International Symposium on Nonlinear Theory and its Applications
Xi'an, PRC, October 7-11, 2002

A Numerical Analysis on the Triangular Arbitrage Chances

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Abstract

Absence of an arbitrage chance is a basic assumption of financial technology. It is not guaranteed, however, for this principle to hold under fine resolution of time. This issue has to be studied more seriously under the circumstances that transactions are mostly done electrically in the near future by means of computer networks. We have done a numerical study on this problem by looking at the tick data of three foreign currency exchange rates and picked up all the arbitrage chances by automatically processing large files in a UNIX workstation. Three facts have been extracted from this line of research. One is the rate of triangular arbitrage chances in the entire quotes. We have found that there exist quite large portion of arbitrage chances out of the entire data The second is the relaxation time of such chances, i.e. the time before such chance vanishes after its appearance. We have identified the probability distribution of this quantity, which tells us that the most frequent value of the relaxation time is 6 seconds with distinctly larger probability of occurrence compared to the minimum value of the relaxation time, 2 seconds. The third is the accumulated probability of the rate of appearance, which lies very close to Zipf's law.

Key words Arbitrage Chance, Triangular Arbitrage, Tick Data, Zipf's Law, HFDF

1. Introduction

Growing number of people have begun studying time series of high-frequency financial data quantitatively, by using various mathematical as well as computational techniques in order to see how the price moves. It is expected from those efforts that the theory of price formation could be renewed from the conventional theory of economics and financial technologies. Early attempts began in Europe that led to the First Conference on High Frequency Data in Finance (HFDF) in 1993. In the process of preparing this, the standard data set called HFDF 93 was established in order to make competitive research possible. In this paper we report our recent analysis on arbitrage chance occurring among three currencies, by using JPY-GM-USD from October 1, 1992 to September 30, 1993 in HFDF93.

It is normally assumed that, under a competitive market, there is no chance to have outrageously high or low price. The reason is if the price is too high then most buyers do not consider buying them and, if the price is too low, many buyers would rush to buy then and the price will not stay as the same level very long. In short the price has to stay in the reasonable range in order to be sold under competition. Investors are supposed to play a perfectly fair gamble under an ideal market. This situation can be compared to physical material in which the temperature is the same everywhere after an equilibrium condition is achieved.

An arbitrage chance is an opportunity to take a merit by simply selling and buying the same thing at different prices. There is no such chance as far as the market is under perfect equilibrium. However, occasionally temperature difference is created locally when the market moves vigorously and stays nonzero until the equilibrium is recovered. We expect to observe such arbitrage chances in high-frequency data such as HFDF93. In this paper we study the rate of occurrence and the duration of relaxation time of such chances by using high-speed computers.

We search for the moments when three foreign exchange rate to satisfy one of the following conditions:

u(t)=(AB.bid)*(BC.bid)/(AC.ask) > 1+TF (1)

v(t)=(AC.bid)/(BC.ask)/(AB.ask) > 1+TF (2)

where TF stands for the transaction fees. Here u(t) represents the rate of gain for triangular transaction from currency A to B then to C at time t, and v(t) means the rate of gain for the inverse transaction, A to C then to B at time t. Since the seller demands higher price than the buyer does, 'ask' quote is always higher than 'bid' quote at the same moment. Note that u(t)<1 and v(t) <1 simultaneously for most times.

2. Observation of Arbitrage Chance

We consider a triangle transaction of the three currencies, Japanese Yen (Y), US Dollar ($) and German Mark (M) from HFDF93. For example, data at the beginning of October 1, 1992 are shown in Table 1.

Here at t=0.01.02, u=0.99983 and v=0.99828 and at t=0.0518 u=0.99972 and v=0.99873. This is a normal situation in that u and v are smaller than one at most times. Occasionally, however, a moment appears in which either u or v exceeds one, which can be regarded as a chance to take arbitrage.

time	¥/$.bid	¥/$.ask	time	mark/$.bid	mark/$.ask	time	¥/mark.bid	¥/mark.ask
0:00:54	119.93	120	0:01:00	1.411	1.412	0:01:02	84.9	84.95
0:01:06	119.93	120.03	0:01:18	1.4115	1.412	0:05:18	84.93	84.96

Table 1 Example of Exchange Rates

Practically, of course, we need consider the effect of transaction time being finite and the transaction fee. We neglect all those in this analysis.

A systematic study of these chances requires computing facility, since more than a million ticks of data are to be processed for one year, although Excel is more convenient to use for the duration of one week or less. The time series of u(t) for one day of October 1, 1992 are shown in Figure 1. This ratio is mostly smaller than one, but we have detected nonzero chances of u(t) > 1.

To be more quantitatively, we examine the probability density distribution of u(t). The mean and the standard distribution calculated from the data are 0.99904 and 0.00072, respectively. The solid line in Figure 2 is the best fitting normal distribution. The mean of and the standard deviation of which are 0.99904 and 0.00052, respectively. The best fitting Levy distribution is $\alpha = 1.9,\ \beta = 0.00000024$.

The portion of arbitrage chance indicated by u>1 is P($u \geq 1$)=0.052, equivalent to 75 minutes of the arbitrage chance was observed on this day. In case if the transaction fee is 0.1% (or 0.2%), then the chance becomes P(u>1.001)=0.011, or P(u>1.002)=0.0054. Without transaction fee, one is supposed to gain 0.1% out of the investment with probability 1%, and gain 0.2% with probability 0.5%.

We now move to study much larger data extending for one year and compare the daily average to the previous result of day average.

The result is shown in Figure 3. The distribution can be fitted by the normal distribution with mean and standard deviation being equal to 0.999153 and 0.00043, respectively. Also the best fitting Levy distribution is $\alpha = 1.5,\ \beta = 0.0000044$. The portion of arbitrage chance represented by the region of u>1 is computed as P($u \geq 1$)=0.035, equivalent to 48 minutes per day in average. Also P(u>1.001)=0.0058, P(u>1.002)=0.0027 indicate that the chance to gain the profit of 0.1 % is 0.6% and the chance to gain of 0.2% is 0.3%.

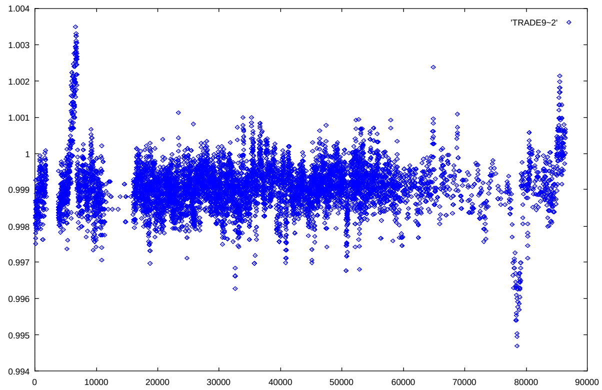

Figure 1: Arbitrage gain u(t) calculated from Eq.(1). Horizontal Axis: Time in second starting midnight of October 1, 1992. Vertical Axis: Arbitrage gain u(t).

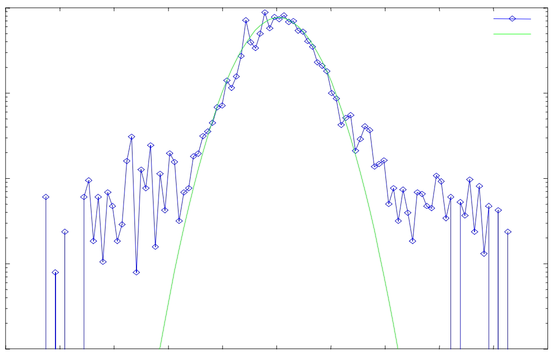

Figure 2 Probability density distribution P(u) of arbitrage gain u(t) for one day on October 1,1992. The solid line is the best fitting normal distribution.

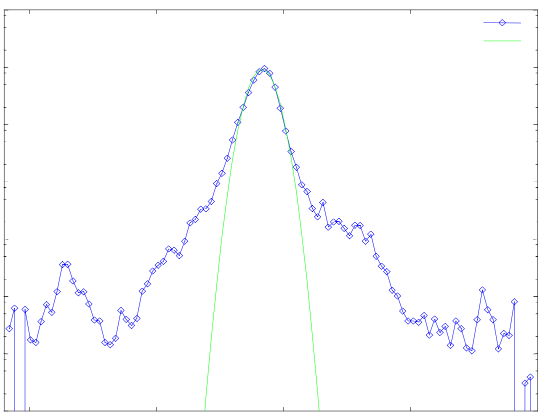

Figure 3 Probability density distribution P(u) of arbitrage rate u(t) for one year from October 1, 1992 to September 30, 1993. The solid line is the best fitting normal distribution (mean=0.99915, standard distribution = 0.00043)

3. Relaxation Time of Arbitrage Chance

We have confirmed the existence of arbitrage chance in triangular trades. However chances that immediately vanish after one second are hard to be used. We now consider how long such a chance lasts once it appears.

Figure 4 shows that most arbitrage chances vanish in a short time. On this particular day 80% of the total chances are shorter than 20 seconds. However there is one example that is longer than 700 seconds (more than ten minutes!) Figure 5 shows the probability P(T) for a triangular arbitrage chance last for T seconds after its appearance. Also Figure 6 indicates the accumulated probability of the same quantity.

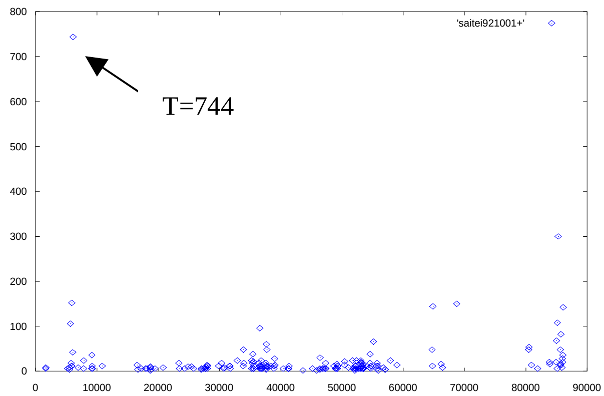

Figure 4 Duration (T) of triangular arbitrage observed on October 1, 1992 as a function of time.

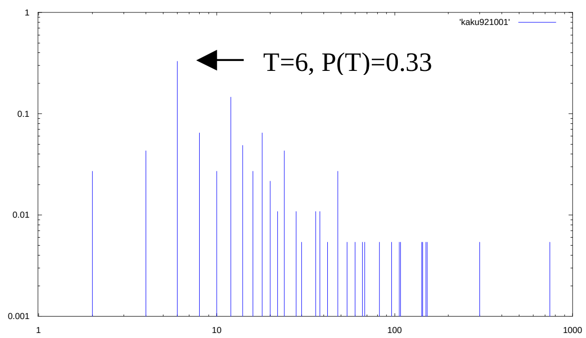

Figure 5 Duration (T in seconds) of arbitrage chance after its appearance as a function of the time t when it started.

At first sight, we are inclined to assume based on our common sense that any arbitrage chance vanishes immediately and the probability of the smallest T would be the largest. Contrary to our expectation, P(6)=0.33

was the largest probability as far as the particular day we considered was concerned. Later we will consider the result of one year.

We now introduce the accumulated probability P(>T) which shows the total probability of T being larger than a particular value T. These are plotted in log-log scale in Figure 6. This graph can be viewed as a straight line. Moreover, the best fitting slope calculated by the least-square fit from T=10 to T=100 is –1.06, which is the Zipf's law !

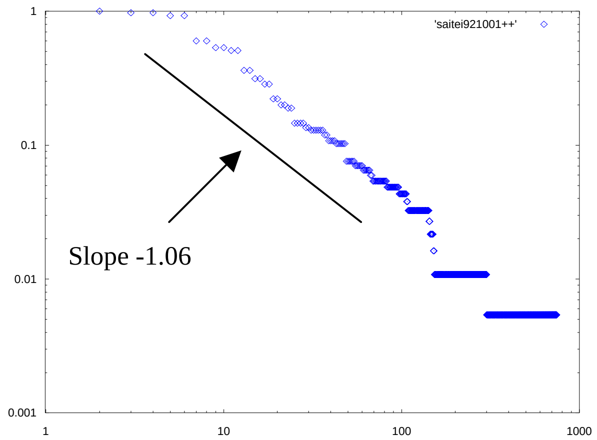

Figure 6 Accumulated probability distribution P(>T) of the time of duration T appeared in one day of October 1, 1992 in log-log plot as a function of T.

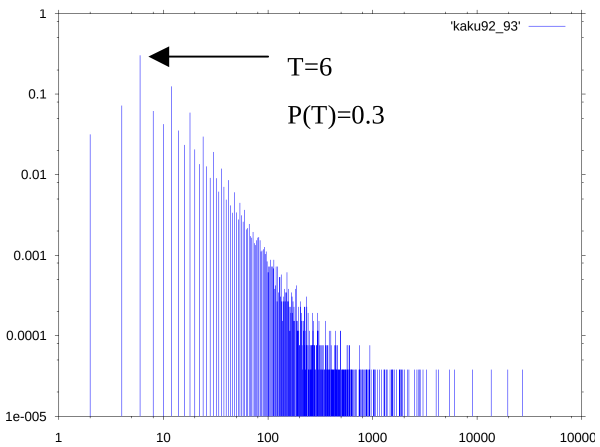

Figure 7 Relaxation time T of arbitrage chances versus its probability distribution P(T) in log-log scale. (Averaged result over a year 1992/10/1-1993/9/30)

The most frequently appeared relaxation time was T=6, with probability P(T=6)=0.302, approximately the same as the particular one day that we have considered. Also the largest T was observed to be 27,134 seconds (approximately 452 minutes, which is equivalent to 7 + half hours). Average relaxation time <T> is 27.5 seconds, which was surprisingly large, considering the wide belief that the arbitrage chance does not exist. The accumulated probability P(>T) for one year is plotted in Figure 8, where the line is more straight but the best fit slope from T=10 to T=100 is –1.25.

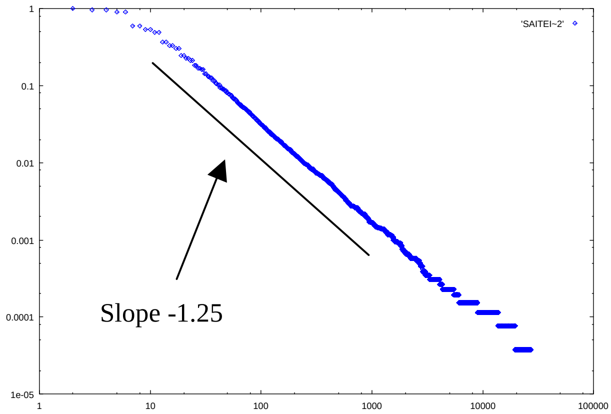

Figure 8 Accumulated probability distribution P(>T) of the time of duration T in log-log plot for one year from October 1, 1992 to September 30, 1993.

4. Conclusion

We have found that the arbitrage chances exist with much greater probabilities if triangle transactions can be performed without transaction time and fee. The result of our numerical analysis shows that surprisingly large amount of chances indeed exist in the tick level of resolution. Moreover, the relaxation time was longer than we naively expect. The most possible relaxation time was 6 seconds, which is amount to one third of the whole chances. The accumulated probability P(>T) approximately follows the Zipf's law.

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