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Design of Robust I-PD Controller based on Multi-objective Genetic Algorithm with Local Search Techniques

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1. Introduction

In practical use, PID-type controller [1] is effective enough to give desirable control performance. However, a design problem of PID-type control system is not always formulated as a concise convex problem. Moreover, taking into account the robustness for the uncertainties of actual plants, the design problem becomes more complex. In the actual applications, settling time and damping ratio are often important factors for the design of PID-type control systems. Since there is a trade-off between the settling time and the damping ratio, a controller is designed by trial and error via observing both properties. Thus, a design method based on the optimization of the settling time and the damping ratio is required to be established.

In this paper, we will formulate the design problem of I-PD controller as a two-objective optimization problem which optimize both the settling time and the damping ratio. The solution of the two-objective optimization problem does not become a single solution, but a set of solutions known as the Pareto-optimal set. If the Pareto-optimal set of this problem can be found efficiently, it will reduce the burden of controller designers. In order to formulate this design problem as a tractable optimization problem, the optimization of the settling time and the damping ratio is considered as a pole assignment problem. The uncertainties of the plant are represented as a polytope of polynomials, and a design cost is reduced by using the edge theorem [2].

In the proposed design method, a genetic algorithm (GA) [3-5] is used for the optimization problem taking into account the advantages of its multiple search property. Moreover, local search techniques are proposed in order to improve the search performance. The proposed local search techniques are concerned with the selecting process of individuals and the crossover operator. In order to demonstrate the effectiveness of the proposed design method, the magnetic levitation system is used as an example of an uncertain plant.

2. Problem Statement

2.1. I-PD Control System

Suppose a plant can be modeled by an all-pole transfer function:

$$g(s) = \frac{1}{a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n} \tag{1}$$

where s is the Laplace operator.

Consider the control system with the I-PD controller [6] shown in Fig. 1.

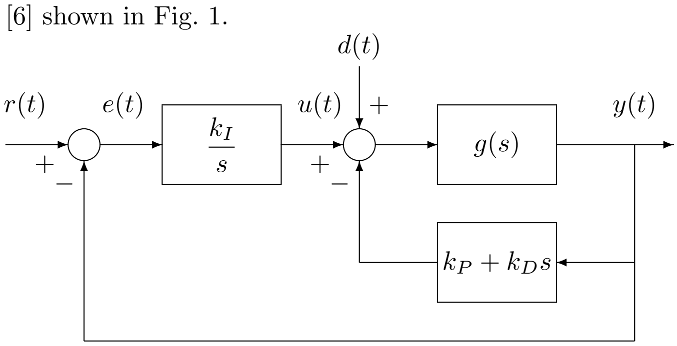

Fig. 1 I-PD control system

From Fig. 1, the I-PD Control system is described by

$$e(t) = r(t) - y(t) \tag{2}$$

$$u(t) = \frac{k_I}{s} e(t) - (k_P + k_D s) y(t) \tag{3}$$

$$y(t) = g(s) u(t) \tag{4}$$

where $r(t)$ is the reference input, $e(t)$ is the error signal, $u(t)$ is the controller output, $y(t)$ is the plant output. The variables k_I, k_P and k_D indicate the I-PD parameters. Hence, the transfer function of the closed-loop system is described as follows:

$$H_{yr}(s) = \frac{g(s) k_I}{s + g(s)(k_I + k_P s + k_D s^2)} \tag{5}$$

$$= \frac{1}{1 + \frac{a_0 + k_P}{k_I}s + \frac{a_1 + k_D}{k_I}s^2 + \cdots + \frac{a_n}{k_I}s^{n+1}} \quad (6)$$

By virtue of using the I-PD control system to the plant which is modeled by an all-pole transfer function, the transfer function of the closed-loop system is also modeled by an all-pole transfer function.

2.2. Representation of Model by Polytope of Polynomials

Each trial of system identification will produce a distinct plant model. Thus, the plant can be represented by a set of models. The set of models is defined as a polytope of denominator polynomials as follows:

$$\mathcal{A}_\lambda(s) = \lambda_1 A_1(s) + \lambda_2 A_2(s) + \cdots + \lambda_{N_0} A_{N_0}(s) \quad (7)$$

$$= \alpha_{\lambda 0} + \alpha_{\lambda 1} s + \alpha_{\lambda 2} s^2 + \cdots + \alpha_{\lambda n} s^n \quad (8)$$

where $A_i(s)$, $i = 1, \ldots, N_0$ denotes a denominator polynomial of the plant model. The coefficient parameter λ_j, $\sum \lambda_j = 1$, $0 \leq \lambda_j \leq 1$, $j = 1, \ldots, n$ represents the uncertainty. By using Eq. (8), the closed-loop system (6) is transformed by

$$H_{yr}(s) = \frac{1}{1 + \frac{\alpha_{\lambda 0} + k_P}{k_I}s + \frac{\alpha_{\lambda 1} + k_D}{k_I}s^2 + \cdots + \frac{\alpha_{\lambda n}}{k_I}s^{n+1}} \quad (9)$$

The generating extreme points [2] of the plant denominator polynomials correspond with those of the polytope of characteristic polynomials.

2.3. Objective Functions

The purpose of the design problem is to find the I-PD parameters which minimize the following objective functions simultaneously:

$$\underset{k_I, k_P, k_D}{\text{minimize}} \; F = (F_1, F_2)^T \quad (10)$$

$$F_1 = \max_{\substack{1 \leq i \leq n \\ \lambda \in \Lambda_N}} [\mathrm{Re}(s_i(\lambda; k_I, k_P, k_D))] \quad (11)$$

$$F_2 = \max_{\substack{1 \leq i \leq n \\ \lambda \in \Lambda_N}} [\theta_i(\lambda; k_I, k_P, k_D)] \quad (12)$$

where s_i indicates a pole of the closed-loop transfer function obtained from (9), θ_i indicates an angle between the real axis and the straight line which connects the pole s_i with the origin. The objective function F_1 is the maximum value of the real part for all poles, which is concerned with the settling time. On the other hand, the objective function F_2 is the maximum value of the angle between the real axis and the straight line which connects the pole s_i with the origin, which is associated with the damping ratio.

Fig. 2 illustrates the objective functions F_1 and F_2 for a typical pole placement of an uncertain system.

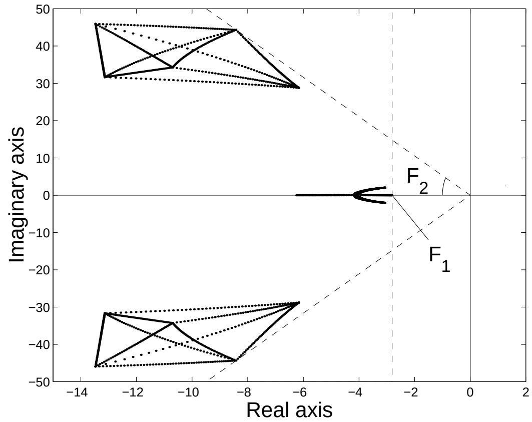

Fig. 2 The objective functions F_1 and F_2 for a typical pole placement of an uncertain system

In the evaluation of the objective functions F_1 and F_2, the maximization over Λ_N should be reduced by using the edge theorem to the maximization over the exposed edges [2].

3. Design Algorithm

The design algorithm of a robust I-PD controller based on the GA is summarized as follows:

Step 1. Generate a plant parameters set from system identifications. Then calculate the generators [2] and the exposed edges from the identifying results.

Step 2. Let S_1 be a set of polytope of polynomials for all generators, let S_2 be a set of polytope of polynomials for all exposed edges.

Step 3. Compute the I-PD parameters set which minimize the objective functions (11) and (12) for each element of the set S_1 by using GA explained following section.

Step 4. By using all I-PD parameters obtained from **Step 3**, check the objective function values for each element of the set S_2. If both objective function values are deteriorated by elements of the set S_2, the elements add to the set S_1. Then go to **Step 3**. Otherwise, go to **Step 5**.

Step 5. Select the desirable I-PD parameters by observing the actual experimental property.

Remark) The example of selecting process of the desirable I-PD parameters can be described as follows. To begin with, a medium I-PD parameter of the solution set is set as an initial point. Then by using the binary search method, select the desirable solution by observing both of the settling time and the damping ratio.

3.1. GA Formulation

To set up a GA for the design of I-PD controller, certain problem-dependent algorithm elements must be defined. These elements have an effect on both the efficiency of the algorithm and the quality of its results.

3.1.1. Representation of Individuals

Each of the I-PD parameters k_I, k_P and k_D is defined as a real number. The individual p_i based on the real-valued encoding method is expressed as follows:

$$p_i \;=\; (b_{i1}, b_{i2}, b_{i3}) \tag{13}$$

where b_{i1}, b_{i2} and b_{i3} correspond with the real number of k_I, k_P and k_D, namely, each population member consists of three real numbers. The individual corresponds to the candidate solution for the problem. In a GA, a population $P(\tau) = \{p_1, p_2, \ldots, p_M\}$ which consists of M individuals is used to solve a given optimization problem.

3.1.2. Procedure of GA

The procedure of GA consists of the following steps.

Step 1. Set a generation number $\tau = 0$. Randomly generate an initial population $P(0)$.

Step 2. Calculate the fitness of each individual in the current population $P(\tau)$ according to the multi-objective ranking method [4].

Step 3. Generate a new population $P'(\tau)$ as follows: select individuals from $P(\tau)$; recombine them using a crossover operator.

Step 4. Apply a mutation operator to the individuals according to the mutation rate.

Step 5. Calculate the fitness both of $P(\tau)$ and $P'(\tau)$. Select top M individuals from all population members on the basis of the fitness.

Step 6. If a terminal condition is satisfied, stop and return the best individual. Otherwise set $\tau = \tau + 1$ and go to **Step 2**.

In this procedure, the current population size is always constant M. Here, to avoid the rapid loss of population diversity, multiple equivalent individuals are eliminated from the current population.

3.1.3. Local Search Techniques

The crossover operator is an important factor to produce desirable individuals. In order to improve the search performance, the selecting process of individuals and the real-valued crossover are proposed.

[Selecting Process] The selecting process of two individuals is effected as follows:

1) All individuals are sorted into the increasing ordered value of the objective functions.

2) Select an individual according to the fitness.

3) On the basis of the sorted order, the other individual is selected from the neighborhood ones.

The aim of this process is to choose two individuals which have similar property for the objective functions.

[Crossover] If individuals p_i and p_j are selected for crossover, two offsprings are calculated as follows:

$$b'_{ik} \;=\; \beta_1 b_{ik} + (1.0 - \beta_1) b_{jk} \tag{14}$$

$$b'_{jk} \;=\; \beta_2 b_{ik} + (1.0 - \beta_2) b_{jk} \tag{15}$$

where the parameters β_1 and β_2 are random numbers generated in $[\beta_{\min}, \beta_{\max}]$. The interval $[\beta_{\min}, \beta_{\max}]$ is defined as follows:

$$[\beta_{\min}, \beta_{\max}] = \begin{cases} [\,0,\,2], & \text{if } r_i = r_j \text{ and } \beta_1 \\ [\text{-}1,\,1], & \text{if } r_i = r_j \text{ and } \beta_2 \\ [\,0,\,2], & \text{if } r_i < r_j \\ [\text{-}1,\,1], & \text{if } r_i > r_j \end{cases} \tag{16}$$

where r_i denotes the rank of the individual p_i obtained from the multi-objective ranking method. The rank of the good individual becomes small. This approach guarantees that parents and offsprings are kept close to each other, making the search smooth toward the trade-off surface.

4. Application to Magnetic Levitation System

In this paper, a magnetic levitation system is used as an example of an uncertain plant. The control objective is to maintain the position of the steel ball to the specified position.

4.1. Plant Model

Suppose the plant can be modeled as an all-pole 3rd order transfer function. The transfer function of the plant is expressed as follows:

$$g(s) = \frac{1}{a_0 + a_1 s + a_2 s^2 + a_3 s^3} \tag{17}$$

The generating extreme points of the polytope of polynomials are obtained from several system identifications.

4.2. A Design of Controller

In the design of the I-PD controller, each parameter k_I, k_P and k_D can be constrained to the range $[0, 10000]$. Concerning the parameters of GA, the population size is 96, the mutation rate is 0.1, the stopping condition is 500 or the number of Pareto-optimal solutions does not update during 100 generations.

Fig. 3 shows the designed result by using the proposed design method. In the figure, the axis of abscissa indicates the value of the objective function F_1, the axis of ordinate indicates the value of the objective function F_2. The dots indicate the solutions obtained from the final population. The solutions are sorted into the increasing ordered value of the objective function F_1. It is clear that the proposed design method can seek for the widely distributed solutions on the trade-off surface. From computational point of view, the designed result is obtained from 30.19 second (183 generations) by using PentiumIII 866MHz personal computer.

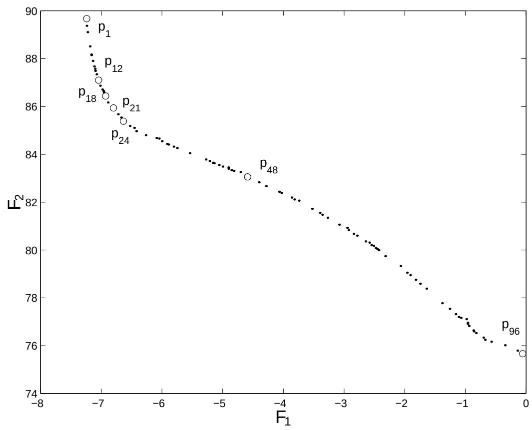

Fig. 3 Distribution of solutions by using the proposed design method

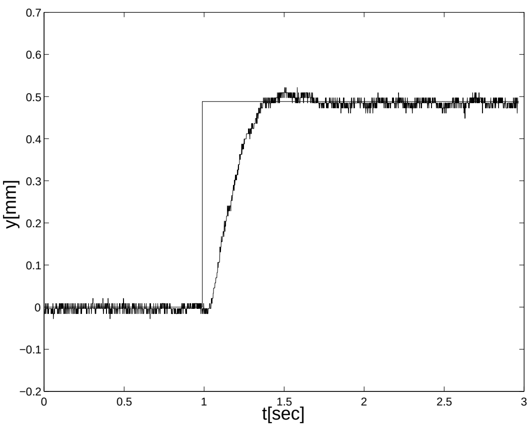

Fig. 4 Actual step response by using the I-PD parameters p_{21}

4.3. Experimental Results

Fig. 4 shows the actual step response by applying the triplet of the I-PD parameters p_{21} to the experiment system. The selecting process based on the binary search method is described as follows. To start with, since the solution p_{48} is ordered in the middle point of the solutions, this is set as an initial search point. Then observe both the settling time and the damping ratio. Because of large settling time, choose the next solution p_{24}. Then observe the actual step response. Since the step response using the solution p_{24} shows still large settling time, choose the next solution p_{12}. By iterating these steps, the controller designer can easily make choice of the desirable solution. Finally, the solution p_{21} can be regarded as the desirable solution.

5. Conclusion

In this paper, we have proposed the new design method of I-PD controller for an uncertain plant. The proposed design method is defined as a two-objective optimization problem based on both of the settling time and the damping ratio. The uncertainties of the plant are represented as a polytope of polynomials, and a design cost is reduced by using the edge theorem. The genetic algorithm with two local search techniques is applied to optimize this problem. The magnetic levitation system is used as an example of an uncertain plant.

From the designed result, the proposed design method can find the widely distributed solutions on the trade-off surface. By using the proposed design method to the practical use, the controller designer can easily find the desirable solution from the Pareto-optimal set.

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