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Analysis and Measurement of Limit Cycles Generated on Neural Networks with Cyclic and Longer Connections

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1. Introduction

The neural networks with asymmetric and cyclic connections are able to generate dynamical output easily. Particularly, it has been reported that stable limit cycles appear in the model with nearest neighbor connections[1][2]. It is important that we make the property of these networks clear in order to realize dynamic information processing system as a new concept information processor like a human brain. However, the property of this type network with longer connections has not been analyzed yet. Therefore it should be meaningful to clarify this problem.

On the other hand it is an important object of research that we implement large-scale neural network systems with hardware to apply to real world problems, for example voice recognition, machine translation and so on. We have already proposed hardware neural network with discrete synapse weight for the purpose of high density implementation[3]. Our target networks can generate various limit cycles with only $-1, +1$ connection weights. Hence we think that these networks can be easily achieved hardware implementation for the dynamic information system.

In this paper, we describe the feature of limit cycles generated in neural networks with asymmetric cyclic connections of wide range, and discuss the mechanism of these limit cycles.

2. Network Model with Asymmetric and Cyclic Connections

Figure 1 shows our target network. Each neuron has a self feedback connection α, $+1$ clockwise connections and -1 contour-clockwise connections as far as the distance of L intervals. The α is an important parameter. If α is negative value, no limit cycles exist and if α is larger than 2 at $L = 1$, all states of combination of -1 and $+1$ are fixed points and no limit cycles exist too. Hence α is set between 0 and 2 to be able to generate limit cycles with various L.

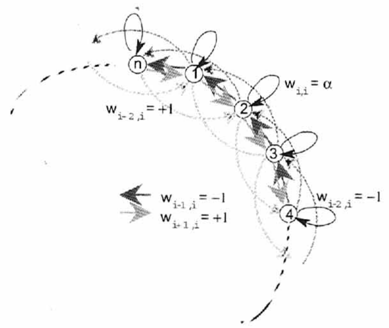

Figure 1: Network ($L = 2$)

Network dynamics is described by Eq. (1) and (2).

$$\tau \frac{du_i}{dt} = -u_i + \alpha x_i + \sum_{j=1}^{L} x_{i-j} - \sum_{j=1}^{L} x_{i+j} \qquad (1)$$

$$x_i = tanh(u_i/T) \qquad (2)$$

Here, u_i ,x_i, τ and T are the membrane potential, the output of $i-th$ neuron, a time constant of this network and the so-called temperature, respectively. When the number of neuron is N, $i + L(> N) = i + L - N$ and $i - L(< 1) = i - L + N$. Because this network is arranged on a ring. In this paper we assume that input-output relation is high gain limit, that is, T is very small.

If we consider discrete time neuron model with connection length $L = 1$, we can analyze fixed points and limit cycles of this network easily [2]. Because the next outputs can be decided by simple rule according to outputs of only neighbor neurons. However, in case of continuous time neuron model it is difficult to analyze limit cycles, because state transition of neuron is strongly affected by movement of other neurons and by an inner state of itself. In case of $L = 1$ with continuous time neurons, approximate expression of waveform of the membrane potential has been derived[4]. But the research of the networks with longer connections is left yet.

3. Stable states of Networks

At first, we consider fixed points of our target networks. Table 1 shows the number of fixed points by simulation.

L	N=3	4	5	6	7	8	9	10	11	12	13
1	2	3	2	3	2	3	2	3	2	3	2
2	-	-	2	5	2	3	4	3	2	5	2
3	-	-	-	-	-	2	6	4	3	2	8
4	-	-	-	-	-	-	-	2	9	2	5
5	-	-	-	-	-	-	-	-	-	2	14

Table 1: The number of fixed points

In case of $L = 1$ the number of fixed points changes regularly. If N is even, three fixed points exist and if N is odd, two exist.

However, two evident type of fixed points exist at any L. Type I is $(-1, -1, -1, \cdots, -1)$ or $(+1, +1, +1, \cdots, +1)$. Type II is $(-1, +1, -1, \cdots, +1)$.

Here, let's assume that membrane potential of every neuron is equilibrium points. In this situation, $i-th$ neuron's output should satisfy the following equation.

$$x_i = tanh(\frac{\alpha x_i + (x_{i-1} - x_{i+1}) + \cdots + (x_{i-L} - x_{i+L})}{T}) \quad (3)$$

If $\frac{\alpha}{T}$ is very large, a solution is that x_{i+L} is equal to x_{i-L}. But we can easily understand that type II is not realized when N is odd because this network has cyclic connections.

Next we consider about limit cycles of this network. In case of $L = 1$, analysis by numerical simulation has been done and the number of limit cycles k is described by

$$k = \begin{cases} \frac{n-1}{2} & if\ n\ is\ odd \\ \frac{n-2}{2} & if\ n\ is\ even. \end{cases} \quad (4)$$

The limit cycles with $L = 1$ have the following characteristic. Here we relocate neurons in one-dimension by cutting of a ring shown in Fig. 2 just for display. We normalize the length of the network to 1. The limit cycles can be defined by "ϕ-type function" as following equations, and shown in Fig. 2.

$$\phi_l(x) = \varphi\{l(x - v_l t)\} \quad (5)$$

$$\varphi(y) = \begin{cases} +1 & (0 \le y \le \frac{1}{2}) \\ -1 & (\frac{1}{2} \le y \le 1) \end{cases} \quad (6)$$

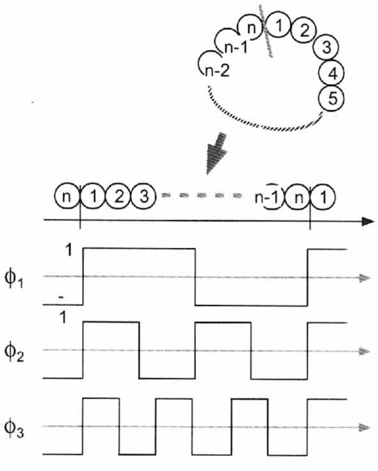

Figure 2: ϕ-type Limit cycles

$$\varphi(y) = \varphi(y + 1) \qquad (0 < y) \quad (7)$$

Here l is wave number, and v_l is the velocity of wave. These ϕ-type function limit cycles move to clockwise at a constant velocity.

We describe the simulation results of limit cycles with $L \ge 2$. Figure 3 shows the typical simulation results of the limit cycles with $L = 2$. In this case we observe two type of limit cycles generated by these network. One is ϕ-type function limit cycle (Fig. 3(a)) as same as $L = 1$ and the other is limit cycle of a complex shape mixed plural waves (Fig. 3(b)).

However, if we add perturbation to complex limit cycles, the limit cycles converge to ϕ-type function. Hence we maintain that the ϕ-type function limit cycle is stable. And the number of ϕ-type function limit cycles increase with N and decrease with L.

We discuss the detail of the generation mechanism of ϕ-type function limit cycles later.

4. Measurement of Neuro-CHIP

In this section we present the measurement of limit cycles on a neuro-chip. Figure 4 shows our used neuro-chip. This neuro-chip is fabricated in the fabrication program of VLSI Design and Education Center(VDEC), the University of Tokyo with the collaboration by Rhom Corporation and Toppan Printing Corporation. And this chip includes 15 neurons, full connected synapses with $-1, 0, +1$, learning circuits. The detailed description is presented in the reference [3].

Figure 5(a) shows the measurement results of this neuro-chip when we set to asymmetric cyclic connections

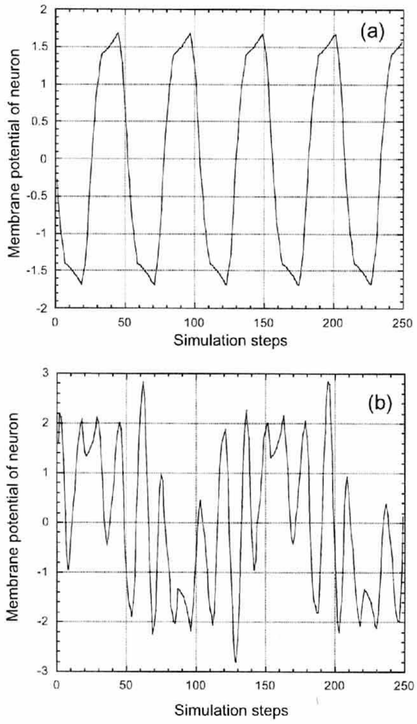

Figure 3: The simulation results of a membrane potential as a function of steps. (a) is a ϕ-type function limit cycle. (b) is a complex limit cycles.

$L = 1$ and $N = 15$. We observe that seven ϕ-type function limit cycles exist as same as the result of numerical simulation. However, waveform is different from simulation. This difference may be caused by the offset of neuron circuit's threshold. Figure 5(b) shows the waveform calculated by numerical simulation including this offset. The mimic waveform is reappeared. Though we should solve a problem of offset, we could generate dynamical patterns on a neuro-chip.

5. Discussion of Generating Mechanizm of Limit Cycle

In this section we consider an infinite one-dimenstional neural network shown in Fig. 6. All neurons have +1 synapse weights to the right hand and -1 to the left hand as far as the distance L intervals.

At first we assume that network has only one bundary between the region of +1 outputs and the region of −1 outputs. Neurons from $i_+ = 0$ to $L-1$ in the +1 region become unstable by receiving negative force as the following equation.

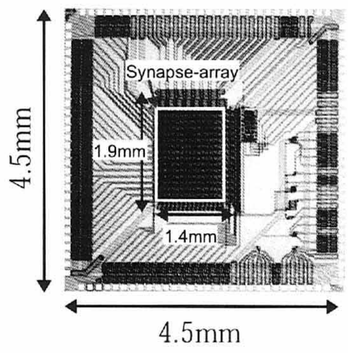

Figure 4: Neuro-chip

$$F(0 \le i_+ < L) = 2(i_+ - L) + \alpha < 0 \tag{8}$$

On the other hand neurons from $i_- = 0$ to $L - 1$ in the −1 region become more stable by receiving negative force as the following equation.

$$F(0 \le i_- < L) = 2(i_- - L) - \alpha < 0 \tag{9}$$

Hence the boundary moves to the right. We can easily understand that the boundary moves to the right even when we exchange +1 region for −1.

The next, let's assume an isolated region of M neurons which output +1 as shown in Fig. 7. Here, M is bigger than $2L$. Neurons from $i_3 = 0$ to $L-1$ become unstable as Eq. (10). And neurons from $i_1 = 0$ to $L-1$ become unstable too.

$$F(0 \le i_3 < L) = 2(L - i_3) - \alpha > 0 \tag{10}$$

$$F(0 \le i_1 < L) = 2(i_1 - L) + \alpha < 0 \tag{11}$$

In this case transit time of these neuron's output are different though $F(i_1) = -F(i_3)$. Because the membrane potential of neurons exponentially close to an equivalent value and the membrane potential of neurons of the left unstable region(denoted by i_1) is more far from 0 than one of the right unstable region (denoted by i_3) when neurons begin changing to unstable states. Hence, this isolated +1 region increases so that the boundary of the right hand move more quickly than the left hand.

Let's assume that the size of an isolated region is m which is less than $2L$. Here, we define $L_d = \left[\frac{m+1}{2}\right] < L$. $[x]$ means to truncate decimal of x. In this case the number of unstable neurons of an isolated region is less than L_d and the number of ones in front of an isolated region is L. Hence, this isolated region increases too.

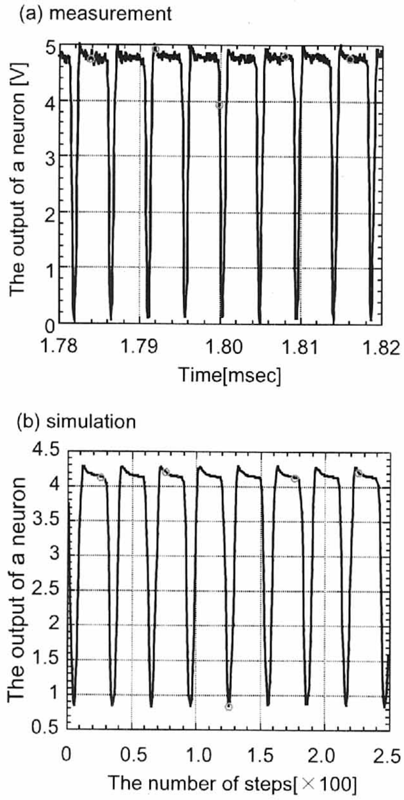

Figure 5: Output of a neuron : (a) is a measurement result. (b) is a simulation result.

Finally we consider the periodic boundary condition. In this case some isolated regions becomes same size as same as infinite one-dimensional neural network. If the number of neurons is equal to 2, all combination patterns of $+1$ and -1 are fixed point from equation (3). Therefore it is necessary that $N \geq 3$ to generate limit cycles.

This is the generate mechanism of ϕ-type function limit cycles.

6. Conclusion

In this paper we present that stable states which are fixed points and limit cycles can be generated by the network with asymmetric and cyclic connections by using both the numerical simulation and the measurement neuro-chip. In particular we found that ϕ-type function limit cycles always exist with various L.

We discussed the generating mechanism of ϕ-type function limit cycles by considering the move of isolated region with infinite one-dimensional neural network. As a result, in case of the periodic boundary condition we made it appear that every isolated region spread to same size during moving to one direction and then stable ϕ-type function limit cycles are generated ultimately.

However we have not yet researched the detail of limit cycles of complex waveform. We think future subject of researches.

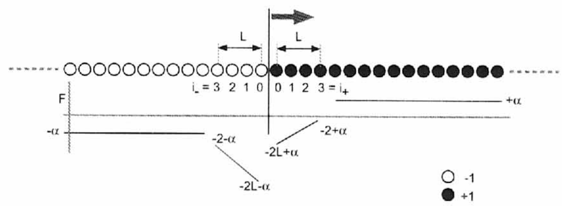

Figure 6: The motion of a boudary

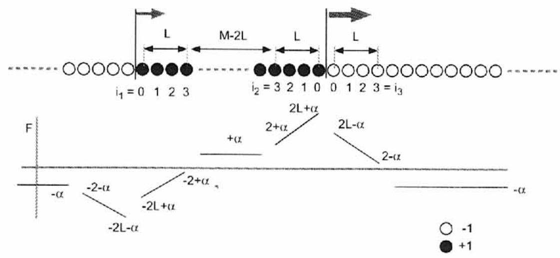

Figure 7: The motion of an isolated region

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