International Symposium on Nonlinear Theory and its Applications
Xi'an, PRC, October 7–11, 2002

Searching for Parameter Values for Non-periodic Dynamics in Chaotic Neural Networks

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1. Introduction

We investigate the possibility of a searching method for non-periodic dynamics in chaotic neural networks. For low dimensional systems, one can use bifurcation diagrams to find sets of parameter values for the dynamics that one wishes, however, it is difficult to apply this method for high dimensional systems like chaotic neural networks[1][2] with many units. Therefore, we apply searching algorithms to find such sets of parameter values for chaotic neural networks.

In this paper, sets of parameter values for an associative chaotic neural network are searched so that it retrieves stored patterns non-periodically. The associative chaotic neural network is composed of chaotic neuron models with synaptic weights that are determined by the symmetric auto-associative matrix of patterns to be stored as memories to the network. It has already been reported that the associative chaotic neural network shows apparently non-periodic dynamics with certain parameter values[3], however, it is relatively difficult to find the sets of parameter values that cause such non-periodic dynamics.

The genetic algorithm (GA) [4] is applied to find sets of parameter values so that the associative chaotic neural network shows dynamical association. Here, the dynamical association means that the retrieval sequence which includes the stored patterns and the output is not a simple periodic sequence but possibly a chaotic one. In order to search for such sets of parameter values, a quasi-energy function and a firing rate are used to determine the fitness for the GA. A high value of the fitness is given when the network does not stay in the same state, which is measured by the quasi-energy function, in successive iterations. The firing rate is used to detect the retrieval of the stored pattern then the high value of the fitness is also given for the network with such parameter values.

2. Associative Chaotic Neural Network

The dynamics of the associative chaotic neural network model is described by the following equations:

$$x_i(t+1) = f\{\eta_i(t+1) + \zeta_i(t+1)\}, \qquad (1)$$

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{16} w_{ij} x_j(t), \qquad (2)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i \qquad (3)$$

where $x_i(t)$ denotes output of the ith neuron at discrete-time t. The variables $\eta_i(t)$ and $\zeta_i(t)$ denote internal states for feedback inputs from the constituent neurons and for refractoriness, respectively. k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively. The parameters w_{ij} and a_i denote the synaptic weights from the jth neuron to the ith neuron and the sum of the threshold and the external inputs to the ith neuron ($a_i = a$ for every neuron in this paper), respectively. Output function of the neuron is denoted by f; in this paper, we use the logistic function represented by

$$f(y) = \frac{1}{1+\exp(-y/\varepsilon)}, \qquad (4)$$

where ε is a parameter for the steepness of the function [1] [2].

The synaptic weights are determined according to the following symmetric auto-associative matrix of three 16-dimensional binary patterns to be stored as memories to the network [5] [6] [7] [8].

$$w_{ij} = \frac{4}{3} \sum_{p=1}^{3} (x_i^{(p)} - \overline{x})(x_j^{(p)} - \overline{x}) \qquad (5)$$

with $w_{ii} = 0$ where $x_i^{(p)}$ is the ith component of the pth stored pattern. $\overline{x}$ denotes spatially averaged value of the stored patterns. In the following numerical experiment, we use the three stored patterns shown in Fig.1.

It has been reported that the network exhibits non-periodic sequential patterns that include the stored patterns when the parameters of the refractoriness are set to certain non-zero values[2], however, it is relatively difficult to find the sets of parameter values that cause such non-periodic dynamics.

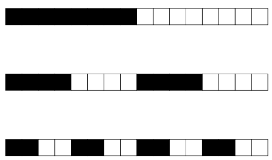

Figure 1: Stored patterns of the associative chaotic neural network. The filled and open squares represent 1 and 0, respectively.

3. Algorithm for the Parameter Search

In this paper we attempt to search for the parameter values for the associative chaotic neural network with 16 chaotic neurons[1][2]. The parameters to be searched for the network in this paper are k_f, k_r, a and α, therefore, the values of these parameters are directly coded into the chromosome. In the following numerical experiment in this paper, the searching ranges of these parameters are determined as $0.0 < k_f < 0.3$, $0.7 < k_r < 1.0$, $0.0 < a < 0.2$, $1.4 < \alpha < 2.0$. These ranges are determined by preliminarily obtained one-parameter bifurcation diagrams of the network.

The genetic operations in the GA are executed as follows. At first, the dynamics of the network with the parameter values, that are represented as the chromosome, are evaluated during 500 iterations. The evaluation is obtained by the following fitness function g,

$$g = \begin{cases} 1/|T-F| & (for\ T \neq F) \\ 2 & (for\ T = F) \end{cases}, \tag{6}$$

where T denotes the target frequency of retrieval of any stored patterns in the evaluation duration. F denotes the frequency of which the network satisfies all the following conditions in the evaluation duration.

Conditons for F

- For the quasi-energy, $E < -4.0$,
- The firing rate R is in the range of $0.4 < R < 0.6$,
- $|E(t) - E(t-1)| > 0.1$,
- $|R(t) - R(t-1)| > 0.01$.

The condition for the quasi-energy E is determined by considering the fact that the lower value of E tends to correspond to the retrieval of the stored patterns. The condition for the firing rate R is determined by the fact that spatial average of each stored pattern is 0.5. The 3rd and 4th conditions are added attempting to put higher score for the dynamical association than the conventional static association. Where, the quasi-energy E and the firing rate R are calculated by the following equations.

$$E(t) = -\frac{1}{2}\sum_{i \neq j}\sum_{j} w_{ij} x_i(t) x_j(t) - \sum_{i} a_i x_i(t), \tag{7}$$

$$R(t) = \frac{1}{16}\sum_{i=1}^{16} h(x_i(t)) \tag{8}$$

where h is a function that binarize the output x_i of the network. In this paper, we use the following function for h,

$$h(x) = \begin{cases} 1 & (for\ x > 0.5) \\ 0 & (otherwise) \end{cases}. \tag{9}$$

The fitness function with $T = 10$ is shown in Fig. 2.

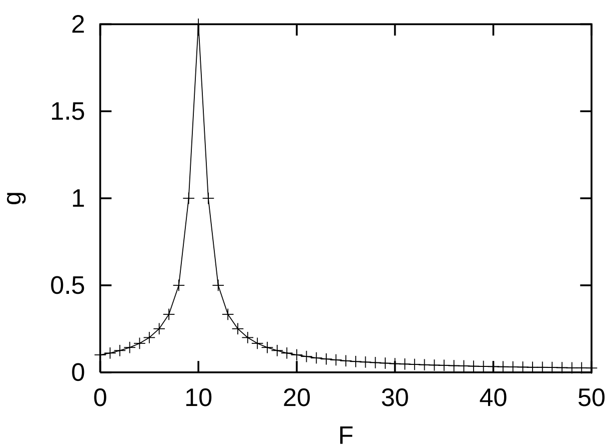

Figure 2: The fitness function with $T = 10$.

The individuals are ranked in descending order of the fitness g. Then the top two individuals are kept as they are in the next generation; i.e., the elite preservation strategy is applied to the population. The other individuals that are not selected as the elites are chosen as parents for the reproduction of the offspring. The frequency to be chosen as the parents are determined according to the value of the fitness g; i.e., the roulette selection is used for the selection process. The offsprings are generated from pairs of the parents by one-point crossover.

Finally, a randomly selected element of the chromosome of the candidate offspring is perturbed; namely, the mutation is applied. The probability of the mutation is fixed to 5% in this paper.

The genetic operations are repeated for predetermined times (generations). Then the finally obtained population is evaluated.

4. Examples of the Search Results

In this paper, 20 individuals and 20 generations are selected as the condition for the GA. Examples of the sets of parameter values obtained by the GA are shown in Table 1. These values are obtained by the GA with $T = 10$ during 500 iterations. Figures 3 and 4 show the time course of distances $d^{(p)}$ between the output pattern of the network and the three stored patterns of Fig.1 with the two sets of parameter values shown in Table 1, respectively. The distance is defined by

$$d^{(p)}(t) = \sum_{i=1}^{16} \left| x_i(t) - x_i^{(p)} \right| \qquad (10)$$

for the pth pattern, where the pth patterns for $p = 1, 2, 3$ represent the patterns of Fig.1, respectively. When the network retrieves the pth exact stored pattern and its reverse one, $d^{(p)}$ becomes 0 and 16, respectively. We see that the retrieval dynamics is non-periodic at least in the duration as shown in Figs.3 and 4. The networks with both sets of parameter values retrieve the stored patterns, however, the retrieval characteristics are different. Namely, Fig.3 shows instantaneous retrieval, on the other hand Fig.4 show that once the network retrieves a stored pattern, the output of the network fluctuates around the pattern for some duration. These examples show that the GA can find the parameter values for different types of dynamical association. Figures 5 and 6 show Lyapunov spectrum of the networks with the sets of parameter values of Example (1) and (2), respectively. We find that both dynamics shown in Figs. 3 and 4 are chaotic since both of the networks have positive Lyapunov exponents as shown in Figs. 5 and 6. Therefore, it is confirmed that the networks with these sets of parameter values realize the dynamical association with chaos.

These parameter values are obtained from ten trials for the GA with different initial conditions for the chromosome. Five trials (including above examples) out of the ten trials find sets of parameter values that causes non-periodic retrieval of at least one of the stored patterns, however, the other five trials fail to find such sets of parameter values.

Table 1: Examples of values of the parameters obtained by GA

Parameters	Example(1)	Example(2)
k_f	0.20000504531431	0.15667807216040
k_r	0.92970558846571	0.89988652911999
a	0.00329995753735	0.08156290184920
α	1.44376670483689	1.81896530364268

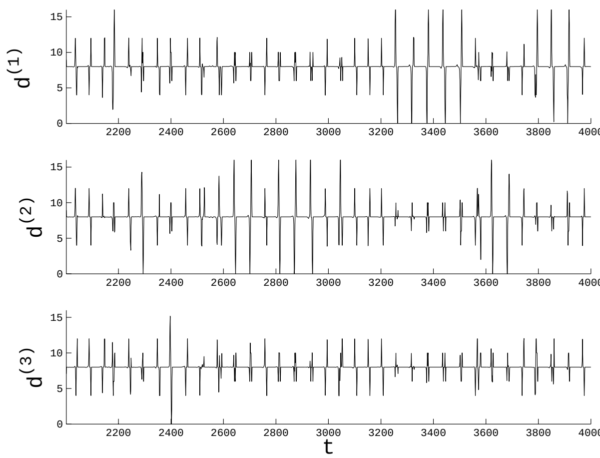

Figure 3: Distances between the output pattern of the network and the three stored patterns with the parameter values found by GA (Example (1)).

5. Conclusions and Discussions

The genetic algorithm[4] is applied to find sets of parameter values so that the associative chaotic neural network shows dynamical association. As the result, several sets of parameter values are found that the network shows dynamics with positive Lyapunov exponents and it includes retrieval of all the stored patterns. The result implies that such a parameter search method for the dynamical neural network models might be better than the conventional method for determining parameter values using bifurcation diagrams.

The parameter search method by GA is not so efficient at this moment. We examined the GA with the other conditions for the target frequency T and duration for the GA trial than the ones shown in this paper. The success rates for finding the parameter values are in between 50% and 70% in these trials. Therefore, further consideration is needed for the fitness functions to achieve higher efficacy of the search method. It is also needed to make comparison between the genetic algorithm based search and the random search.

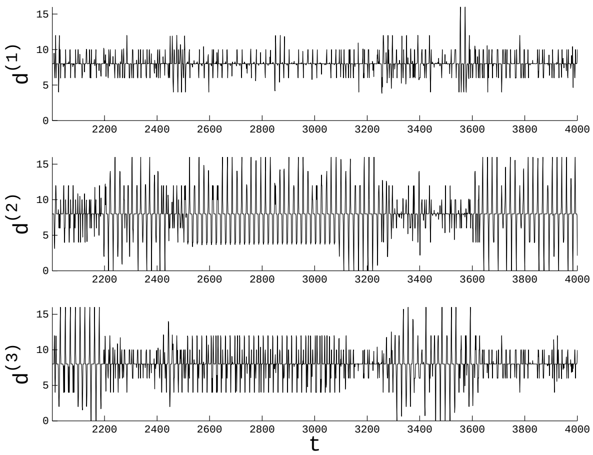

Figure 4: Distances between the output pattern of the network and the three stored patterns with the parameter values found by GA (Example (2)).

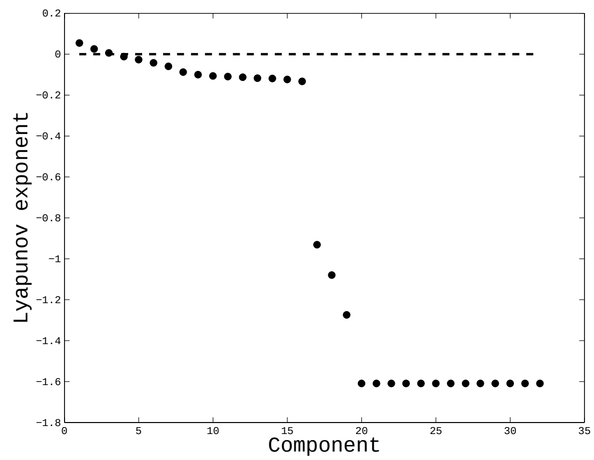

Figure 5: Lyapunov spectrum of the network with the parameter values found by GA (Example (1)). The dashed line denotes zero level.

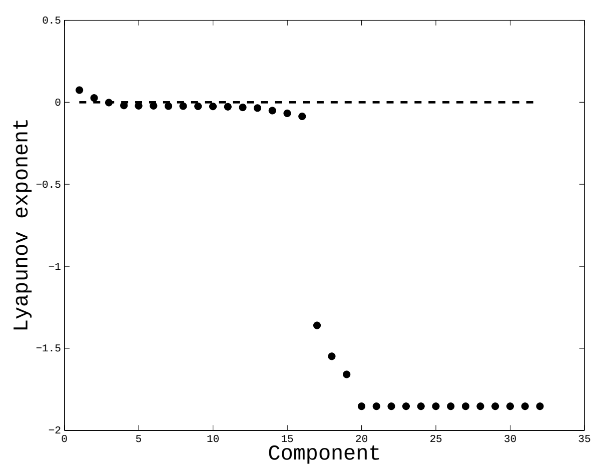

Figure 6: Lyapunov spectrum of the network with the parameter values found by GA (Example (2)).

Acknowledgement: The present work is partially supported by the Research Institute for Technology of Tokyo Denki University under Grant Q01S–01 (to MA).

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