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A Coding Scheme for Asynchronous Pulse Transmission

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Abstract – Recent physiological experiments and theoretical researches imply a possibility of information coding with spatio-temporal patterns of action potential spikes (spatio-temporal coding) in a brain. In this coding, timing of the pulse propagates in a network is very important. When the pulse timings should be preserved, the number of the synaptic connections will limit the size of the network on integrated circuits. In this paper, an asynchronous coding scheme for the synaptic communication is proposed, which preserves the timing information of each pulse while small number of wires is required. In the proposed scheme, the firing time and the source of an action potential are coded separately. The source information is expressed with a pulse sequence. When pulse sequences overlap each other, an error occurs. Error-rate and bit-rate are examined. The proposed coding scheme is suitable for implementing a spatio-temporal coding network with analog electronic circuit.

1 Introduction

Recent physiological experiments and theoretical researches imply a possibility of information coding with spatio-temporal pattern of action potential spikes (sp-atio-temporal coding) in a brain [1, 2]. The state variables and parameters of the spatio-temporal coding network such as time, the internal state of the neuron, the nonlinear output function at the axon hillock and synaptic delay and weight, are all continuous. These continuous variables enable the network to possess an analog information processing ability [3]. Several asynchronous pulse neural network models suitable for analog circuit implementation have been proposed [4]. However, it is difficult to realize a complex asynchronous neural network structure. That is, the number of synaptic connections limits the size of the network on integrated circuits since the fine pulse timings should be preserved.

In this paper, a coding scheme for inter-neuron communications with action potentials is proposed, which preserves the timing information of each pulse while using a small number of wires. In the proposed coding scheme, the followings should be achieved. (1) A fine timing information of a pulse emission should be retained, because timing of the pulse propagates in a network is very important in spatio-temporal coding. (2) The number of wiring should be kept as small as possible. This is because the structure of an analog integrated neural network circuit is usually two-dimensional and the number of pins of the IC package is limited. (3) Transmission error caused by collisions of pulses should be low. If some neurons share a common channel, collisions of pulses arise. But the coincidence detector neuron should be able to receive some simultaneous pulses in spatio-temporal coding network.

In order to achieve the above points, a firing time of an action potential and the source of it are transmitted through separate wires. The firing time of one action potential is expressed with one short pulse in order to preserve the fine timing information. The source of the action potential is specially coded with corresponding pulse trains. A coding scheme similar to the optical orthogonal coding is used [5, 6].

A construction method of the codes for the source information is very simple. With the coding, receiver neuron is poss ible to distinguish several simultaneous action potentials on one communication line.

In Section 2, a transmitter block and a receiver block are introduced. Section 3 proposes the coding methods. A theoretical analysis of the code is discussed in Section 4.

2 An Asynchronous Pulse Transmission System

In this section, transmitter block and receiver block are introduced. Figure 1 shows a diagram of the transmitter block. The action potential of the neuron with pulse duration T_f is split into two by the pulse splitter. One output is an input to a source encoder. Another one is an input to a timing information encoder. In the source information encoder, the input pulse is encoded with a corresponding pulse train pattern with length, F. Here, the length F represents the number of 0's and 1's, and weight W is the number of 1's in the sequence. We define a chip duration T_c as $T_c = T_f/F$, where a frame duration T_f is the same length as the width of an action potential. Namely, frame consist of F pieces of chips (Fig. 1). The resulting pulse sequence, which represents the source of the corresponding single action potential, propagates through the source information transmission line.

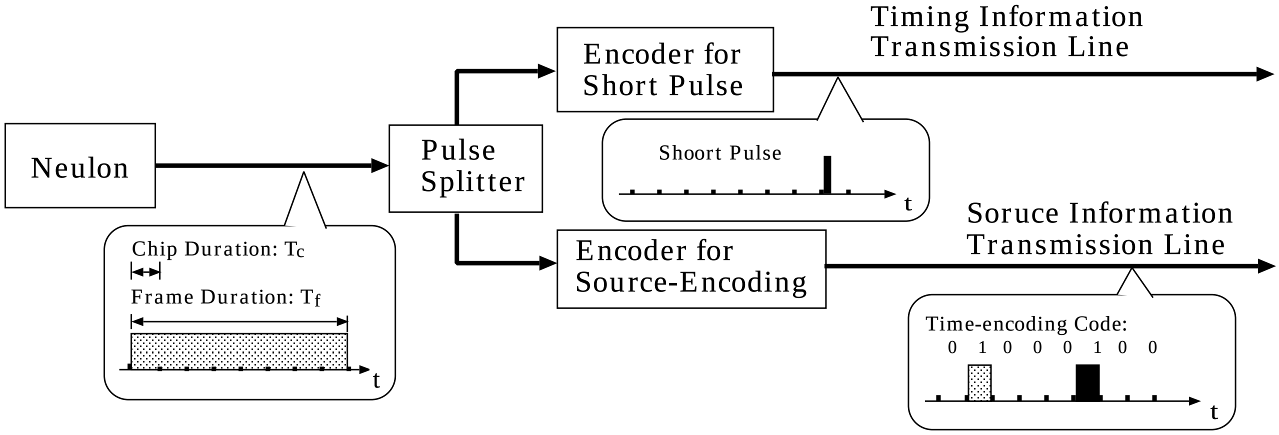

Figure 1: Transmitter block.

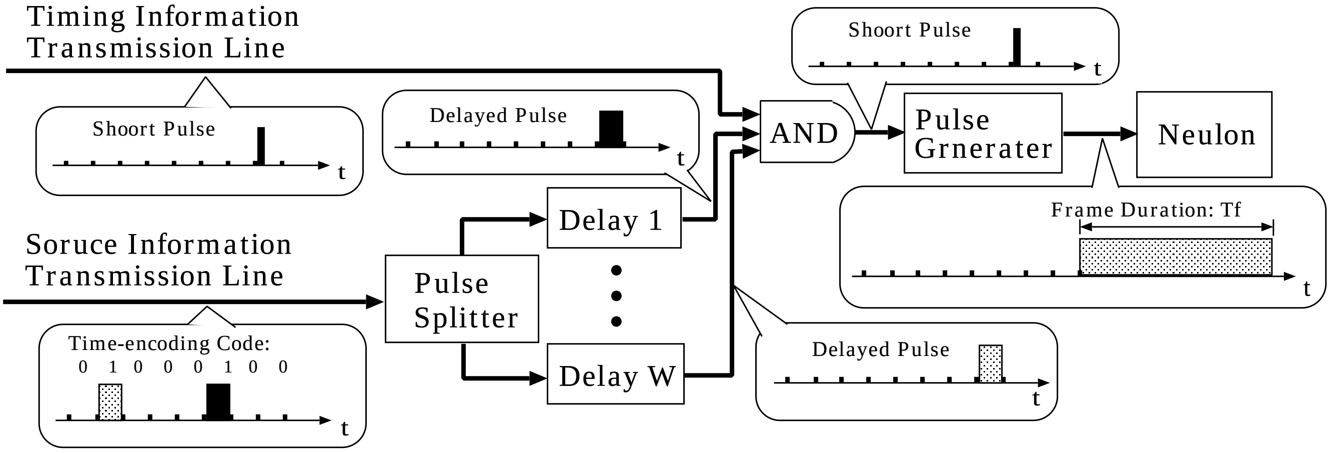

Figure 2: Receiver block.

The width of the short pulse output from the timing information encoder is much shorter than the duration of the chip (T_c) The short pulse is generated on the beginning of the final chip (Fig. 1).

Figure 2 shows a diagram of the receiver block. The input pulse stream from the source information transmission line is copied into W. Each pulse passes stream through a delay element with different delay time. The delays and AND gate form a matched filter. If the short pulse reaches to AND gate at that time, the generator restores the action potential.

3 Design of the Code

We assign each neuron a different code. Therefore, the number of codes is the same as that of neurons, N, in a transmission group. The code, $C(N, W)$, is constructed as follows. Here, the code C is a family of $(0, 1)$ sequences of length F and weight W ($1 < W < N$).

For example, Fig. 3 shows the code $C(4, 3)$. The first 1's bit of each code is sifted one by one. The interval between the first 1's bit and the second 1's bit on the code 1 is 7 bit. This interval is calculated from $N(W-1)-(W-2)$. The corresponding interval of the second code is 1 bit larger than that of the first code. In the same way, the interval of the third code's is $N(W-1)-(W-2)+2$. The fourth code's is $N(W-1)-(W-2)+3$. Therefore, that interval of ith code is $N(W-1)-(W-2)+(i-1)$.

The interval of the second 1's bit and third 1's bit on the code 1 is 11 bit. This interval is determined with $N(W-1)-(W-2)+N$. That is, this interval is N bit larger than the interval between the first 1's bit and the second 1's bit. Therefore, that of second code is 12 bit. The third code is 13 bit. The fourth code is 14 bit.

In the case of $C(4, 3)$, the interval between the third 1's bit and the final bit on the fourth code is 12 bit. This interval is given by $N(W-1)-(W-2)+N(W-1)-N$ In this way, each code overlaps one 1's bit at most.

This sequence satisfies the following two conditions. (1) Each sequence can be distinguished from shifted

	12345	67890	12345	67890	12345	67890	12345	6789
Code 1	10000	00100	00000	00010	00000	00000	00000	0000
Code 2	01000	00001	00000	00000	01000	00000	00000	0000
Code 3	00100	00000	01000	00000	00001	00000	00000	0000
Code 4	00010	00000	00010	00000	00000	00100	00000	0000

Figure 3: Code for N = 4, W = 3.

versions of itself. (2) Each sequence can be distinguished from a possible shifted versions of every other sequences in the set. That is, one 1's bit is overlapped at most. In the ith code, jth 1's bit number S_{ij} is

$$
\begin{aligned}
S_{ij} &= i + \sum_{k=0}^{j-2} \{N(W-1) + i - W + 1 + Nk\} \\
&= i + (j-1)\left\{N(\frac{j}{2} + W - 2) + i - W + 1\right\}. \quad (1)
\end{aligned}
$$

The code length F is

$$
\begin{aligned}
F &= S_{NW} + N(W-1) - (W-2) + N(W-2) \\
&= N\left\{\frac{3}{2}W^2 - \frac{1}{2}W - 1\right\} - W^2 - W - 1. \quad (2)
\end{aligned}
$$

Therefore, the code length F is in proportion to the number of code N, and quadratic function of the code weight W. Table 1 shows the F for the number of neurons N is 10, 100 and 1000.

Table 1: Code Length (F) for N = 10, 100 and 1000.

W	F ($N = 10$)	F ($N = 100$)	F ($N = 1000$)
2	33	393	3993
3	97	1087	10987
4	189	2079	20979
5	309	3369	33969
6	457	4957	49957
7	633	6843	68943
8	837	9027	90927
9	1069	11509	115909

4 Theoretical Analysis

In this section, we examine an error rate, ER, that is a rate of undesirable receiving, and a bit-rate, BR, which is the maximum number of action potentials that are transmitted per second. Here let us assume that the action potentials are transmitted periodically with period T_f. Namely, the action potentials are transmitted continuously. This case gives the worst ER. When the short pulse is generated on the timing information transmission line, a occurence probability of undesirable receiving P_i is

$$
P_i = \left\{1 - \frac{1}{NW}\right\} P(N-1, W) \left\{\frac{W}{F}\right\}^W, \quad (3)
$$

where $P(x, y)$ is the permutations of y chosen from x. This applies all neurons but the neuron that emits the short pulse. So, ER is

$$
\begin{aligned}
ER &= 1 - (1 - P_i)^{N-1} \\
&= 1 - \left\{1 - \frac{\{1 - \frac{1}{NW}\}(N-1)!}{(N-1-W)!}\left\{\frac{W}{F}\right\}^W\right\}^{N-1} \quad (4)
\end{aligned}
$$

Figure 4 shows the relations between W and BR, with $N = 10, 100$ and 1000.

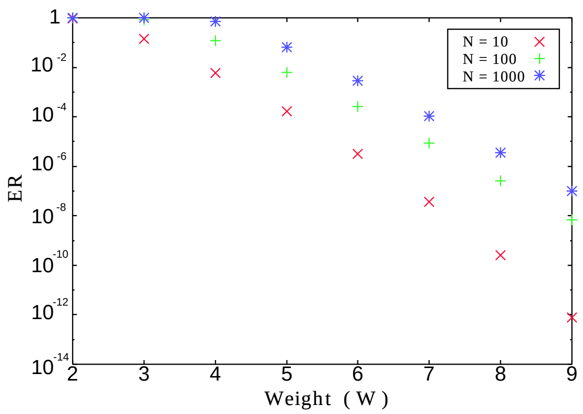

Figure 4: ER versus W.

Next, we calculate BR. BR is the number of action potential received par second without error. BR is given by

$$
\begin{aligned}
BR &= \frac{N}{T_f}(1 - ER) \\
&= \frac{N}{FT_c}(1 - ER). \quad (5)
\end{aligned}
$$

Table 2 shows the BR for the number of neurons N is 10, 100 and 1000. As the W increases, ER decrease exponentially. Even if N becomes large, ER doesn not increase much. Indeed a code length becomes very long with large N, and BR per one neuron is

Table 2: BR $[1/T_c]$ for $N = 10, 100$ and 1000.

W	BR (N = 10)	BR (N = 100)	BR (N = 1000)
2	1.91×10^{-2}	6.89×10^{-14}	2.44×10^{-126}
3	9.05×10^{-2}	1.25×10^{-2}	1.20×10^{-10}
4	5.26×10^{-2}	4.25×10^{-2}	1.29×10^{-2}
5	3.24×10^{-2}	2.95×10^{-2}	2.75×10^{-2}
6	2.19×10^{-2}	2.02×10^{-2}	2.00×10^{-2}
7	1.58×10^{-2}	1.46×10^{-2}	1.45×10^{-2}
8	1.20×10^{-2}	1.10×10^{-2}	1.10×10^{-2}
9	9.36×10^{-3}	8.68×10^{-3}	8.63×10^{-3}

seriously limited. But BR of all the network is the same if N increases. The spatio-temporal network doesn't need a high-speed data link. Therefore, this coding scheme may make asynchronous simultaneous communication possible in a large size network.

5 Conclusions

A coding scheme for the asynchronous synaptic communication with action potentials has been proposed. The firing time and the source of an action potential are coded separately. With the proposed coding scheme, several simultaneous pulses can be transmitted through one communication line set. The transmitter block and receiver block have been proposed. The error rate and bit-rate of the proposed scheme have been analyzed.

The proposed coding scheme may be suitable for low-speed asynchronous simultaneous communication in the spatio-temporal coding network. In order to implement the proposed scheme with electric circuits, the characteristics of the transmission line, the effect of noise and so on, should be considered.

References

[1] M. Tsukada, T. Aihara,H.Saito, and H. Kato, "Hippocampal LTP depends on spatio and temporal correlation of input," INNS Neural Networks, vol. 9, no. 8, pp. 1357–1365, 1996.

[2] H. Fujii, H. Ito, K. Aihara, N. Ichinose, and M. Tsukada, "Dynamical cell assembly hypothesis– Theoretical possibility of spatio-temporal coding in the cortex," INNS Neural Networks, vol. 9, no. 8, pp. 1303–1350, 1996.

[3] N. Ichinose, and K. Aihara, "An analysis on propagation dynamics in asynchronous chaotic neural networks," Trans. of IEICE A, vol. J78-A no. 3, pp. 373–380, 1995, (in Japanese) ; Electronics and communications in Japan, Part 3, 78(11), 52–59 Scripta Technica, Inc. New York: John Wiley, 1995, (the Enghish translation)

[4] M. Hanagata, Y. Horio,and K. Aihara; "Asynchronous pulse neural network model for VLSI implementation," Proc. IEICE, Trans. Fundamentals, vol. E81-A, no. 9, pp. 1853–1859, 1998.

[5] F.R.K. Chaung, J.A. Salehi, and V.K. Wei, "Optical orthogonal codes: Design, analysis and applications," IEEE Trans. Info. Theory, vol. 37, no. 8, pp. 595–604, May 1989.

[6] J.A. Salehi and C.A. Brackett, "Code division multiple-access techniques in optical fiber network-part I : Fundamental principles," IEEE Trans. Commun., vol. 37, pp. 824–833, Aug., 1989.