mamdani and Takagi-Sugeno type have been introduced for nonlinear or uncertain system. The fuzzy PID controllers have also been introduced recently [2, 3]. A fuzzy sliding mode approach for the two-mass system [4] has also reported in the literature.

Since the conventional PID controller consists of three terms: proportional term, integral term and derivative term, the three inputs of the fuzzy PID controller will be needed. This will lead to the difficulty and complexity in designing the fuzzy control rules and membership functions.

In order to avoid those difficulty and complexity in designing fuzzy PID controller, this paper proposes a controller called the Mamdani fuzzy PI+PD controller, which is the modification of the conventional PID structure. The conventional PID structure is first divided into two parts, PI controller part and PD controller part. Then the structure of the PID controller after rearranging is a parallel structure. The structure of PI and PD controller will be constructed to be a fuzzy PI controller and fuzzy PD controller. Consequently, the inputs of the fuzzy PI and fuzzy PD controllers will consist of two inputs, error and change of error. The merit points of the Mamdani fuzzy PI+PD controller can be stated as follows:

(1) The two inputs of each part of the fuzzy controller consist of the same inputs. That is error and change of error.

(2) The same control rules and membership functions of the two fuzzy controllers can be designed.

(3) The stability of a closed-loop system can be guaranteed by utilizing the small gain theorem.

Hence, it is easily to adjust the Mamdani fuzzy PI+PD controller in order to meet the desired system performances and stability.

2. Structure of Mamdani Fuzzy PI+PD Control System

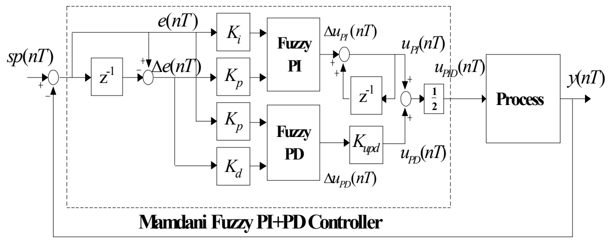


Fig. 1. Structure of Mamdani Fuzzy PI+PD Control System.

The block diagram of a Mamdani fuzzy PI+PD control system consisting of a process and the proposed Mamdani fuzzy PI+PD controller is illustrated in Fig. 1. The structure of the proposed controller is shown in the dashed line. The integral gain K_i and proportional gain K_p are the fuzzy control gains for the Mamdani fuzzy PI controller. The proportional gain K_p, derivative gain K_d and output gain K_{upd} are the fuzzy control gains for the Mamdani fuzzy PD controller. The control signal $u_{PID}(nT)$ of the Mamdani fuzzy PI+PD controller is then proposed as

$$u_{PID}(nT) = \frac{u_{PI}(nT) + u_{PD}(nT)}{2} \tag{1}$$

where

$$u_{PI}(nT) = \Delta u_{PI}(nT) + u_{PI}(nT - T) \tag{2}$$

and

$$u_{PD}(nT) = \Delta u_{PD}(nT) K_{upd} \tag{3}$$

are the new outputs of the fuzzy PI and fuzzy PD controllers, respectively. The derivation of the fuzzy PI and fuzzy PD controller structures are described in the following subsections.

2.1. Structure of Fuzzy PI Controller

In order to find the structure of the Mamdani fuzzy PI+PD controller, which is constructed from fuzzy PI and fuzzy PD controllers, the control signal of the conventional PID controller is first given by

$$u_{PID}^{c}(t) = K_c\left[e(t) + \frac{1}{T_i}\int_0^t e(t)dt + T_d\frac{de(t)}{dt}\right] \tag{4}$$

where K_c is the proportional gain, T_i is the integral time, T_d is the derivative time and $e(t)$ is the error signal. By rearranging the structure of the above conventional PID controller, it becomes

$$u_{PID}^{c}(t) = \frac{u_{PI}^{c}(t) + u_{PD}^{c}(t)}{2} \tag{5}$$

where

$$u_{PI}^{c}(t) = K_c\left[e(t) + \frac{2}{T_i}\int_0^t e(t)dt\right] \tag{6}$$

and

$$u_{PD}^{c}(t) = K_c\left[e(t) + 2T_d\frac{de(t)}{dt}\right]. \tag{7}$$

Approximating the integral term of equation (6) by the trapezoidal summation yields

$$u_{PI}^{c}(nT) = K_i e(nT) + K_P\Delta e(nT) + u_{PI}^{c}(nT - T) \tag{8}$$

where the error $e(nT) = sp(nT) - y(nT)$, the change of error $\Delta e(nT) = e(nT) - e(nT - T)$, $K_i = \frac{2T}{T_i}K_c$, $K_p = \left(1 - \frac{T}{T_i}\right)K_c$ and T is the sampling time. When the term $K_i e(nT) + K_p\Delta e(nT)$ is replaced by $\Delta u_{PI}(nT)$, the new output fuzzy PI controller can be obtained as

$$u_{PI}(nT) = \Delta u_{PI}(nT) + u_{PI}(nT - T) \tag{9}$$

where $e(nT)$ and $\Delta e(nT)$ are the inputs and $\Delta u_{PI}(nT)$ is the output change of the fuzzy PI controller.

2.2. Structure of Fuzzy PD Controller

The structure of fuzzy PD controller can be found from equation (7) by approximating the derivative term with two-point difference form. Hence ,

$$u_{PD}^{c}(nT) = \left(K_p e(nT) + K_d\Delta e(nT)\right)K_{upd}\ , \tag{10}$$

where $K_d = \frac{2T_d}{T}K_p$ and $K_{upd} = \frac{T_i}{T_i - T}$. When the term of $K_p e(nT) + K_d\Delta e(nT)$ is replaced by $\Delta u_{PD}(nT)$, the new output fuzzy PD controller becomes

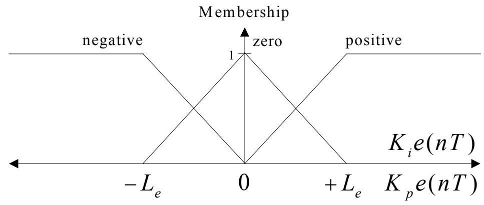
$$u_{PD}(nT) = \Delta u_{PD}(nT)K_{upd}\ , \tag{11}$$

where $e(nT)$ and $\Delta e(nT)$ are the inputs and $\Delta u_{PD}(nT)$ is the output change of the fuzzy PD controller.

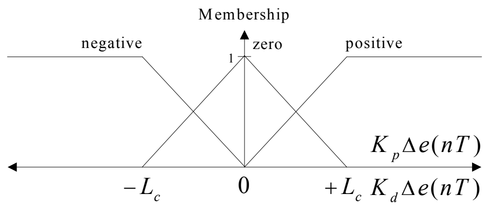
3. Mamdani Fuzzy PI+PD Controller Design

3.1. Fuzzification

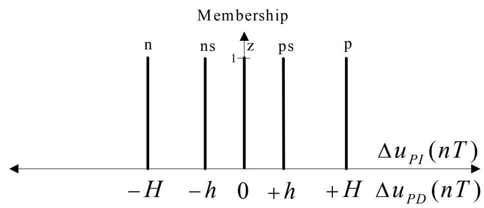
In the fuzzifcation step, the error $e(nT)$ and the change of error $\Delta e(nT)$ of the fuzzy PI controller and fuzzy PD controller are employed. The input membership functions and the output change membership functions are shown in Fig. 2 (a), (b) and (c) respectively, where L_e, L_c, H and h are the arbitrary constants.



(a) Error membership functions.



(b) Change of error membership functions.



(c) Output change membership functions.

Fig. 2. Membership functions.

In Fig. 2 (c), terminology is **n** for negative, **ns** for negative small, **z** for zero, **ps** for positve small and **p** for positive.

3.2. Fuzzy Control Rules

Based on these membership functions, the fuzzy control rules that used for designing the fuzzy PI or PD controllers are shown in Table 1**.** The number of the control rules is 9.

Table 1 Fuzzy control rules

error \ change of error	negative	zero	positive
negative	n	ns	z
zero	ns	z	ps
positive	z	ps	p

3.3. Defuzzification

The output change $\Delta u_{PI}(nT)$ and $\Delta u_{PD}(nT)$ of the fuzzy PI controller and the fuzzy PD controller, respectively, are obtained by employing the centroid defuzzifier as

$$\Delta u_{PI}(nT)=\frac{\sum_{r=1}^{9}\mu_{PI_r}\Delta u_{PI_r}}{\sum_{r=1}^{9}\mu_{PI_r}} \text{ and } \Delta u_{PD}(nT)=\frac{\sum_{r=1}^{9}\mu_{PD_r}\Delta u_{PD_r}}{\sum_{r=1}^{9}\mu_{PD_r}}, \quad (12)$$

where μ_{PI_r} and μ_{PD_r} are the membership values at the r^{th} rule, Δu_{PI_r} and Δu_{PD_r} are the singleton output fuzzy sets at the r^{th} rule of the fuzzy PI and fuzzy PD controllers.

3. Stability Analysis

In this section, the stability analysis of Mamdani fuzzy PI+PD control system illustrated in Fig.1 is shown by using the Small Gain Theorem [5], which is a generally applicable tool in nonlinear control theory.

Let C_{PI}, C_{PD} and P denote the fuzzy PI controller, the fuzzy PD controller and a process respectively. Then the process output is $P(u_{PID}(nT))$. When the fuzzy PI controller is treated as a nonlinear operator mapping inputs $e(nT)$ and $\Delta e(nT)$ to the output change of fuzzy PI controller $\Delta u_{PI}(nT)$, therefore, $\Delta u_{PI}(nT) = C_{PI}e(nT)$ can be obtained. In the same manner, when the fuzzy PD controller is treated as a nonlinear operator mapping inputs $e(nT)$ and $\Delta e(nT)$ to the output change of fuzzy PD controller $\Delta u_{PD}(nT)$, then $\Delta u_{PD}(nT) = C_{PD}(e(nT))$.

In order to analyze the stability of the Mamdani fuzzy PI+PD control system, the $\Delta u_{PI}(nT)$ and $\Delta u_{PD}(nT)$ using defuzzification algorithm (12) will be found by decomposing the input space of the input membership functions into 28 different input combinations (ICs) as shown in Fig. 3.

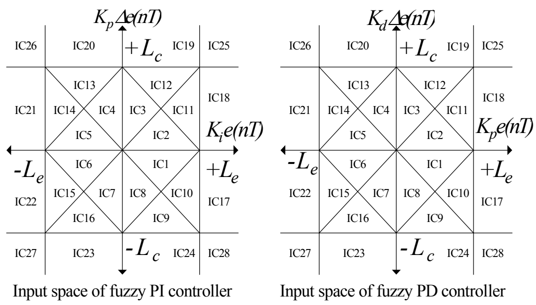


Fig. 3. Input space of fuzzy PI and fuzzy PD controller.

Next, determine $\|\Delta u_{PI}(nT)\|$ and $\|\Delta u_{PD}(nT)\|$ so that the maximum value α_{PI} and α_{PD} of each IC can be obtained and summarized in Table 2. Then the stability condition is expressed as

$$\left(\frac{\alpha_{PI}+\alpha_{PD}}{2}\right)\cdot\|P\| < 1 \quad (13)$$

where $\|P\|$ is operator norms of a given process.

Table 2 α_{PI} and α_{PD} for fuzzy PI and fuzzy PD controllers

Group	IC#	α_{PI}	α_{PD}
A1	IC1, IC4, IC5, IC8	$\frac{\lvert mhK_i + hK_p\rvert}{2mL_e}$	$\frac{\lvert mhK_p + hK_d\rvert}{2mL_e}K_{upd}$
A2	IC2, IC6	$\frac{\lvert mhK_i + (H+h)K_p\rvert}{2mL_e}$	$\frac{\lvert mhK_p + (H+h)K_d\rvert}{2mL_e}K_{upd}$
A3	IC3, IC7	$\frac{\lvert m(H+h)K_i + hK_p\rvert}{2mL_e}$	$\frac{\lvert m(H+h)K_p + hK_d\rvert}{2mL_e}K_{upd}$
A4	IC9, IC10, IC13, IC14	$\frac{\lvert mhK_i + hK_p\rvert}{mL_e}$	$\frac{\lvert mhK_p + hK_d\rvert}{mL_e}K_{upd}$
A5	IC11, IC15	$\frac{\lvert -mhK_i + (H-h)K_p\rvert}{mL_e}$	$\frac{\lvert -mhK_p + (H-h)K_d\rvert}{mL_e}K_{upd}$
A6	IC12, IC16	$\frac{\lvert m(H-h)K_i - hK_p\rvert}{mL_e}$	$\frac{\lvert m(H-h)K_p - hK_d\rvert}{mL_e}K_{upd}$
A7	IC17, IC21	$\frac{\lvert hK_p\rvert}{mL_e}$	$\frac{\lvert hK_d\rvert}{mL_e}K_{upd}$
A8	IC18, IC22	$\frac{\lvert (H-h)K_p\rvert}{mL_e}$	$\frac{\lvert (H-h)K_d\rvert}{mL_e}K_{upd}$
A9	IC19, IC23	$\frac{\lvert (H-h)K_I\rvert}{L_e}$	$\frac{\lvert (H-h)K_p\rvert}{L_e}K_{upd}$
A10	IC20, IC24	$\frac{\lvert hK_I\rvert}{L_e}$	$\frac{\lvert hK_p\rvert}{L_e}K_{upd}$
A11	IC25, IC26, IC27, IC28	0	0

The α_{PI} and α_{PD} for the stability judgment are chosen according to the parameters of K_p, K_i, K_d, K_{upd}, H, h and $m = L_c/L_e$.

4. Experimental Results

The experimental results in controlling the motor speed ω_m of the two-inertia system by using the Mamdani fuzzy PI+PD controller is shown in this section. The block diagram of the two-inertia system to be controlled is shown in Fig. 4.

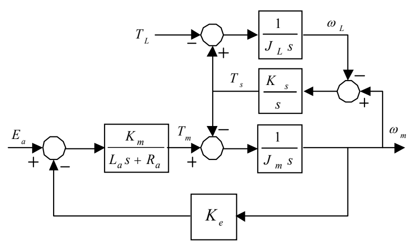


Fig. 4. Block diagram of the two-inertia system.

The transfer function from the reference input to the motor speed is given by

$$P(s)=\frac{\omega_m(s)}{E_a(s)}=\frac{a_2s^2+a_0}{b_4s^4+b_3s^3+b_2s^2+b_1s+b_0} \quad (14)$$

where $a_0=\frac{K_mK_s}{J_mJ_LL_a}$, $a_2=\frac{K_m}{J_mL_a}$, $b_0=\frac{K_mK_eK_s}{J_mJ_LL_a}$, $b_1=\frac{K_sR_a(J_m+J_L)}{J_mJ_LL_a}$, $b_2=\frac{K_sL_a(J_m+J_L)+K_mK_eJ_L}{J_mJ_LL_a}$, $b_3=\frac{R_a}{L_a}$, $b_4=1$ and the parameters of the two-inertia system are shown in Table 3.

Table 3 Parameters of the two-inertia system

R_a Ω	L_a mH	K_e $v/krpm$	K_m $N-cm/A$	J_m $kg-cm^2$	J_L $kg-cm^2$	K_s $N-m/rad$
0.25	0.10	4	3.8	0.7	0.7	15

By taking the z-transform to equation (14) and using sampling time $T = 0.001$ second, yields

$$P(z) = \frac{0.01354z^3 - 0.02098z^2 + 0.001418z + 0.006066}{z^4 - 2.949z^3 + 3.085z^2 - 1.167z + 0.08208}. \quad (15)$$

Hence,

$$\|P(z)\| = \sup_{|z|=1} |P(z)| = 0.25$$

can be obtained.

The parameters of the conventional PID controller K_c, T_i and T_d are first obtained from the Ziegler-Nichols tuning method and are tuned later until the system response has no overshoot. Hence, K_c, T_i and T_d are 0.09, 0.9×10^{-2} second and 0.094×10^{-3} second respectively. By using parameters of PID controller thus obtained and $T = 0.001$ second, the gains of Mamdani fuzzy PI+PD controller become $K_p = 0.08$, $K_i = 0.02$, $K_d = 0.015$ and $K_{upd} = 1.125$. The constants H, h and $m = L_c/L_e$ are chosen to be $L_e = L_c = 0.5$, $H = 4$ and $h = 1$. Therefore, the Mamdani fuzzy PI+PD control system is stable according to equation (13), when $\|P(z)\| = 0.25$, $\alpha_{PI}^{\max} = 0.48$ and $\alpha_{PD}^{\max} = 0.6$.

5.1. Step Response with Disturbance Torque

The Mamdani fuzzy PI+PD controller and the PID controller designed in section 5 will be implemented to control the speed of the two-inertia system at 1 krpm. The controlled systems have been also applied the disturbance torque T_L at 1.4 seconds. The experimental results are shown in Fig. 5.

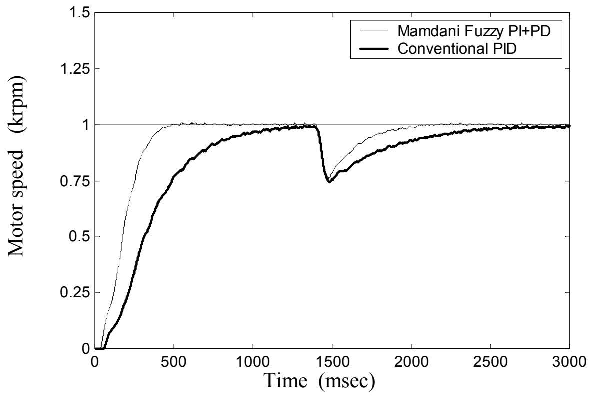


Fig. 5. Step responses with disturbance torque.

It is seen that the response of the proposed fuzzy PI+PD control system is faster the response of the PID control system. However, both control systems can reduce the vibration of the two-inertia system. It is also observed that the effect of the disturbance torque can be fast rejected by the Mamdani fuzzy PI+PD controller.

5.2. Step Response due to Motor Speed Change

In this section, the Mamdani fuzzy PI+PD controller is used to control the speed of the two-inertia system at 1.3 krpm and 0.7 krpm without changing the controller parameters in order to show the effectiveness of the controller.

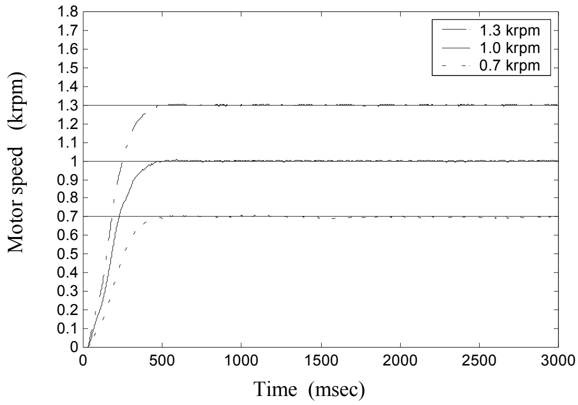


Fig. 6. Step responses for reference speed has been changed.

The experimental results are shown in Fig. 6 and are seen that the proposed controller has the ability to control the different speed without overshoots and steady state errors of the step responses.

6. Conclusions

The Mamdani fuzzy PI+PD controller has been proposed and implemented to control motor speed of the two-inertia system. The simplicity in designing the proposed controller to meet the desired control system, and stability have also been shown. The effectiveness of the proposed controller has been demonstrated by the experimental results.

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